Int. J. Adv. Appl. Math. and Mech. 4(3) (2017) 7 – 13 (ISSN: 2347-2529)
Journal homepage: www.ijaamm.com

International Journal of Advances in Applied Mathematics and Mechanics

Approximation properties of general gamma type operators in polynomial weighted space

Research Article

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Received 12 November 2016; accepted (in revised version) 19 December 2016

Abstract: In this paper we give direct approximation theorems for general Gamma type operators in polynomial weighted spaces of functions of one variable. The results are given in terms of some Ditzian-Totik moduli of smoothness.

MSC: 41A25 • 26A15 • 40A35

Keywords: Gamma type operators • Rate of convergence • Steklov means • Polynomial weighted space

1. Introduction

In [1], Lupas and M*ü*ller defined and studied some approximation properties of a sequence of positive and linear operators $\{G_n\}$ defined as

$$G_n(f;x) = \int_0^\infty g_n(x,u) f\left(\frac{n}{u}\right) du,$$

where

$$g_n(x,u) = \frac{x^{n+1}}{n!} e^{-xu} u^n, \quad x \in (0,\infty).$$

Approximation properties of $\{G_n\}$ in some function spaces were studied in many papers (see [1–4]). The above operators were modified by several researchers (see [5–7]), which showed that new operators have similar approximation properties to $\{G_n\}$ (see [8–14]).

In 2007, Mao [15] defined the following operators

$$M_{n,k}(f;x) = \frac{(2n-k+1)!}{n!(n-k)!} x^{n+1} \int_0^\infty \frac{t^{n-k}}{(x+t)^{2n-k+2}} f(t)\,dt, \; x,t \in (0,\infty) \tag{1}$$

for any f for which the above integral is convergent.

Some approximation properties of $\{M_{n,k}\}$ were studied in these papers (see [16–19]).

Recently, Alok Kumar [20] obtained the following result.

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Lemma 1.1 ([20]).
If r^{th} derivative $f^{(r)}(r=0,1,2...)$ exists continuously, then we get

$$M^{(r)}_{n,k}(f;x)=\beta_n x^{n+1-r}\int_0^\infty \frac{t^{n-k+r}}{(x+t)^{2n-k+2}}f^{(r)}(t)dt,\ x\in(0,\infty),$$

where

$$\beta_n=\frac{(2n-k+1)!}{n!(n-k)!}.$$

The Voronovskaja type theorem and the local rate of convergence for operators $M^{(r)}_{n,k}$ were given in [20].

The aim of this paper is to study approximation properties of $M^{(r)}_{n,k}$ in C_p, $p\in N_0$(set of non-negative integers), where C_p is a polynomial weighted space with the weight function μ_p

$$\mu_0(x)=1,\ \mu_p(x)=\frac{1}{1+x^p},\ p\geq 1, \tag{2}$$

and C_p is the set of all real valued continuous functions f for which $f\mu_p$ is bounded and uniformly continuous on $[0,\infty)$.
The norm on C_p is defined by

$$\|f\|_p=\sup_{x\in[0,\infty)}\mu_p(x)|f(x)|,\ f\in C_p.$$

We also consider the modulus of smoothness of $f\in C_p$,

$$\omega^2_p(f;\delta):=\sup_{h\in(0,\delta]}\|\Delta^2_h f\|_p,$$

and the modulus of continuity of $f\in C_p$,

$$\omega_p(f;\delta):=\sup_{h\in(0,\delta]}\|\Delta_h f\|_p,$$

where

$$\Delta^2_h f(x):=f(x+2h)-2f(x+h)+f(x),\ \Delta_h f(x):=f(x+h)-f(x)$$

for $x,h\in[0,\infty)$.

2. Auxiliary results

In this section we give some preliminary results which will be used in the main part of this paper.
Let

$$b(n,k,r)=\beta_n x^{n+1-r}\int_0^\infty \frac{t^{n-k+r}}{(x+t)^{2n-k+2}}dt=\frac{(n-r)!(n-k+r)!}{n!(n-k)!}.$$

We define the sequence of linear and positive operators $\{M^*_{n,k,r}\}$ as

$$M^*_{n,k,r}(g;x)=\frac{\beta_n}{b(n,k,r)}x^{n+1-r}\int_0^\infty \frac{t^{n-k+r}}{(x+t)^{2n-k+2}}g(t)dt, \tag{3}$$

for each $x,t\in(0,\infty)$, $r\in N_0$ (see [20]).

