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Complete syncronization anti syncronization and hybrid syncronization of restricted three body problem when bigger primary is oblate spheroid and smaller primary is ellipsoid

Research Article

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Abstract: In this article we have investigated the complete synchronization, anti synchronization and possible cases of hybrid synchronization behavior of the restricted three body problem by taking into consideration the bigger primary is oblate spheroid and smaller primary is an ellipsoid evolving from deferent initial conditions using active control technique based on the Lyapunov-stability theory and Routh-Hurwitz criteria. Numerical simulations are performed to plot time series analysis graphs of the master system and the slave system which further illustrate the effectiveness of the proposed control techniques.

MSC: 70E55 • 34K20

Keywords: Space dynamics • Restricted three body problem • Complete synchronization • Hybrid synchronization • Lyapunov stability theory • Routh-Hurwitz criteria

1. Introduction

In the last few years considerable research has been done in non-linear dynamical systems and their various properties. One of the most important properties is synchronization which is an important topic in the nonlinear dynamics. Chaotic synchronization did not attract much attention until Pec-ora and Carroll [1] introduced a method to synchronize two identical chaotic systems with deferent initial conditions in (1990) and they demonstrated that chaotic synchronization could be achieved by driving or replacing one of the variables of a chaotic system with a variable of another similar chaotic device. From then on, chaos synchronization has been widely explored in a variety of fields including ecological systems, secure communications, chemical systems, physical systems, etc.

Chaos synchronization using active control has recently been widely accepted as an efficient technique for synchronizing chaotic systems. In Ayub Khan and Priyamvada Tripathi [2] have investigated the synchronization behavior of a restricted three body problem under the eïňĂect of radiation pressure. In an another paper the Complete synchronization, anti-synchronization and hybrid synchronization of two identical parabolic restricted three body problem have been studied by Ayub Khan and Rimpi pal[3] . Anti-synchronization between two different hyperchaotic systems systems by using active control have been studied by M. Mossa Al-sawalha and M.S.M. Noorani [4]. Arif. Mohd. [5] have studied the Complete synchronization and anti synchronization behavior of a planer magnetic binaries problem when one primary is ellipsoid. In javid et al. [6] have been studied the restricted three body problem when both primaries are ellipsoid. R.K. Sharma and Subbarao [7] have discussed the collinear equilibria and their characteristic exponents in the restricted three body problem when the primaries are oblate spheroids.

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In this article, active control techniques base on the Lyapunov stability theory and Routh-Hurwitz criteria have been used to study the complete synchronization , anti synchronization and hybrid synchronization behavior of planar restricted three body problem by taking into consideration the bigger primary is oblate spheroid and smaller primary is an ellipsoid. The system under consideration is chaotic for some values of parameter involved in the system. Here two systems (master and slave) are not only complete synchronized, but they also anti and hybrid synchronized start with deferent initial conditions. Hence the slave chaotic system completely traces the dynamics of the master system in the course of time.

2. Equation of motion

In formulating the problem we shall assume that the two primaries one is in the shape of ellipsoid and other is oblate spheroid participate in the circular motion around their centre of mass O see Fig. 1. The motion of a particle P of mass m defined by its radius vector r will be referred to a frame of reference $Oxyz$ that rotates in the same direction and the same angular velocity ω as the primaries, which in this frame are taken to stay at rest on x-axis. Here we assumed that the distance between the primaries as the unit of distance and the coordinate of one primary is $(\mu, 0, 0)$ then the other is $(\mu - 1, 0, 0)$. We also assumed that the sum of their masses as the unit of mass. If mass of the one primaries μ then the mass of the other is $(1-\mu)$. The unit of time in such a way that the gravitational constant G has the value unity. The equation of motion of the particle P may be written as:

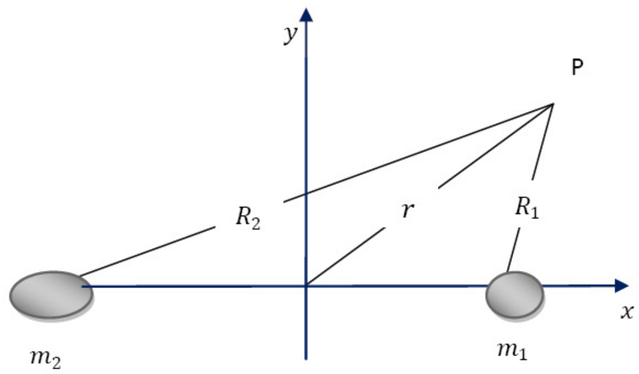


Fig. 1. Schematics diagram of the problrm.

