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Numerical solution of nonlinear mixed integral equations with singular Volterra kernel

Research Article

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Abstract: The purpose of this paper is to obtain the approximate solution of the nonlinear mixed integral equations of type Volterra-Fredholm with singular kernel in the space $L_p(\Omega)\times C[0,T]$ by utilized the combined Laplace-Adomian decomposition method. The technique be a convergent sequence of functions, which approximates the exact solution with few iterations. Numerical results and a comparison with the exact solution are given, which uncover its efficiency. The efficiency and precision of the proposed technique is clear by three numerical examples.

MSC: 45B05 • 45E10

Keywords: Singular integral equation • Linear and nonlinear Volterra-Fredholm integral equation • Adomian decomposition method • Carleman kernel • Logarithmic kernel and Cauchy kernel

1. Introduction

The mixed Volterra-Fredholm integral equations arises from parabolic boundary value problems, the mathematical modeling of the spatio-temporal development of an epidemic [1] and various physical and biological problems.

We consider the nonlinear mixed Volterra-Fredholm integral equation of the form

$$\mu u(x,t) = f(x,t) + \lambda \int_0^t \int_\Omega F(|\, t-\tau\,|)k(x,y)\gamma(y,\tau,u(y,\tau))\,dy\,d\tau \tag{1}$$

In general μ defines the kind of the integral equation, where $\mu = 0$ for the first kind, $\mu = const \neq 0$ for the second kind and $\mu = \mu(x,t)$ for the third kind. The functions $k(x,y)$, $F(|\, t-\tau\,|)$ and $f(x,t)$ are given and called the kernel of Fredholm integral term, Volterra integral term and the free term respectively and λ is a real parameter (may be complex and has physical meaning). Also, Ω is the domain of integration with respect to position, and the time $t,\tau \in [0,T]$, $T<\infty$. While $u(x,t)$ is the unknown function to be determined in the space $L_p(\Omega)\times C[0,T]$.Hacia [2] obtained an approximation solution for the linear case of Eq. (1) by using projection method in studying the problem numerically. Kauthen [3] used continuous time collocation method to analyze the global discrete convergence properties. In [4] Euler-Nystrom and trapezoidal Nystrom methods were implemented to handle the linear case of Eq. (1). Brunner [5] applied the collocation method to investigate the nonlinear Volterra-Fredholm integral equation. Abdou et al. [6] considered the integral equation with singular Volterra kernel and used two different numerical methods (Toeplitz matrix and product Nystrom methods) to obtain the solution.

Recently there has been renewed interest in the Volterra-Fredholm integral equations where Maleknejad and Hadizadeh [1] treated Eq. (1) by using the Adomian decomposition method [7, 8, 11].

In this paper we will discuss the combined Laplace Adomian decomposition method to approximate solutions with high degree of accuracy for the mixed integral equations of type Volterra-Fredholm with singular Volterra kernel.

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2. The existence and uniqueness of the solution [6]

Consider the integral Eq. (1). In order to guarantee the existence of a unique solution of Eq. (1) we assume through this work the following conditions:

(i) The kernel of position $k(x,y)$,

$$x=\tilde{x}(x_1,x_2,...,x_n),\, y=\tilde{y}(y_1,y_2,...,y_n),$$

satisfies the condition

$$\left[\int_\Omega\left[\int_\Omega |\,k(x,y)\,|^p\,dx\right]^{\frac{q}{p}}dy\right]^{\frac{1}{q}}=c^*,\quad \left(p>1, c^*\text{is a constant}\right).$$

(ii) The kernel of time $F(|\,t-\tau\,|)$ satisfies the following : for every continuous function $h(\tau)$ and all $0\leqslant\tau_1\leqslant\tau_2\leqslant t$, the integrals

$$\int_{\tau_1}^{\tau_2}F(|\,t-\tau\,|)h(\tau)d\tau \quad\text{and}\quad \int_0^t F(|\,t-\tau\,|)h(\tau)d\tau$$

are continuous functions of t which mean
$\left|\int_0^t F(|\,t-\tau\,|)\,h(\tau)d\tau\right|\leqslant M^*$,M^* is a constant, $\forall t,\tau\in[0,T], 0\leqslant\tau\leqslant t\leqslant T<\infty$.