Let us consider

$$e_m(t)=t^m,\ \varphi_{x,m}(t)=(t-x)^m,\ m\in N_0,\ x,t\in(0,\infty).$$

Lemma 2.1 ([20]).
For any $m\in N_0$, $m+r\leq n$ and $r\leq n$ we have

$$M^*_{n,k,r}(e_m;x)=\frac{(n-r-m)!(n-k+r+m)!}{(n-r)!(n-k+r)!}x^m$$

and

$$M^*_{n,k,r}(\varphi_{x,m};x)=\left(\sum_{j=0}^m(-1)^j\binom{m}{j}\frac{(n-r-m+j)!(n-k+r+m-j)!}{(n-r)!(n-k+r)!}\right)x^m$$

for each $x\in(0,\infty)$.

Lemma 2.2 ([20]).
For $m = 0,1,2,3,4$, one has

(i) $M^*_{n,k,r}(\varphi_{x,0};x) = 1$,

(ii) $M^*_{n,k,r}(\varphi_{x,1};x) = \dfrac{2r-k+1}{n-r}x$,

(iii) $M^*_{n,k,r}(\varphi_{x,2};x) = \dfrac{4r^2+4r(2-k)+2n+k^2-5k+4}{(n-r)(n-r-1)}x^2$,

(iv) $M^*_{n,k,r}(\varphi_{x,3};x) = \dfrac{c_{n,k,r}}{(n-r)(n-r-1)(n-r-2)}x^3$,

(v) $M^*_{n,k,r}(\varphi_{x,4};x) = \dfrac{d_{n,k,r}}{(n-r)(n-r-1)(n-r-2)(n-r-3)}x^4$,

where $c_{n,k,r} = 8r^3 + r^2(36-2k) + r(51+14n-42k+6k^2) - k^3 + 12k^2 - 34k - n^2 + n(17-6k-6k^2+2kr) + 21$
and
$d_{n,k,r} = 16r^4+r^3(128-32k)+r^2(348+48n-216k+24k^2)+r(366+177n+k(6n^2-54n-440)+120k^2-8k^3)+k^4+k^3(4n-22)+139k^2-k(245+116n)+24n^2+131n+100.$

Let $p, r \in N_0$. By C^r_p, we denote the space of all functions $f \in C_p$ such that $f', f''...f^{(r)} \in C_p$.

Theorem 2.1.
For the operator $M^{(r)}_{n,k}$ and for fixed $p \in N_0, n \in N$ and $f \in C^r_p$, there exists a positive constant $\mathcal{K}^1_{p,k,r}$ depending only on the parameters p, k and r such that

$$\left\| \frac{1}{b(n,k,r)} M^{(r)}_{n,k}(f;.) \right\|_p \le \mathcal{K}^1_{p,k,r} \, \|f^{(r)}\|_p \tag{4}$$

which guarantees that $M^{(r)}_{n,k}$ maps C^r_p into C^r_p.

Proof. For $p = 0$, the proof follows immediately.
Let $p \in N$. By Lemma 2.2, we can find a positive constant $\mathcal{K}^1_{p,k,r}$ such that

$$\begin{aligned}\mu_p(x)M^*_{n,k,r}\left(\frac{1}{\mu_p(t)};x\right) &= \mu_p(x)\{M^*_{n,k,r}(e_0;x)+M^*_{n,k,r}(e_p;x)\}\\ &= \mu_p(x)\left\{1+\frac{(n-r-p)!(n-k+r+p)!}{(n-r)!(n-k+r)!}x^p\right\}\\ &\le \mathcal{K}^1_{p,k,r}\mu_p(x)\{1+x^p\} = \mathcal{K}^1_{p,k,r},\end{aligned}$$

where

$$\mathcal{K}^1_{p,k,r} = \max\left\{\sup_n \frac{(n-r-p)!(n-k+r+p)!}{(n-r)!(n-k+r)!}, 1\right\}.$$