$$\ddot{x} - 2\omega = U_x \tag{1}$$

$$\ddot{y} + 2\omega \dot{x} = U_y \tag{2}$$

$$\dot{x}^2 + \dot{y}^2 = 2U - C \tag{3}$$

where

$$U = \frac{\omega^2}{2}(x^2 + y^2) + \frac{(1-\mu)}{R_1} + \frac{1}{2R_1^2} + V \tag{4}$$

$$R_1^2 = (x-\mu)^2 + y^2, R_2^2 = (x+1-\mu)^2 + y^2$$

$$V = \frac{3\mu}{2}\left[\left\{1 + \frac{y^2 - (x+1-\mu)^2}{(a^2-b^2)}\right\}\frac{F(\varphi,k)}{\sqrt{(a^2-c^2)}} + \left\{\frac{(x+1-\mu)^2}{(a^2-b^2)} + \frac{(c^2-a^2)\,y^2}{(a^2-b^2)(b^2-c^2)}\right\}\frac{E(\varphi,k)}{\sqrt{(a^2-c^2)}} + \frac{\sqrt{(\gamma+c^2)}\,y^2}{(b^2-c^2)\sqrt{(\gamma+a^2)(\gamma+b^2)}}\right]$$

$$F(\varphi,k) = \int_0^{\varphi} \frac{d\theta}{\sqrt{1-k^2\sin^2\theta}} \quad \text{(Elliptic integral of first kind)}$$

$$E(\varphi,k) = \int_0^{\varphi} \sqrt{1-k^2\sin^2\theta}\,d\theta \quad \text{(Elliptic integral of second kind1)}$$

$$k=\sqrt{\frac{(a^2-b^2)}{(a^2-c^2)}}\quad 0\leqslant k^2\leqslant 1,\quad \varphi=\sin^{-1}\sqrt{\frac{(a^2-c^2)}{(\gamma+c^2)}}\quad 0\leqslant\varphi\leqslant\frac{\pi}{2},$$

$$\gamma=\frac{1}{2}\left[(x+1-\mu)^2+y^2-p_1+\sqrt{\left\{(x+1-\mu)^2+y^2-p_1\right\}^2+4\left\{p_3(x+1-\mu)^2+p_4y^2-p_2\right\}}\right],$$

$p_1=a^2+b^2+c^2, p_2=a^2b^2+b^2c^2+a^2c^2, p_3=b^2+c^2, p_4=a^2+c^2, p_5=a^2+b^2$, a, b and c are the axes of the ellipsoid. I is the moment of inertia of oblate body. ω is the mean motion of the primaries.

$$\omega=1+\frac{3}{10}\frac{\mu}{(1-\mu)}\left[2a^2-b^2-c^2\right]+\frac{3I}{2\mu}$$

3. Complete synchronization via Active Control

Let $x=x_1, \dot{x}=x_2, y=x_3, \dot{y}=x_4, \dot{y}=x_4$, then the Eqs. (1) and (2) can be written as:

$$\dot{x}_1=x_2 \tag{5}$$

$$\dot{x}_2=2\omega x_4+\omega^2x_1+A_1 \tag{6}$$

$$\dot{x}_3=x_4 \tag{7}$$

$$\dot{x}_4=-2\omega x_2+\omega^2x_3+A_2 \tag{8}$$

where

$$A=-(x_1-\mu)\left\{\frac{1-\mu}{r_1^3}+\frac{3I)}{2r_1^5}\right\}-3\mu(x_1+1-\mu)\times$$

$$\left[\frac{E(\varphi,k)-F(\varphi,k)}{p_6p_8}-\left\{1-k^2\sin^2\varphi\frac{(x_1+1-\mu)^2}{p_6}+\left(\frac{1}{p_6}+\frac{1-k^2\sin^2\varphi}{p_9}\right){x_3}^2\right\}\times\right.$$
$$\left.\frac{\gamma_1+p_3}{2(\gamma_1+a^2)(2\gamma_1+p_1-r_2^2)\sqrt{(\gamma_1+c^2)}\sqrt{1-k^2\sin^2\varphi}}-\frac{(2c^2\gamma_1+p_{11}+{\gamma_2}^2)(\gamma_1+p_3){x_3}^2}{2p_7(2\gamma_1+p_1-r_2^2)\sqrt{(\gamma_1+c^2)}(p_{10}+p_5\gamma_1+{\gamma_1}^2)^{\frac{3}{2}}}\right]$$

$$r_1^2=?(x_1-\mu)^2+{x_3}^2, r_2^2=(x_1+1-\mu)^2+{x_3}^2, p_6=a^2-b^2, p_7=b^2-c^2, p_8=(a^2-c^2),$$
$$p_9=\frac{(a^2-b^2)(b^2-c^2)}{(c^2-a^2)}/, p_{10}=b^2a^2, p_{11}=p_{10}-c^2p_5,$$

$$\gamma_1=\frac{1}{2}\left[(x_1+1-\mu)^2+{x_3}^2-p_1+\sqrt{\left\{(x_1+1-\mu)^2+{x_3}^2-p_1\right\}^2+4\left\{p_3(x_1+1-\mu)^2+p_4{x_3}^2-p_2\right\}}\right]$$

$$A_2=-x_3\left\{\frac{1-\mu}{r_1^3}+\frac{3I)}{2r_1^5}\right\}-3\mu x_3$$

$$\times\left[\frac{E(\varphi,k)}{p_9p_8}+\frac{F(\varphi,k)}{p_6p_8}-\left\{1-k^2\sin^2\varphi\frac{(x_1+1-\mu)^2}{p_6}+\left(\frac{1}{p_6}+\frac{1-k^2\sin^2\varphi}{p_9}\right){x_3}^2\right\}\times\right.$$
$$\frac{\gamma_1+p_4}{2(\gamma_1+a^2)(2\gamma_1+p_1-r_2^2)\sqrt{(\gamma_1+c^2)}\sqrt{1-k^2\sin^2\varphi}}-\frac{(2c^2\gamma_1+p_{11}+{\gamma_1}^2)(\gamma_1+p_4){x_3}^2}{2p_7(2\gamma_1+p_1-r_2^2)\sqrt{(\gamma_1+c^2)}(p_{10}+p_5\gamma_1+{\gamma_1}^2)^{\frac{3}{2}}}$$
$$\left.+\frac{\sqrt{(\gamma_1+c^2)}}{p_7\sqrt{(p_{10}+p_5\gamma_1+{\gamma_1}^2)}}\right]$$

The system (5)-(8) is the master system. The state orbits of this master system are shown in Fig. 2 and this figure shows that the system is chaotic.

