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Numerical modeling of creeping flow past a square cylinder using Finite Volume Method

Research Article

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Abstract: A Finite Volume Method has been implemented in Matlab to investigate creeping flow problem past a square cylinder kept between parallel plates. To model the stress-deformation constitutive equation proposed by Papanastasiou is used. Appling SIMPLE algorithm under Finite Volume Method, governing equations are solved on staggered grid. Yielded and unyielded regions and velocity has been studied over different ranges of Bingham number. The model results are found to be in good agreement with existing results. The results show a strong dependence of the position and shape of the yielded and unyielded regions on Bingham number.

MSC: 65K05 • 76A05 • 76A10 • 76D05 • 68U20

Keywords: Bingham number • Creeping flow • Yield stress • Yielded/Unyielded regions • SIMPLE algorithm

1. Introduction

The study of viscoplastic flow(like fresh concrete and food paste) is important due to its wide application in everyday life. Viscoplasticity is characterized by yielded and unyielded regions. Contrary to yielded zone, the unyielded zone, where the stress falls the yield stress, considered to be rigid. To model the viscoplastic flow problem several researchers have studied 2D creeping flow past circular and square cylinder numerically or experimentally. To model the stress deformation several constitutive models include the Bingham plastic model, the Herschel-Bulkley model, the Papanastasiue model and the Casson model have been adopted by the authors. Employing variation principles, Adachi and Yoshioka [1] obtained the stress field from a maximum principle and the velocity field from minimum principle. Over the range $0.01 \leqslant B_n \leqslant 1000$, Mitsoulis [11] Extensive results on the location and size of unyielded regions, has reported the effect of the Bingham number on the size of various unyielded regions around the cylinder.

Tokpavi et al. [18] adopted Papanastasious regularisation with finite element methods to study flow around a circular cylinder, they have been identified and characterised the rigid zones over the wide range of Oldroyd number. Using the finite element method, Nirmalkar et al. [13] simulated the 2D-creeping flow around square cylinder over a wide range of Bingham number as $1 \leqslant B_n \leqslant 10^5$. They are identified three zones of unyielded regions in the vicinity of the cylinder, all zones expanded by increasing Bingham number.

Das and Bagheri [5] used compact scheme to simoulation flow in channel, Patil and Krishna Prasad [16] used Finite Volume Method to solve multidimensional unsteady state heat conduction equation with Dirichlet boundary conditions, Messelmi [8] studied the diffusion-reaction problem for the Bingham fluid and proved the existence of local strong solution.

In this study Finite Volume Method adopted to simulation 2D-creeping flow past a square cylinder. Constitutive equation proposed by Papanastasiou [20] is used to model the stress deformation. Semi Implicit Method for Pressure-Linked Equation(SIMPLE) is used for coupling the pressure and velocity.

In particular, the governing differential equations are solved numerically together with the Bingham fluid model within the framework of Papanastasiou[20]. Extensive re-

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sults on the velocity on yielded and unyielded regions are presented over the range of Bingham number $10 \leqslant B_n \leqslant 10^5$. The objective of this work is implement Finite Volume Method for Non-Newtonian flow problems. The results are validated with numerical results of Nirmalkar et al.[13].

2. Mathematical problem formulation

Consider the problem of the 2D-creeping flow of a viscoplastic material past a square cylinder of side d. Here the flowing is very slow then the inertia forces are negligible. The length of the domain is $2L$ and cylinder is situated halfway between two flat plates $2L$ apart. Because of the creeping flow assumption ($Re = 0$)there is symmetry in both x-direction and y-direction in a Cartesian coordinate system (x, y). Therefore, it is necessary to consider only one quadrant of the flow domain, as was done previously by Beris et al. [3] in the case of flow around a sphere. Fig. 1 shows the solution domain and boundary conditions. The boundary conditions are:

- symmetry along $AF(u = 0, \frac{\partial v}{\partial x} = 0)$
- symmetry along $DC(u = 0, \frac{\partial v}{\partial y} = 0)$
- on the boundary AB and BC ($u = 0, v = V$)
- no slip at the cylinder surface along $FD(u = v = 0)$
- the reference pressure is set to zero at point C