Observe that for every $f \in C^{(r)}_p$ and $x \in (0,\infty)$, we get

$$\begin{aligned}\mu_p(x)\left|\frac{1}{b(n,k,r)}M^{(r)}_{n,k}(f;x)\right| &\le \mu_p(x)\frac{\beta_n}{b(n,k,r)}x^{n+1-r}\int_0^\infty \frac{t^{n-k+r}}{(x+t)^{2n-k+2}}|f^{(r)}(t)|\frac{\mu_p(t)}{\mu_p(t)}dt\\ &\le \|f^{(r)}\|_p\mu_p(x)M^*_{n,k,r}\left(\frac{1}{\mu_p(t)};x\right)\\ &\le \mathcal{K}^1_{p,k,r}\|f^{(r)}\|_p.\end{aligned}$$

Taking supremum over $x \in (0,\infty)$, we get desired result. □

Lemma 2.3.
For the operators $M^*_{n,k,r}$ and for fixed $p, r \in N_0$, there exists a positive constant $\mathcal{K}^2_{p,k,r}$ such that

$$\mu_p(x)M^*_{n,k,r}\left(\frac{\varphi_{x,2}}{\mu_p(t)};x\right) \le \mathcal{K}^2_{p,k,r}\frac{x^2}{n}$$

for all $x \in (0,\infty)$ and $n \in N$.

Proof. Let $p = 0$. Using Lemma 2.2, we can write

$$\mu_0(x) M^*_{n,k,r}\left(\frac{\varphi_{x,2}}{\mu_0(t)}; x\right) = \frac{4r^2 + 4r(2-k) + 2n + k^2 - 5k + 4}{(n-r)(n-r-1)} x^2$$
$$= \frac{n(4r^2 + 4r(2-k) + 2n + k^2 - 5k + 4)}{(n-r)(n-r-1)} \frac{x^2}{n}$$
$$\leq \mathcal{K}^2_{k,r} \frac{x^2}{n}.$$

Now, let $p > 0$. Then, we get from Lemma 2.2

$$M^*_{n,k,r}\left(\frac{\varphi_{x,2}}{\mu_p(t)}; x\right) = M^*_{n,k,r}(e_{p+2}; x) - 2x M^*_{n,k,r}(e_{p+1}; x) + x^2 M^*_{n,k,r}(e_p; x) + M^*_{n,k,r}(\varphi_{x,2}; x)$$
$$= \frac{(n-r-p-2)!(n-k+r+p)!}{(n-r)!(n-k+r)!} x^{p+2} [(n-r-p)(n-r-p-1)$$
$$-2(n-r-p-1)(n-k+r+p+1) + (n-k+r+p+2)$$
$$(n-k+r+p+1)] + \frac{4r^2 + 4r(2-k) + 2n + k^2 - 5k + 4}{(n-r)(n-r-1)} x^2$$
$$= \left(1 + \left\{1 + \frac{8rp + 4p^2 + 8p - 4kp - 1}{4r^2 + r(8-4k) + 2n + k^2 - 5k + 4}\right\} \frac{(n-r-p-2)!(n-k+r+p)!}{(n-r-2)!(n-k+r)!} x^p\right)$$
$$\frac{4r^2 + r(8-4k) + 2n + k^2 - 5k + 4}{(n-r)(n-r-1)} x^2$$
$$\leq \mathcal{K}^2_{p,k,r} \frac{x^2}{n}(1 + x^p).$$

Hence, the proof is completed. □

3. Approximation Properties

Theorem 3.1.
For every $p, r \in N_0$ and $g \in C^1_p$, there exists a positive constant $\mathcal{K}^3_{p,k,r}$ depending only on the parameters p, k and r such that

$$\mu_p(x)|M^*_{n,k,r}(g; x) - g(x)| \leq \mathcal{K}^3_{p,k,r} \|g'\|_p \frac{x}{\sqrt{n}}$$

for all $x \in (0, \infty)$ and $n \in N$.

Proof. Let $x \in (0, \infty)$ be fixed. We have

$$g(t) - g(x) = \int_x^t g'(v)\,dv, \quad t \in (0, \infty).$$

By using the linearity of $M^*_{n,k,r}$ and $M^*_{n,k,r}(1; x) = 1$, we get

$$M^*_{n,k,r}(g; x) - g(x) = M^*_{n,k,r}\left(\int_x^t g'(v)\,dv; x\right). \tag{5}$$

Observe that

$$\left|\int_x^t g'(v)\,dv\right| \leq \|g'\|_p \left|\int_x^t \frac{dv}{\mu_p(v)}\right| \leq \|g'\|_p |t - x| \left(\frac{1}{\mu_p(t)} + \frac{1}{\mu_p(x)}\right).$$