Corresponding to master system (5)-(8), the identical slave system is

$$\dot{y}_1=y_2+u_1(t) \tag{9}$$

$$\dot{y}_2=2\omega y_4+\omega^2y_1+A_3+u_2(t) \tag{10}$$

$$\dot{y}_3=y_4+u_3(t) \tag{11}$$

$$\dot{y}_4=-2\omega y_2+\omega^2y_3+A_4+u_4(t) \tag{12}$$

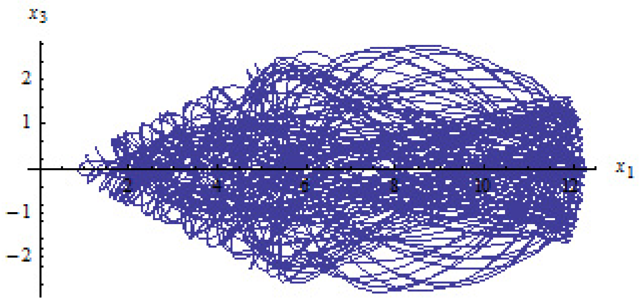


Fig. 2. State orbits of this master system .

where

$$r_{11}{}^2 = (y_1-\mu)^2 + y_3{}^2, r_{21}{}^2 = (y_1+1-\mu)^2 + y_3{}^2$$

$$A_3 = -(y_1-\mu)\left\{\frac{1-\mu}{r_{11}^3} + \frac{3I}{2r_{11}^5}\right\} - 3\mu(y_1+1-\mu)\times$$

$$\left[\frac{E(\varphi,k)-F(\varphi,k)}{p_6 p_8} - \left\{1-k^2\sin^2\varphi\frac{(y_1+1-\mu)^2}{p_6} + \left(\frac{1}{p_6}+\frac{1-k^2\sin^2\varphi}{p_9}\right)y_3{}^2\right\}\times \frac{\gamma_2+p_3}{2(\gamma_2+a^2)(2\gamma_2+p_1-r_{21}^2)\sqrt{(\gamma_2+c^2)}\sqrt{1-k^2\sin^2\varphi}} - \frac{(2c^2\gamma_2+p_{11}+\gamma_2{}^2)(\gamma_2+p_3)y_3{}^2}{2p_7(2\gamma_2+p_1-r_{21}^2)\sqrt{(\gamma_2+c^2)}(p_{10}+p_5\gamma_2+\gamma_2{}^2)^{\frac{3}{2}}}\right]$$

$$A_4 = -y_3\left\{\frac{1-\mu}{r_{11}^3} + \frac{3I}{2r_{11}^5}\right\} - 3\mu y_3\times$$

$$\left[\frac{E(\varphi,k)}{p_9 p_8} + \frac{F(\varphi,k)}{p_6 p_8} - \left\{1-k^2\sin^2\varphi\frac{(y_1+1-\mu)^2}{p_6} + \left(\frac{1}{p_6}+\frac{1-k^2\sin^2\varphi}{p_9}\right)y_3{}^2\right\}\times \frac{\gamma+p_4}{2(\gamma_2+a^2)(2\gamma_2+p_1-r_{21}^2)\sqrt{(\gamma_2+c^2)}\sqrt{1-k^2\sin^2\varphi}} - \frac{(2c^2\gamma_2+p_{11}+\gamma_2{}^2)(\gamma_2+p_4)y_3{}^2}{2p_7(2\gamma_2+p_1-r_{21}^2)\sqrt{(\gamma_2+c^2)}(p_{10}+p_5\gamma_2+\gamma_2{}^2)^{\frac{3}{2}}} + \frac{\sqrt{(\gamma_2+c^2)}}{p_7\sqrt{(p_{10}+p_5\gamma_2+\gamma_2{}^2)}}\right]$$

$$\gamma_2 = \frac{1}{2}\left[(y_1+1-\mu)^2 + y_3{}^2 - p_1 + \sqrt{\left\{(y_1+1-\mu)^2+y_3{}^2-p_1\right\}^2 + 4\left\{p_3(y_1+1-\mu)^2+p_4 y_3{}^2-p_2\right\}}\right]$$

where $u_i(t); i = 1,2,3,4$ are control functions to be determined. Let $e_i = y_i - x_i; i = 1,2,3,4$ be the synchronization errors [8]. From (5) to (12), we obtain the error dynamics as follows:

$$\dot{e}_1 = e_2 + u_1(t) \tag{13}$$
$$\dot{e}_2 = 2\omega e_4 + \omega^2 e_1 + A_3 - A_1 + u_2(t) \tag{14}$$
$$\dot{e}_3 = e_4 + u_3(t) \tag{15}$$
$$\dot{e}_4 = -2\omega e_2 + \omega^2 e_3 + A_4 - A_2 + u_4(t) \tag{16}$$