(iii) The given function $f(x,t)$ with its partial derivatives are continuous in $L_p(\Omega)\times C[0,T]$
where $\|\,f(x,t)\,\|_{L_p(\Omega)\times C[0,T]}=\max\limits_{0\le t\le T}\left|\int_0^t\left[\int_\Omega|\,f(x,\tau)\,|^p\,dx\right]^{\frac{1}{p}}d\tau\right|=G^*$, ($G^*$ is a constant)

(iv) The known continuous function $\gamma(x,t,u(x,t))$, for the constants $Q>P_1$ and $Q>Q_1$, satisfies the following conditions:

(a) $\max\limits_{0\le t\le T}\left|\int_0^t\left[\int_\Omega|\,\gamma(x,\tau,u(x,\tau))\,|^p\,dx\right]^{\frac{1}{p}}d\tau\right|\le Q_1\,\|\,u(x,t)\,\|_{L_p(\Omega)\times C[0,T]}$

(b) $|\,\gamma(x,t,u_1(x,t))-\gamma(x,t,u_2(x,t))\,|\le N(x,t)\,|\,u_1(x,t)-u_2(x,t)\,|$

where

$$\|\,N(x,t)\,\|_{L_p(\Omega)\times C[0,T]}=\max_{0\le t\le T}\left|\int_0^t\left[\int_\Omega|\,N(x,\tau)\,|^p\,dx\right]^{\frac{1}{p}}d\tau\right|=P_1<\infty.$$

The unknown function $u(x,t)$ behaves as the known function $f(x,t)$ in the space $L_p(\Omega)\times C[0,T]$.

3. Adomian decomposition method (ADM) [12]

In this section, we will present the Adomian decomposition method. We consider the differential equation

$$Lu+Ru+Nu=g(x) \tag{2}$$

where L is the highest order derivative which is assumed to be easily invertible, R the linear differential operator of order less than L, Nu represents the nonlinear terms, and g is the source term. Applying the inverse operator L^{-1} to both side Eq. (2), and using the given condition we obtain

$$u=f(x)-L^{-1}(Ru)-L^{-1}(Nu) \tag{3}$$

where the function $f(x)$ represents the terms arising from integrating the source term $g(x)$ and from using the given conditions, all of which are assumed to be prescribed. The nonlinear term

$$Nu=\gamma(u) \tag{4}$$

is usually represented by an infinite series of the so-called Adomian polynomials

$$\gamma(u)=\sum_{n=0}^{\infty}A_n$$

The polynomials A_n are generated for all kinds of nonlinearity so that A_0 depends only on u_0, A_1 depends on u_0 and u_1, and so on. The Adomian polynomial [7, 9] , $A_n(u_0, u_1, u_2, ..., u_n)$, is given by

$$A_n = \frac{1}{n}\frac{d^n}{d\alpha^n}\left[\gamma(y,\tau,\sum_{i=0}^{\infty}\alpha^i u_i)\right]_{\alpha=0}, \quad n = 0,1,2,\cdots \tag{5}$$

The (ADM) Adomian [10] defines the solution $u(x)$ by the series

$$u(x) = \sum_{i=0}^{\infty} u_i(x) \tag{6}$$

where the components $u_0, u_1, u_2, ...,$ are usually determined recursively by

$$u_0(x) = f(x)$$
$$u_{i+1}(x) = -L^{-1}(Ru_i) - L^{-1}(A_i), \quad i \geq 0$$

This lead to the determination of the components of u. Having determined the components $u_0, u_1, u_2, ...,$ the solution u in a series form defined by Eq. (6) follows immediately,

$$u = u_0 + u_1 + u_2 + \cdots$$

It is important to note that the decomposition method suggests that the zeroth component u_0 be defined by the function $f(x)$ as described above.

4. The Application of ADM to mixed Volterra-Fredholm integral equation with singular Volterra Kernel [14]

Consider the integral equation

$$\mu u(x,t) = f(x,t) + \lambda \int_0^t \int_\Omega F\big(|\, t-\tau\,|\big)k(x,y)\gamma(y,\tau,u(y,\tau)\big)\,dy\,d\tau \tag{7}$$

The Adomian decomposition method introduces the following expression

$$u(x,t) = \sum_{i=0}^{\infty} u_i(x,t) \tag{8}$$

for the solution $u(x,t)$ of Eq. (7) , where the components $u_i(x,t)$ will be determined recurrently. Moreover, the method defines the nonlinear function $\gamma(y,\tau,u(y,\tau))$ by an infinite series of polynomials

$$\gamma\big(y,\tau,u(y,\tau)\big) = \sum_{n=0}^{\infty} A_n \tag{9}$$

where A_n are the so-called Adomian polynomials that represent the nonlinear term $\gamma(y,\tau,u(y,\tau))$ and can be calculated for various classes of nonlinear operators according to specific algorithms set by Adomian [7, 8] . A new algorithm for calculating these polynomials was established by Wazwaz [15] .