Using (5) we obtain

$$\mu_p(x)|M^*_{n,k,r}(g; x) - g(x)| \leq \|g'\|_p \left(M^*_{n,k,r}(|\varphi_{x,1}|; x) + \mu_p(x) M^*_{n,k,r}\left(\frac{|\varphi_{x,1}|}{\mu_p(t)}; x\right)\right).$$

Applying the Cauchy-Schwarz inequality we can write

$$M^*_{n,k,r}(|\varphi_{x,1}|; x) \leq \sqrt{M^*_{n,k,r}(\varphi_{x,2}; x)} \times \sqrt{M^*_{n,k,r}(\varphi_{x,0}; x)},$$

$$M^*_{n,k,r}\left(\frac{|\varphi_{x,1}|}{\mu_p(t)}; x\right) \leq \sqrt{M^*_{n,k,r}\left(\frac{1}{\mu_p(t)}; x\right)} \times \sqrt{M^*_{n,k,r}\left(\frac{\varphi_{x,2}}{\mu_p(t)}; x\right)}.$$

Finally, using Lemma 2.2, Lemma 2.3 and Theorem 2.1, we obtain

$$\mu_p(x)|M^*_{n,k,r}(g; x) - g(x)| \leq \mathcal{K}^3_{p,k,r} \|g'\|_p \frac{x}{\sqrt{n}}.$$

□

Theorem 3.2.
Let $p, r \in N_0$ and $n \in N$. If

$$W_{n,k,r}(f;x) = M^*_{n,k,r}(f;x) - f\left(x + \frac{2r-k+1}{n-r}x\right) + f(x), \tag{6}$$

then there exists a positive constant $\mathcal{K}^4_{p,k,r}$ depending only on the parameters p, k and r such that for all $x \in (0,\infty)$, we have

$$\mu_p(x)\left|W_{n,k,r}(f;x) - f(x)\right| \le \mathcal{K}^4_{p,k,r}\|f''\|_p \frac{x^2}{n}$$

for any function $f \in C^2_p$.

Proof. From Lemma 2.2, we observe that the operators $W_{n,k,r}$ are linear and reproduce the linear functions. Hence

$$W_{n,k,r}(\varphi_{x,1};x) = 0.$$

For $f \in C^2_p$ and $t \in (0,\infty)$, we have

$$f(t) = f(x) + (t-x)f'(x) + \int_x^t (t-v)f''(v)dv.$$

Then,

$$\begin{aligned}|W_{n,k,r}(f;x) - f(x)| &= |W_{n,k,r}(f(t) - f(x);x)| = \left|W_{n,k,r}\left(\int_x^t (t-v)f''(v)dv;x\right)\right| \\ &= \left|M^*_{n,k,r}\left(\int_x^t (t-v)f''(v)dv;x\right) - \int_x^{x+\frac{2r-k+1}{n-r}x}\left(x + \frac{2r-k+1}{n-r}x - v\right)f''(v)dv\right|.\end{aligned}$$

Notice that

$$\left|\int_x^t (t-v)f''(v)dv\right| \le \frac{\|f''\|_p(t-x)^2}{2}\left(\frac{1}{\mu_p(x)} + \frac{1}{\mu_p(t)}\right)$$

and

$$\left|\int_x^{x+\frac{2r-k+1}{n-r}x}\left(x + \frac{2r-k+1}{n-r}x - v\right)f''(v)dv\right| \le \frac{\|f''\|_p}{2\mu_p(x)}\left(\frac{2r-k+1}{n-r}x\right)^2.$$

Using the above inequality, we get

$$\mu_p(x)|W_{n,k,r}(f;x) - f(x)| \le \frac{\|f''\|_p}{2}\left(M^*_{n,k,r}(\varphi_{x,2};x) + \mu_p(x)M^*_{n,k,r}\left(\frac{\varphi_{x,2}}{\mu_p(t)};x\right)\right) + \frac{\|f''\|_p}{2}\left(\frac{2r-k+1}{n-r}x\right)^2.$$

Hence, by Lemma 2.3 we obtain

$$\mu_p(x)|W_{n,k,r}(f;x) - f(x)| \le \mathcal{K}^4_{p,k,r}\|f''\|_p \frac{x^2}{n}$$

for every $f \in C^2_p$. □

Theorem 3.3.
Let $p, r \in N_0$ and $n \in N$. If $f \in C^{(r)}_p$, then there exists a positive constant $\mathcal{K}^5_{p,k,r}$ such that

$$\mu_p(x)\left|\frac{1}{b(n,k,r)}M^{(r)}_{n,k}(f;x) - f^{(r)}(x)\right| \le \mathcal{K}^5_{p,k,r}\omega^2_p\left(f^{(r)}, \frac{x}{\sqrt{n}}\right) + \omega_p\left(f^{(r)}, \frac{2r-k+1}{n-r}x\right),$$

for all $x \in (0,\infty)$.