This above error system to be controlled is a linear system with control functions. Thus, let us redefine the control functions so that the terms in (13) to (16) which cannot be expressed as linear terms in e_i's are eliminated:

$$u_1(t) = v_1(t)$$
$$u_2(t) = -A_3 + A_1 + v_2(t)$$
$$u_3(t) = v_3(t)$$
$$u_4(t) = -A_4 + A_2 + v_4(t)$$

The new error system can be expressed as:

$$\left.\begin{aligned}\dot{e}_1 &= e_2 + v_1(t)\\ \dot{e}_2 &= 2\omega e_4 + \omega^2 e_1 + v_2(t)\\ \dot{e}_3 &= e_4 + v_3(t)\\ \dot{e}_4 &= -2\omega e_2 + \omega^2 e_3 + v_4(t)\end{aligned}\right\} \tag{17}$$

The error system (17) to be controlled is a linear system with a control input $v_i(t)(i = 1,...4)$ as function of the error states $e_i(i = 1,...4)$. As long as these feedbacks stabilize the system $e_i(i = 1,...4)$ converge to zero as time t tends to infinity. This implies that master and the slave system are synchronized with active control. There are many possible choice for the control $v_i(t)(i = 1,...4)$. We choose,

$$\begin{bmatrix} v_1(t)\\ v_2(t)\\ v_3(t)\\ v_4(t)\end{bmatrix} = A \begin{bmatrix} e_1\\ e_2\\ e_3\\ e_4\end{bmatrix} \tag{18}$$

Here A is a 4×4 constant matrix to be determined. As per Lyapunov stability theory and Routh-Hurwitz criterion, in order to make the closed loop system (18) stable, proper choice of elements of A has to be made so that the system (18) must have all eigen values with negative real parts. Choosing

$$A = \begin{bmatrix} -1 & -1 & 0 & 0\\ -\omega^2 & -1 & 0 & -2\omega\\ 0 & 0 & -1 & -1\\ 0 & 2\omega & -\omega^2 & -1\end{bmatrix} \tag{19}$$

and defining a matrix B as

$$\begin{bmatrix} \dot{e}_1\\ \dot{e}_2\\ \dot{e}_3\\ \dot{e}_4\end{bmatrix} = B \begin{bmatrix} e_1\\ e_2\\ e_3\\ e_4\end{bmatrix} \tag{20}$$

where B is

$$B = \begin{bmatrix} -1 & 0 & 0 & 0\\ 0 & -1 & 0 & 0\\ 0 & 0 & -1 & 0\\ 0 & 0 & 0 & -1\end{bmatrix} \tag{21}$$

Clearly, B has eigen values with negative real parts. This implies $\lim_{t\to\infty} |e_i| = 0; i = 1,2,3,4$ and hence, complete synchronization is achieved between the master and slave systems. Simulation results for uncoupled system are presented in Figs. 3, 5, 7, 9 and that of controlled system are shown in Figs. 4, 6, 8 and 10 for respectively.

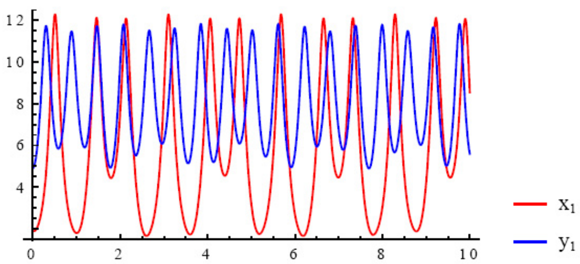


Fig. 3.

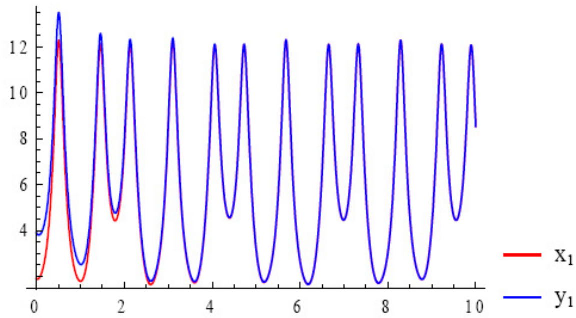


Fig. 4.

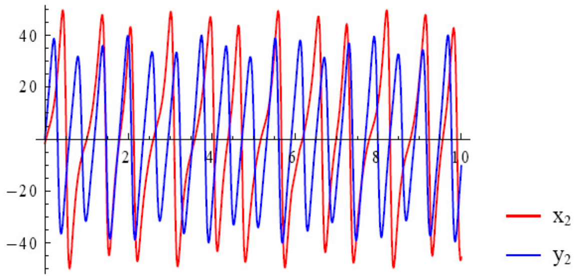


Fig. 5.

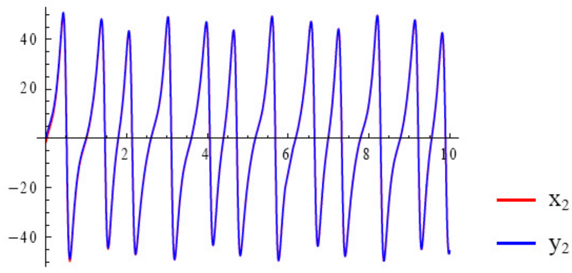


Fig. 6.