Substituting Eq. (8) and Eq. (9) into Eq. (7) yields

$$\sum_{i=0}^{\infty} u_i(x,t) = \frac{1}{\mu}f(x,t) + \frac{\lambda}{\mu}\int_0^t \int_\Omega F(|\, t-\tau\,|)\,k(x,y)\sum_{i=0}^{\infty} A_i\,dy\,d\tau \tag{10}$$

The components $u_i(x,t)$,$i \geq 0$ are computed using the following recursive relations

$$\begin{aligned} u_0(x,t) &= \frac{1}{\mu}f(x,t), \\ u_{i+1}(x,t) &= \frac{\lambda}{\mu}\int_0^t \int_\Omega F(|\, t-\tau\,|)k(x,y)A_i\,dy\,d\tau, \quad i \geqslant 0 \end{aligned} \tag{11}$$

5. Laplace ADM applied to mixed Volterra-Fredholm integral equation with singular Volterra Kernel [16–18]

We assume that the kernel $F(|t-\tau|)$ of Eq. (1) takes the form

a - $F(|t-\tau|)=|t-\tau|^{-\upsilon}$

b - $F(|t-\tau|)=\ln|t-\tau|$

c - $F(|t-\tau|)=\dfrac{1}{(t-\tau)}$

Applying the Laplace transform to both sides of Eq. (1) gives:

$$L\{u(x,t)\}=\frac{1}{\mu}L\{f(x,t)\}+\frac{\lambda}{\mu}L\left[\{F(|t-\tau|)\}\{k(x,y)\}\{\gamma(y,\tau,u(y,\tau))\}\right] \tag{12}$$

The Adomian decomposition method can be used to handle Eq. (12). We represent the linear term $u(x,t)$ from Eq. (8)and the nonlinear term $\gamma(y,\tau,u(y,\tau))$ will be represented by the Adomian polynomials from Eq. (9).

Substituting Eq. (8) and Eq. (9) into Eq. (12) leads to

$$L\left\{\sum_{i=0}^{\infty}u_i(x,t)\right\}=\frac{1}{\mu}L\{f(x,t)\}+\frac{\lambda}{\mu}L\left[\{F(|t-\tau|)\}\{k(x,y)\}\left\{\sum_{i=0}^{\infty}A_i\right\}\right] \tag{13}$$

The Adomian decomposition method introduces the recursive relation

$$\begin{aligned}L\{u_0(x,t)\}&=\frac{1}{\mu}L\{f(x,t)\},\\ L\{u_{i+1}(x,t)\}&=\frac{\lambda}{\mu}L\left[\{F(|t-\tau|)\}\{k(x,y)\}\{A_i\}\right],\quad i\geqslant 0\end{aligned} \tag{14}$$

Applying the inverse Laplace transform to the first part of Eq. (14) gives $u_0(x,t)$, that will define A_0. Using A_0 will enable us to evaluate $u_1(x,t)$. The determination of $u_0(x,t)$ and $u_1(x,t)$ leads to the determination of A_1 that will allows us to determine $u_2(x,t)$, and so on. This in turn will lead to the complete determination of the components of $u_i, i\geq 0$ upon using the second part of Eq. (14). The series solution follows immediately after using Eq. (8). The obtained series solution may converge to an exact solution if such a solution exists.

6. Numerical examples

We consider three examples for the integral equation

$$\mu u(x,t)=f(x,t)+\lambda\int_0^t\int_0^1 F(|t-\tau|)x^2y^2u^l(y,\tau)\,dy\,d\tau \tag{15}$$

$\mu=1,\lambda=0.001,N=20,$ the exact solution $u(x,t)=x^2t$

We consider the linear and nonlinear cases: $l=1, l=3$ respectively, for the Carleman kernel $F(|t-\tau|)=|t-\tau|^{-\upsilon}$ and the computing results are obtained when $\upsilon=0.01, 0.32$ where υ is called Poisson's coefficient, $0<\upsilon<1$, and using the recursive relation (14), while the kernel in the second example takes the logarithmic kernel $F(|t-\tau|)=\ln|t-\tau|$. Finally, in three example the kernel takes the Cauchy kernel $F(|t-\tau|)=\dfrac{1}{(t-\tau)}$ and the results are computing, using Maple 17 at $t=0.001, t=0.7$ and $N=20$.