Proof. Let $f \in C_p^r$. We consider the Steklov means $\tilde{f}_h^{(r)}, h > 0$ of $f^{(r)}$ as (see p. 317, [21])

$$\tilde{f}_h^{(r)}(x) = \frac{4}{h^2} \int_0^{h/2} \int_0^{h/2} \left(2f^{(r)}(x+s+t) - f^{(r)}(x+2(s+t))\right) dsdt,$$

for $x, h \in (0,\infty)$. We have

$$f^{(r)}(x) - \tilde{f}_h^{(r)}(x) = \frac{4}{h^2} \int_0^{h/2} \int_0^{h/2} \Delta_{s+t}^2 f^{(r)}(x) dsdt,$$

which gives

$$\left\| f^{(r)} - \tilde{f}_h^{(r)} \right\|_p \leq \omega_p^2 \left(f^{(r)}, h\right). \tag{7}$$

Remark that

$$\tilde{f}_h^{(r+2)}(x) = \frac{1}{h^2}\left(8\Delta_{h/2}^2 f^{(r)}(x) - \Delta_h^2 f^{(r)}(x)\right)$$

and

$$\left\| \tilde{f}_h^{(r+2)} \right\|_p \leq \frac{9}{h^2} \omega_p^2 \left(f^{(r)}, h\right). \tag{8}$$

From (7) and (8), we conclude that $\tilde{f}_h^{(r)} \in C_p^2$ if $f^{(r)} \in C_p$.
Observe that

$$\begin{aligned}\left|M_{n,k,r}^*(f^{(r)};x) - f^{(r)}(x)\right| &\leq W_{n,k,r}\left(\left|f^{(r)} - \tilde{f}_h^{(r)}\right|;x\right) + \left|f^{(r)}(x) - \tilde{f}_h^{(r)}(x)\right| \\ &\quad + \left|W_{n,k,r}\left(\tilde{f}_h^{(r)};x\right) - \tilde{f}_h^{(r)}(x)\right| + \left|f^{(r)}\left(x + \frac{2r-k+1}{n-r}x\right) - f^{(r)}(x)\right|,\end{aligned}$$

where $W_{n,k,r}$ is defined in (6). Since $\tilde{f}_h^{(r)} \in C_p^2$, it follows from Theorem 2.1 and Theorem 3.2 that

$$\begin{aligned}\mu_p(x)\left|\frac{1}{b(n,k,r)} M_{n,k}^{(r)}(f;x) - f^{(r)}(x)\right| &= \mu_p(x)\left|M_{n,k,r}^*(f^{(r)};x) - f^{(r)}(x)\right| \\ &\leq (\mathcal{K}+5)\left\|f^{(r)} - \tilde{f}_h^{(r)}\right\|_p + \mathcal{K}_{p,k,r}^4 \left\|\tilde{f}_h^{(r+2)}\right\|_p \frac{x^2}{n} \\ &\quad + \mu_p(x)\left|f^{(r)}\left(x + \frac{2r-k+1}{n-r}x\right) - f^{(r)}(x)\right|\end{aligned}$$

Using (7) and (8), we get

$$\mu_p(x)\left|\frac{1}{b(n,k,r)} M_{n,k}^{(r)}(f;x) - f^{(r)}(x)\right| \leq \mathcal{K}_{p,k,r}^5 \omega_p^2(f,h)\left\{1 + \frac{1}{h^2}\frac{x^2}{n}\right\} + \omega_p\left(f^{(r)}, \frac{2r-k+1}{n-r}x\right).$$

Thus, choosing $h = \frac{x}{\sqrt{n}}$ we get the desired result. □

Acknowledgement

The author(s) are very grateful to the referee for making valuable comments leading to the overall improvement of the paper.

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