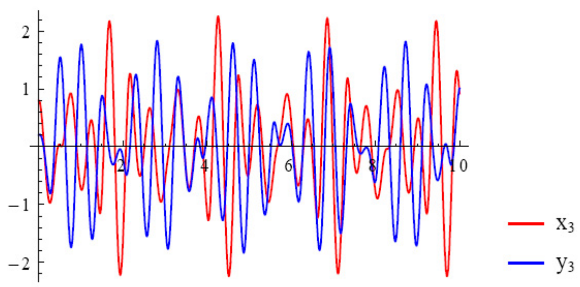
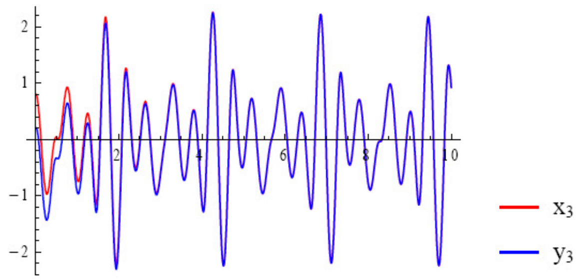


Fig. 7.



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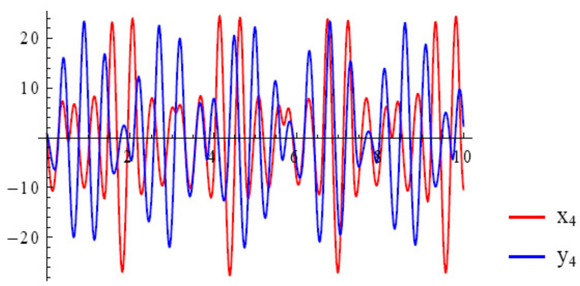

Fig. 9.

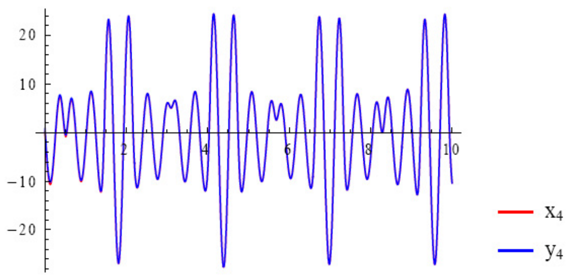

Fig. 10.

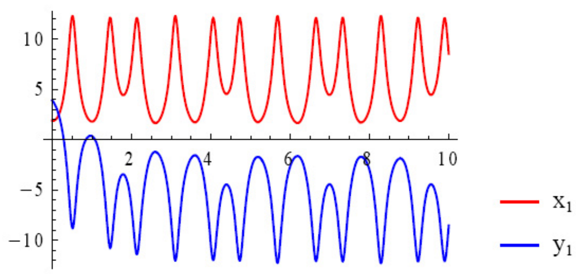

Fig. 11.

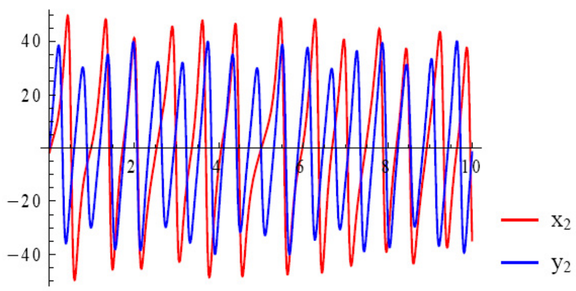

Fig. 12.

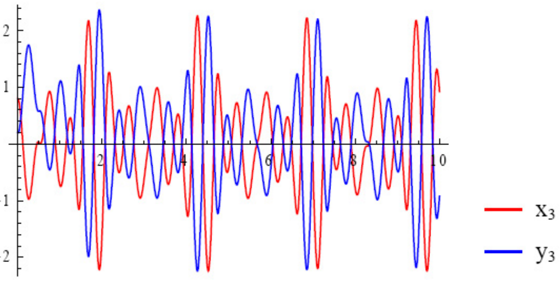

Fig. 13.

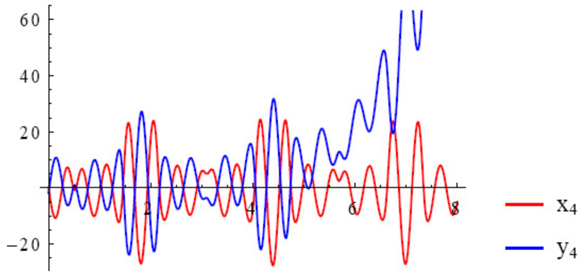

Fig. 14.

Case I

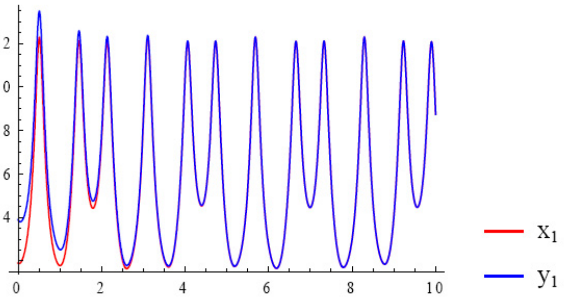

Fig. 15.