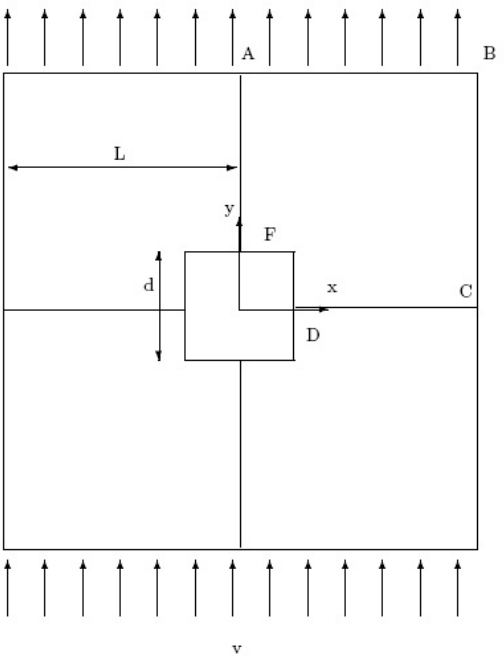

Fig. 1. Flow field: Schematic representation of the problem.

The flow is governed by the usual conservation equations of mass and momentum. For an incompressible fluid under isothermal, creeping flow conditions. Governing equation can be written as:

$$\nabla \cdot \overline{v} = 0 \tag{1}$$

$$-\nabla P + \nabla \cdot \overline{\overline{\tau}} = 0 \tag{2}$$

where $\overline{v}$ is the velocity vector, $\overline{\overline{\tau}}$the extra-stress tensor, and p is the scalar pressure. In simple shear flow the Bingham constitutive equations can be written as:

$$\begin{aligned} \tau &= \tau_y + \mu\dot{\gamma} \quad for \quad |\tau| > \tau_y \\ \dot{\gamma} &= 0 \qquad\quad for \quad |\tau| \leqslant \tau_y \end{aligned} \tag{3}$$

where τ is the shear stress,$\dot{\gamma}$ the shear rate, τ_y the yield stress, and μ is the constant plastic viscosity. Since an unyielded structure is formed,a material parameter is introduced to modify the equations in order to avoid the discontinuity in viscoplastic model, which was proposed by Papanastasio[14] . The Bingham model is then modified as

$$\tau = \tau_y[1 - \exp(-m\dot{\gamma})] + \mu\dot{\gamma} \tag{4}$$

where m is the stress growth exponent. If m is great enough (typical values verify $m > 100$) the regularized low fits well the real constitutive equation. The validation and effectiveness of the Papanastasiou model has been studied by several researchers [1, 11, 13]. The constitutive Eq. 4 can be written as:

$$\overline{\overline{\tau}} = \left(\mu + \tau_y \frac{1 - \exp(-m\dot{\gamma})}{\dot{\gamma}}\right)\overline{\overline{\dot{\gamma}}}. \tag{5}$$

Here the rate-of-strain tensor, $\dot{\gamma}$, is given by:

$$\dot{\gamma} = (\nabla V + \nabla V^T) \tag{6}$$

The magnitude of the rate of the deformation tensor and the deviatoric stress tensor are given by:

$$|\dot{\gamma}| = \sqrt{\frac{1}{2} tr(\dot{\gamma}^2)} \qquad\qquad |\tau| = \sqrt{\frac{1}{2} tr(\tau^2)} \tag{7}$$

The Bingham number which is the ratio of the yield stress of viscous stress, can be written as:

$$B_n = \frac{\tau_y d}{\mu_B V} \tag{8}$$

The variables appearing in the governing equations and boundary conditions are rendered dimensionless using d, V and τ_0 as scaling variables for length, velocity and stress components respectively.

2.1. Unyielded and yielded zone

Unyielded zones refer to all areas where inequality $\tau \leqslant \tau_0$ is satisfied, otherwise is called yielded zone and consider them as a fluid. The boundary between unyielded and yielded zones is called yield surface. Three distinct solid zones, Fig. 2, with their corresponding yield surfaces are observed by Nirmalkar et al.[13].