Example 6.1 ([6]).
Consider the mixed integral equation with Carleman kernel

$$\mu u(x,t)=f(x,t)+\lambda\int_0^t\int_0^1 |t-\tau|^{-\upsilon}x^2y^2u^l(y,\tau)\,dy\,d\tau \tag{16}$$

$\mu=1,\lambda=0.001,N=20,$ the exact solution $u(x,t)=x^2t$

Using Maple 17, we obtain Table 1 and Table 2

Table 1. Linear case ($l = 1$)

x	Exact	App.(u)	Error	App.(u)	Error
		t=0.001			
		v=0.01		v=0.32	
0.00E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
2.00E-01	4.00000E-05	4.00000E-05	0.0000E+00	4.00000E-05	0.0000E+00
4.00E-01	1.60000E-04	1.60000E-04	0.0000E+00	1.60000E-04	0.0000E+00
6.00E-01	3.60000E-04	3.60000E-04	0.0000E+00	3.60000E-04	0.0000E+00
8.00E-01	6.40000E-04	6.40000E-04	0.0000E+00	6.40002E-04	2.00000E-09
1.00E+00	1.00000E-03	1.00000E-03	0.0000E+00	1.00001E-03	1.00000E-08
		t=0.7			
		v=0.01		v=0.32	
0.00E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
2.00E-01	2.80000E-02	2.80000E-02	0.0000E+00	2.80000E-02	0.0000E+00
4.00E-01	1.12000E-01	1.12001E-01	1.00000E-06	1.12002E-01	2.00000E-06
6.00E-01	2.52000E-01	2.52012E-01	1.20000E-05	2.52022E-01	2.20000E-05
8.00E-01	4.48000E-01	4.48065E-01	6.50000E-05	4.48126E-01	1.26000E-04
1.00E+00	7.00000E-01	7.00250E-01	2.50000E-04	7.00481E-01	4.81000E-04

Table 2. Nonlinear case ($l = 3$)

x	Exact	App.(u)	Error	App.(u)	Error
		t=0.001			
		v=0.01		v=0.32	
0.00E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
2.00E-01	4.00000E-05	4.00000E-05	0.0000E+00	4.00000E-05	0.0000E+00
4.00E-01	1.60000E-04	1.60000E-04	0.0000E+00	1.60000E-04	0.0000E+00
6.00E-01	3.60000E-04	3.60000E-04	0.0000E+00	3.60000E-04	0.0000E+00
8.00E-01	6.40000E-04	6.40000E-04	0.0000E+00	6.40000E-04	0.0000E+00
1.00E+00	1.00000E-03	1.00000E-03	0.0000E+00	1.00000E-03	0.0000E+00
		t=0.7			
		v=0.01		v=0.32	
0.00E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
2.00E-01	2.80000E-02	2.80000E-02	0.0000E+00	2.80000E-02	0.0000E+00
4.00E-01	1.12000E-01	1.12000E-01	0.0000E+00	1.12000E-01	0.0000E+00
6.00E-01	2.52000E-01	2.52000E-01	0.0000E+00	2.52001E-01	1.00000E-06
8.00E-01	4.48000E-01	4.48007E-01	7.00000E-06	4.48015E-01	1.50000E-05
1.00E+00	7.00000E-01	7.00062E-01	6.20000E-05	7.00143E-01	1.43000E-04

Example 6.2 ([6]).
Consider the mixed integral equation with logarithmic kernel

$$\mu u(x,t) = f(x,t) + \lambda \int_0^t \int_0^1 \ln|t-\tau|\, x^2 y^2 u^l(y,\tau)\, dy\, d\tau \tag{17}$$

$\mu = 1, \lambda = 0.001, N = 20,$ the exact solution $u(x,t) = x^2 t$

Using Maple 17, we obtain Table 3 and Table 4

Table 3. Linear case ($l = 1$)

x	Exact	App.(u)	Error
		t=0.001	
0.00E+00	0.0000E+00	0.0000E+00	0.0000E+00
2.00E-01	4.00000E-05	4.00000E-05	0.0000E+00
4.00E-01	1.60000E-04	1.60000E-04	0.0000E+00
6.00E-01	3.60000E-04	3.60000E-04	0.0000E+00
8.00E-01	6.40000E-04	6.39999E-04	1.00000E-09
1.00E+00	1.00000E-03	9.99996E-04	4.00000E-09
		t=0.7	
0.00E+00	0.0000E+00	0.0000E+00	0.0000E+00
2.00E-01	2.80000E-02	2.80000E-02	0.0000E+00
4.00E-01	1.12000E-01	1.11998E-01	2.00000E-06
6.00E-01	2.52000E-01	2.51979E-01	2.10000E-05
8.00E-01	4.48000E-01	4.47881E-01	1.19000E-04
1.00E+00	7.00000E-01	6.99545E-01	4.55000E-04