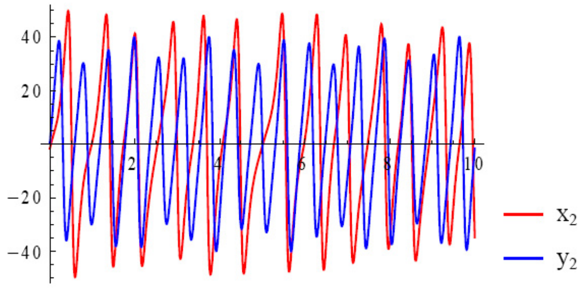

Fig. 16.

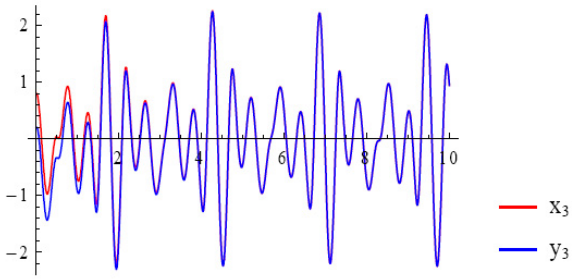

Fig. 17.

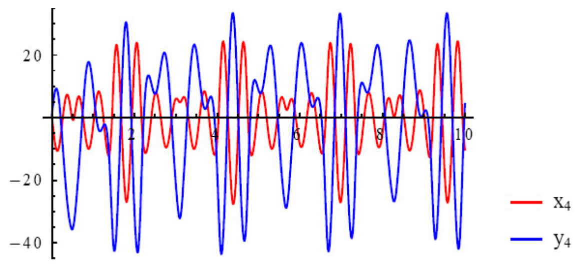

Fig. 18.

Case II

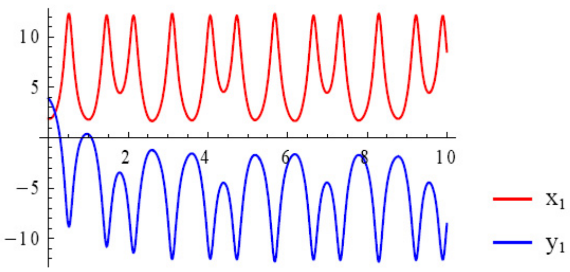

Fig. 19.

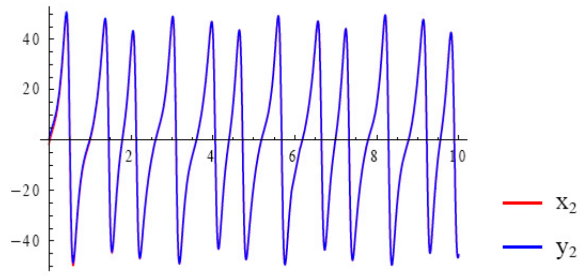

Fig. 20.

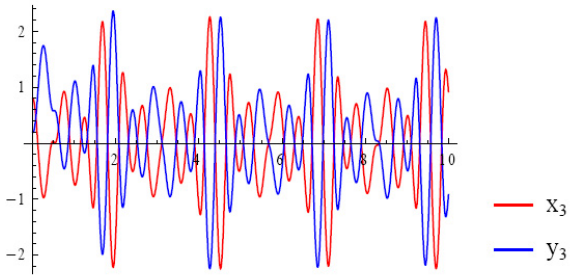

Fig. 21.

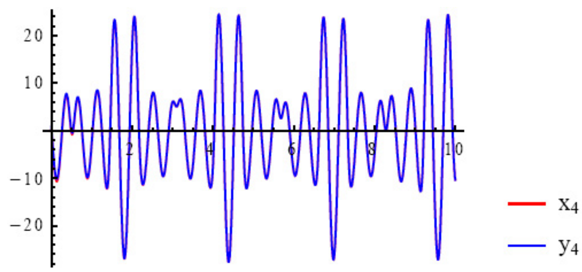

Fig. 22.

4. Anti synchronization via Active Control

To observe anti-synchronization between the master system and the slave system, let $E_i = y_i + x_i; i = 1,2,3,4$ be the synchronization errors. Now from (5) to (12), we obtain the error dynamics as.

$$\dot{E}_1 = E_2 + u_{11}(t) \tag{22}$$
$$\dot{E}_2 = 2\omega E_4 + \omega^2 E_1 + A_3 + A_1 + u_{21}(t) \tag{23}$$
$$\dot{E}_3 = E_4 + u_{31}(t) \tag{24}$$
$$\dot{E}_4 = -2\omega E_2 + \omega^2 E_3 + A_4 + A_2 + u_{41}(t) \tag{25}$$

This above error system to be controlled is a linear system with control functions. Thus, let us redefine the control functions so that the terms in (22) to (25) which cannot be expressed as linear terms in E_i's are eliminated:

$$u_{11}(t) = v_{11}(t)$$
$$u_{21}(t) = -A_3 - A_1 + v_{21}(t)$$
$$u_{31}(t) = v_{31}(t)$$
$$u_{41}(t) = -A_4 - A_2 + v_{41}(t)$$