- A far-field Dynamic rigid zone(Zr1)
- Two static rigid zone with triangular shape connected to the cylinder(Zr2)
- Two dynamic zone on both sides of the cylinder equidistant from it(Zr3)
- The yield surface S_1, boundary between Zr1 and fluid zone.
- The yield surface S_2, boundary between Zr2 and fluid zone.
- The yield surface S_3, boundary between Zr3 and fluid zone.

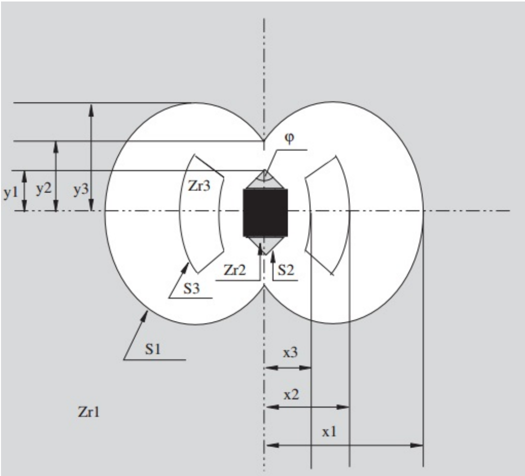

Fig. 2. Definition of rigid zones, fluid zones and yield structures.

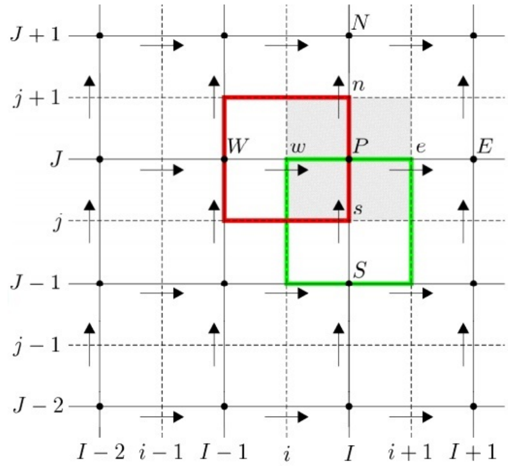

Fig. 4. Staggered grid, u control volume, v control volume and scalar control volume

3. Computational procedures and algorithm

The SIMPLE algorithm under Finite Volume Method on a staggered grid adopted. The scheme developed here in rectangular Cartesian coordinate system, as shown in Fig. 3 the staggered nodes are achieved by shift a half-mesh in $x-$ and $y-$ spatial direction.

The pressure p, stored in the cell midpoint and, the velocities u and v placed on the vertical and horizontal cell interface respectively. To simplicity numbering I and J used instead $i+\frac{1}{2}$ and $j+\frac{1}{2}$ respectively. The constitutive equation must be solved together with the momentum equations and the continuity equations. Fig. 4 shows the stagered grid, there are three type of control volume, the u-control volume is the center of the index (i,J), the v-control volume (I,j) and the scalar-control volume (I,J). Using the center difference approximation, the u-momentum and v-momentum discretized on the u-control volume and v-control volume respectively, the continuity equation and the constitutive equation discretized on scalar-control volume.

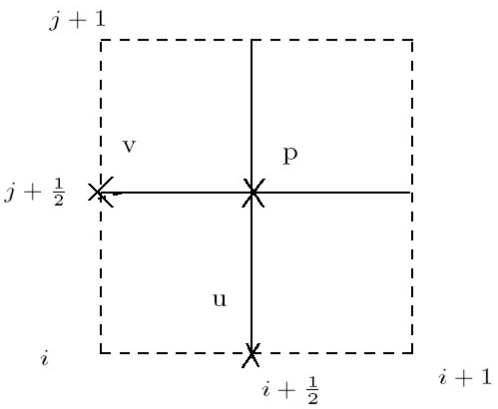

Fig. 3. Control volume cell.