Table 4. Nonlinear case ($l = 3$)

x	Exact	App.(u)	Error
		t=0.001	
0.00E+00	0.0000E+00	0.0000E+00	0.0000E+00
2.00E-01	4.00000E-05	4.00000E-05	0.0000E+00
4.00E-01	1.60000E-04	1.60000E-04	0.0000E+00
6.00E-01	3.60000E-04	3.60000E-04	0.0000E+00
8.00E-01	6.40000E-04	6.40000E-04	0.0000E+00
1.00E+00	1.00000E-03	1.00000E-03	0.0000E+00
		t=0.7	
0.00E+00	0.0000E+00	0.0000E+00	0.0000E+00
2.00E-01	2.80000E-02	2.80000E-02	0.0000E+00
4.00E-01	1.12000E-01	1.12000E-01	0.0000E+00
6.00E-01	2.52000E-01	2.51999E-01	1.00000E-06
8.00E-01	4.48000E-01	4.47984E-01	1.60000E-05
1.00E+00	7.00000E-01	6.99854E-01	1.46000E-04

Example 6.3 ([6]).
Consider the mixed integral equation with Cauchy kernel

$$\mu u(x,t) = f(x,t) + \lambda \int_0^t \int_0^1 \frac{1}{(t-\tau)} x^2 y^2 u^l(y,\tau)\,dy\,d\tau \tag{18}$$

$$\mu = 1, \lambda = 0.001, N = 20, \quad \text{the exact solution} \quad u(x,t) = x^2 t$$

Using Maple 17, we obtain Table 5 and Table 6

Table 5. Linear case ($l = 1$)

x	Exact	App.(u)	Error
		t=0.001	
0.00E+00	0.0000E+00	0.0000E+00	0.0000E+00
2.00E-01	4.00000E-05	4.00000E-05	0.0000E+00
4.00E-01	1.60000E-04	1.60000E-04	0.0000E+00
6.00E-01	3.60000E-04	3.60000E-04	0.0000E+00
8.00E-01	6.40000E-04	6.40000E-04	0.0000E+00
1.00E+00	1.00000E-03	1.00000E-03	0.0000E+00
		t=0.7	
0.00E+00	0.0000E+00	0.0000E+00	0.0000E+00
2.00E-01	2.80000E-02	2.80000E-02	0.0000E+00
4.00E-01	1.12000E-01	1.12001E-01	1.00000E-06
6.00E-01	2.52000E-01	2.52012E-01	1.20000E-05
8.00E-01	4.48000E-01	4.48067E-01	6.70000E-05
1.00E+00	7.00000E-01	7.00257E-01	2.57000E-04

Table 6. Nonlinear case ($l = 3$)

x	Exact	App.(u)	Error
		t=0.001	
0.00E+00	0.0000E+00	0.0000E+00	0.0000E+00
2.00E-01	4.00000E-05	4.00000E-05	0.0000E+00
4.00E-01	1.60000E-04	1.60000E-04	0.0000E+00
6.00E-01	3.60000E-04	3.60000E-04	0.0000E+00
8.00E-01	6.40000E-04	6.40000E-04	0.0000E+00
1.00E+00	1.00000E-03	1.00000E-03	0.0000E+00
		t=0.7	
0.00E+00	0.0000E+00	0.0000E+00	0.0000E+00
2.00E-01	2.80000E-02	2.80000E-02	0.0000E+00
4.00E-01	1.12000E-01	1.12000E-01	0.0000E+00
6.00E-01	2.52000E-01	2.52000E-01	0.0000E+00
8.00E-01	4.48000E-01	4.48006E-01	6.00000E-06
1.00E+00	7.00000E-01	7.00053E-01	5.30000E-05

7. Conclusion

In this work the combined Laplace-Adomian method is utilized to solve mixed integral equations of type Volterra-Fredholm with singular Volterra kernel. This method introduces a helpful way to develop computational technique for solving these sorts of mixed integral equations with singular kernel. The efficiency and precision of the proposed method is clear by three numerical examples and compared the results with their exact solutions. The given numerical examples showed the accuracy and effortlessness this technique.

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