The new error system can be expressed as:

$$\left.\begin{aligned} \dot{E}_1 &= E_2 + v_1(t) \\ \dot{E}_2 &= 2\omega E_4 + \omega^2 E_1 + v_{21}(t) \\ \dot{E}_3 &= E_4 + v_{31}(t) \\ \dot{E}_4 &= -2\omega E_2 + \omega^2 E_3 + v_{41}(t) \end{aligned}\right\} \tag{26}$$

The error system (26) is same as the system given by (17). Thus, it follows that following the same steps henceforth and with same choice of matrix A, by Active Control Technique, we obtain

$$\begin{bmatrix} \dot{E}_1 \\ \dot{E}_2 \\ \dot{E}_3 \\ \dot{E}_4 \end{bmatrix} = B \begin{bmatrix} E_1 \\ E_2 \\ E_3 \\ E_4 \end{bmatrix} \tag{27}$$

Here B is given by (21). This implies $\lim_{t\to\infty} |E_i| = 0; i = 1,2,3,4$ and hence, complete anti synchronization is achieved between the master and slave systems. Time Series Analysis graphs of the above are shown next to each via Figs. 11 to 14.

5. Hybrid synchronization via Active Control

In order to observe the hybrid synchronization behavior between the master system and the slave system, we note that the co-existence of synchronization and anti-synchronization in above system can occur in more than one way and there are 14 different hybrid synchronization phenomena possible. Here we design active controllers so that two pairs of states are synchronized and the other two pairs are anti-synchronized. Numerical results verify that both synchronization and anti-synchronization can co-exist.

Here we consider two of the above mentioned cases and the hybrid synchronization errors are defined as:
For case I

$$e_i = y_i - x_i, i = 1,3$$

$$E_i = y_i + x_i, i = 2,4$$

and for Case II

$$e_i = y_i - x_i, i = 2,4$$
$$E_i = y_i + x_i, i = 1,3$$

Corresponding error dynamics for case I is given as:

$$\dot{e}_1 = e_2 + U_1(t) \tag{28}$$
$$\dot{E}_2 = 2\omega E_4 + \omega^2 E_1 + A_3 + A_1 + U_2(t) \tag{29}$$
$$\dot{e}_3 = e_4 + U_3(t) \tag{30}$$
$$\dot{E}_4 = -2\omega E_2 + \omega^2 E_3 + A_4 + A_2 + U_4(t) \tag{31}$$

And for case II as:

$$\dot{E}_1 = E_2 + U_1(t) \tag{32}$$
$$\dot{e}_2 = 2\omega e_4 + \omega^2 e_1 + A_3 - A_1 + U_2(t) \tag{33}$$
$$\dot{E}_3 = E_4 + U_3(t) \tag{34}$$
$$\dot{e}_4 = -2\omega e_2 + \omega^2 e_3 + A_4 - A_2 + U_4(t) \tag{35}$$

Let us redefine the control functions so that the terms in (28) to (35) which cannot be expressed as linear terms in e_i's and E_i's are eliminated :
For case I

$$U_1(t) = V_1(t)$$
$$U_2(t) = -A_3 - A_1 + V_2(t)$$
$$U_3(t) = V_3(t)$$
$$U_4(t) = -A_4 - A_2 + V_4(t)$$

The new error system can be expressed as:

$$\left.\begin{aligned} \dot{e}_1 &= e_2 + V_1(t) \\ \dot{E}_2 &= 2\omega E_4 + \omega^2 E_1 + V_2(t) \\ \dot{e}_3 &= e_4 + V_3(t) \\ \dot{E}_4 &= -2\omega E_2 + \omega^2 E_3 + V_4(t) \end{aligned}\right\} \tag{36}$$

For case II

$$U_1(t) = V_1(t)$$
$$U_2(t) = -A_3 + A_1 + V_2(t)$$
$$U_3(t) = V_3(t)$$
$$U_4(t) = -A_4 + A_2 + V_4(t)$$

The new error system can be expressed as:

$$\left.\begin{aligned} \dot{E}_1 &= E_2 + V_1(t) \\ \dot{e}_2 &= 2\omega e_4 + \omega^2 e_1 + V_2(t) \\ \dot{E}_3 &= E_4 + V_3(t) \\ \dot{e}_4 &= -2\omega e_2 + \omega^2 e_3 + V_4(t) \end{aligned}\right\} \tag{37}$$

The error system (36) and (37) are equivalent to the linear error system (17) and with the same choice of matrices A and B as given by (19) and (21) respectively, it can be proved by Lyanupov Stability theory and Routh-Hurwitz criterion that $\lim_{t\to\infty} |e_i| = 0; i = 1,3$ and $\lim_{t\to\infty} |E_i| = 0; i = 2,4$ for case I and $\lim_{t\to\infty} |E_i| = 0; i = 2,4$ and $\lim_{t\to\infty} |E_i| = 0; i = 1,3$ for case II. Hence here we achieved the hybrid synchronization where x_1 and x_3 are completely synchronized and x_2 and x_4 are anti synchronized for case I and for case II x_1 and x_3 are anti synchronized and x_2 and x_4 are completely synchronized. Time Series Analysis graphs of the above system are shown next to each via Figs. 15 to 22.

6. Application

Let us consider an example of Jupiter-Earth system in the restricted three body problem in which the bigger primary is taken as the Jupiter and small primary is the Earth. From the astrophysical data we have.