3.1. Solution algorithm

- Step1: Initialize the velocity and pressure
- Step 2: Evaluate the apparent viscosity by using Eq. (5)

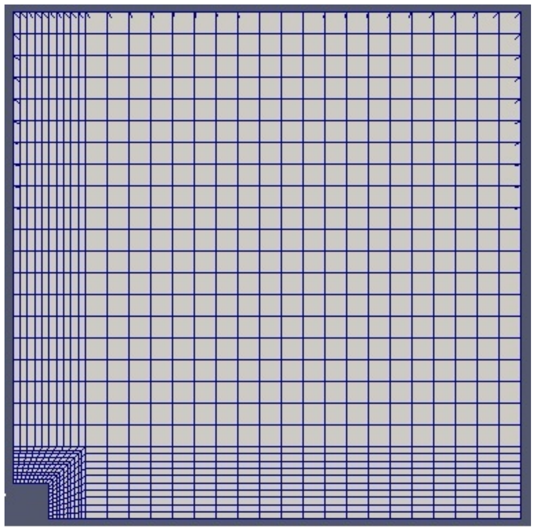

Fig. 5. Full view of the finite volume grid for $L/d = 7$

- Step 3: Set initial guess for p^*, u^*, v^*
- Step 4: Evaluate u^* by solving discretized $u-$momentum equation

$$a_{i,J}u^*_{i,J} = a_{i-1,J}u^*_{i-1,J} + a_{i+1,J}u^*_{i+1,J} + a_{i,J-1}u^*_{i,J-1} + a_{i,J+1}u^*_{i,J+1} + S_u\Delta V - \frac{p_{I,J}-p_{I-1,J}}{\Delta x}\Delta V + (1-\alpha_u)u^{n-1}_{i,J} \quad (9)$$

- Step 5: Evaluate v^* by solving $v-$momentum equation approximated

$$a_{I,j}v^*_{I,j} = a_{I,j-1}v^*_{I,j-1} + a_{I+1,j}v^*_{I+1,j} + a_{I-1,j}v^*_{I-1,j} + a_{I-1,j}v^*_{I-1,j} + S_v\Delta V - \frac{p_{I,J}-p_{I,J-1}}{\Delta y}\Delta V + (1-\alpha_v)v^{n-1}_{I,j} \quad (10)$$

- Step 6: Evaluate p' by solving Possion equation

$$a_{I,J}p'_{I,J} = a_{I+1,J}p'_{I+1,J} + a_{I-1,J}p'_{I-1,J} + a_{I,j+1}p'_{I,j+1} + a_{I,J-1}p'_{I,J-1} + b'_{I,J} \quad (11)$$

- Step 7: Correct pressures and velocities using under-relaxation

$$p^n = p^{*n-1} + \alpha_p p'^n$$

$$u'_{i,J} = -(p'_{I,J} - p'_{I-1,J})\frac{\Delta x_{i,J}}{a_{i,J}}, \quad v'_{I,j} = -(p'_{I,J} - p'_{I,J-1})\frac{\Delta x_{I,j}}{a_{I,j}}$$

$$u^n = \alpha_u(u^{*n} + u'^n) + (1-\alpha_u)u^{n-1},$$

$$v^n = \alpha_v(v^{*n} + v'^n) + (1-\alpha_v)v^{n-1}$$

- Step 8: Increase iteration step, if iteration step < max iteration, go to step 2

4. Result and discussion

In this problem, a square cylinder takes place between two plates, the flow regime is creeping with yield stress. The momentum equation and continuity equation are solved together with constitutive equation proposed by Papanastasiou [20]. The algorithm developed here is second order accurate in space, the FVM by using SIMPLE algorithm to coupling pressure-velocity is used. The schematic diagram of the 2D-creeping flow past a square cylinder is shown in Fig. 1. The side of the cylinder is d, the length of the domain is $2L$ and the cylinder is situated half-way between two plates $2L$ apart, as inertia force is neglected, two plates of symmetry is considered. Thus, the domain is reduced to the quarter of the flow field. The computational domain comprises $L \times L$, the ratio length by cylinder diameter is considered $L/d = 7$. The symmetry boundaries are considered at the AF and DC. The velocity u and v at the AB and BC are specified as zero and V respectively. The no slip boundary is specified at the cylinder. Consider to have $nX_p \times nY_p$ cells to store the pressure values, $nX_p - 1 \times nY_p$ to store the u-velocity values and $nX_p \times nY_p - 1$ cells to store the v-velocity values. As shown in the Fig. 5 near the cylinder a finer mesh is used. The FVM is used for numerical simulation for 1900 cells, the corresponding grid size $\Delta x = \Delta y = 0.005$ and $\Delta x = \Delta y = 0.01$ are used near cylinder and far from cylinder respectively. The iterative Gauss Seidel (GS) has been adopted to solve descritize $u-$ and $v-$ momentum equations, iteration is continue until residual criteria is met, $eps = 1e^{-5}$. Under relaxation factor are considered $\alpha_u = \alpha_v = 0.6$. The Possion equation solved in a same manner with $eps = 1e^{-5}$ and $\alpha_p = 0.1$ Fig. 5 shows the Finite Volume grid used in the simulation for ratio of $L/d = 7$. Fig. 6 shows the yield surface at $B_n = 10, 100, 1000, 10000$ and 100000. The comparison of numerical and result of Nirmulkar [13] shows good agreement about, 3 – 4 percent difference was observed with their result. Fig. 7 shows a comparison of yield surface S_1 for different Bingham number 10, 100, 1000, 10000. S_1 decreasing while the Bingham number increasing. Fig. 8 shows a comparison of yield surface S_2 for a Bingham number 10,100,1000,100000. S_2 increasing while the Bingham number increasing. As can be seen from Table 1 the length of the rigid zone increased by increasing the Bingham number. for large value of Bingham number changes is insignificant. Fig. 9 shows the velocity profile at the symmetry line in the y-direction, the constant velocity in represents the rigid zones. Fig. 10 shows the velocity profile along the line of symmetry in the x-direction Once again, a thin viscoplastic boundary layer fluid zone observed at the beginning of domain. At the downstream, constant velocity observed.

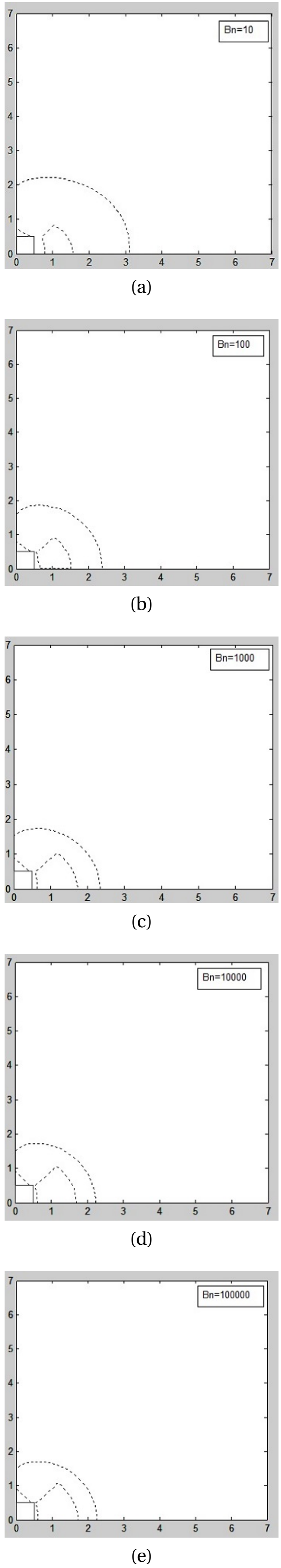

Fig. 6. Location of yield structures.

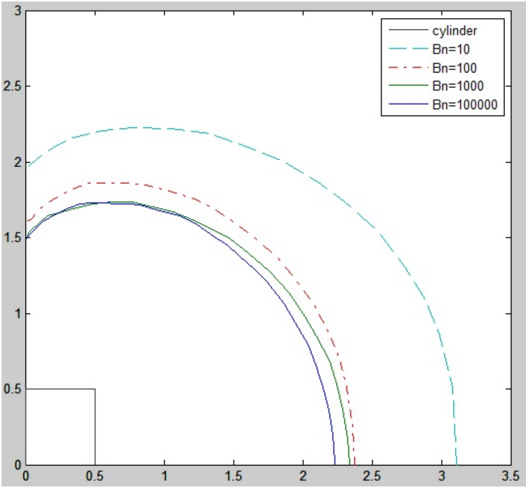

Fig. 7. Effective of Bingham number on the location of yield surfaces, S_1.

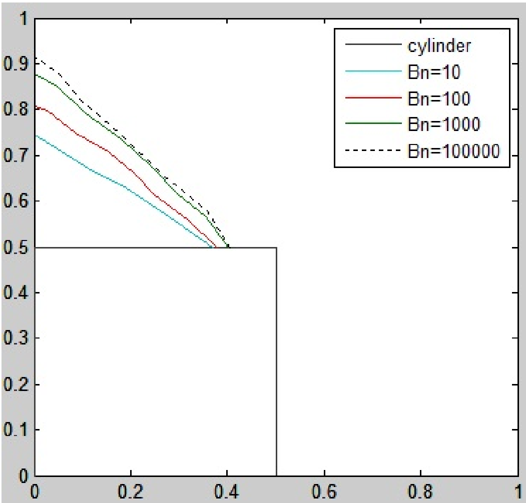

Fig. 8. Effect of Bingham number on the location of yield surface, S_2.

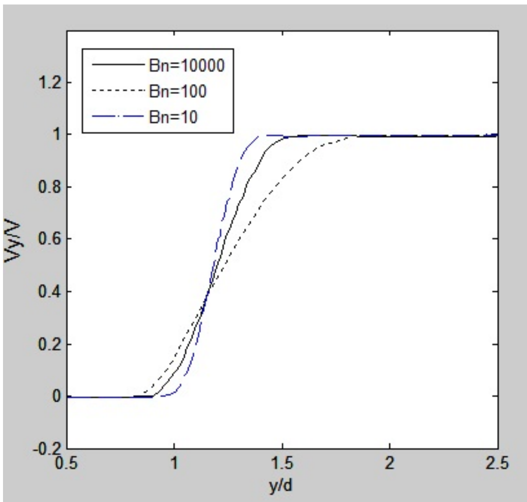

Fig. 9. Velocity profile along $x = 0$

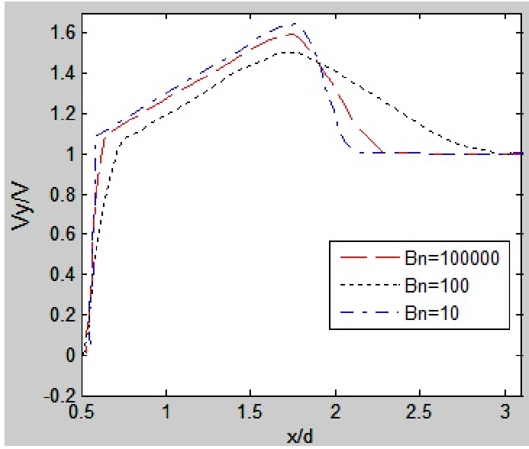

Fig. 10. Velocity profile along $y = 0$

Table 1. Effect of Bingham number on the size of rigid zones.

B_n	10	1000	10000	100000
x_1	3.11	2.33	2.25	2.24
x_2	1.55	1.73	1.74	1.75
x_3	0.781	0.64	0.63	0.60
y_1	0.74	0.87	0.91	0.92
y_2	2	1.54	1.49	1.48
y_3	2.22	1.73	1.71	1.69

5. Conclusion

In this paper, a Finite Volume Method has been implemented for creeping flow past square cylinder. Governing equations with Papanastasiou regularisation equation discritized on a staggered grid by adopting SIMPLE algorithm. The numerical results are compared with the numerical result. The present model could be useful for solving creeping flow problems and could be a good alternative to the existing method like finite volume and finite difference.

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