Mass of the Jupiter $m_1 = 1.89712 \times 10^{27}$

Mass of the Earth $m_2 = 5.977414 \times 10^{24}$

Distance between the Jupiter and Earth = 6286×10^8 km. Then $\mu = .996859$

Semi-major axis of Jupiter $a = .000010147$. Semi-minor axis of Jupiter $b = .0000101126$. Semi-minor axis of Jupiter $c = 0.00000112576$.

Moment of Inertia of Earth $I = 9.8100826 \times 10^{-13}$.

Then simulation results for uncoupled system are presented in Figs. 23, 25, 27, 29 and that of controlled system are shown in Figs. 24, 26, 28 and 30 for respectively. Time Series Analysis graphs of the anti-synchronization are shown next to each via Figs. 31 to 34.

Again the time Series Analysis graphs of the hybrid-synchronization behavior are shown in the following Figs. 35 to 42.

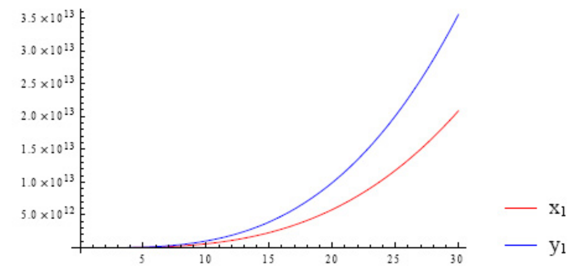

Fig. 23.

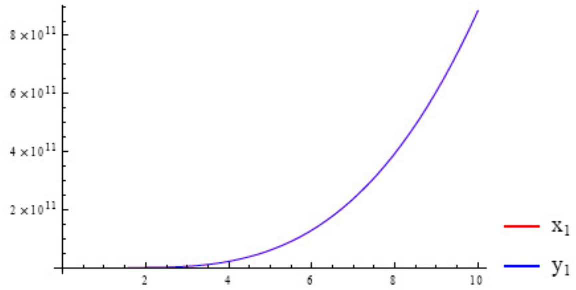

Fig. 24.

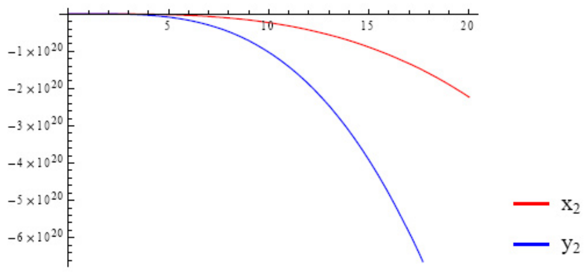

Fig. 25.

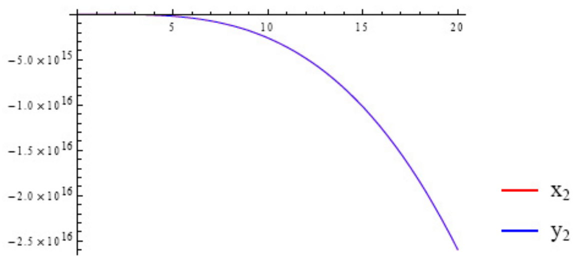

Fig. 26.

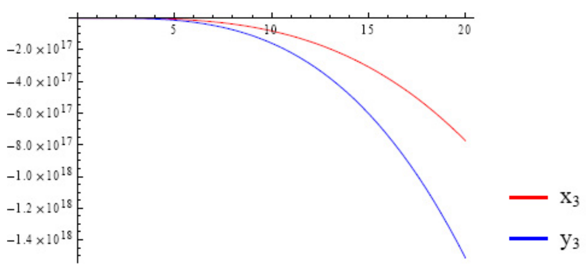

Fig. 27.

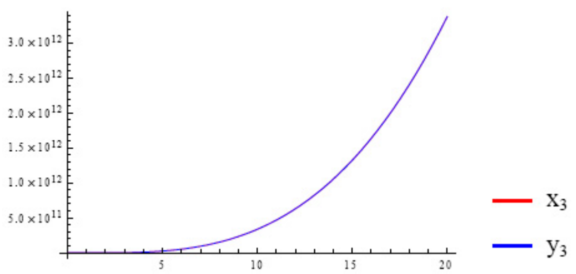

Fig. 28.

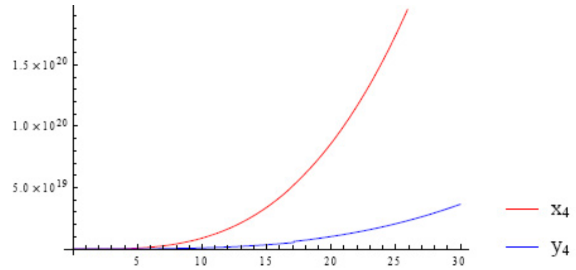

Fig. 29.

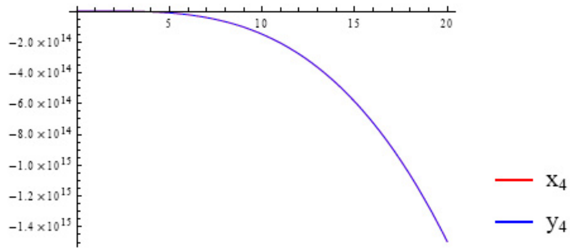

Fig. 30.

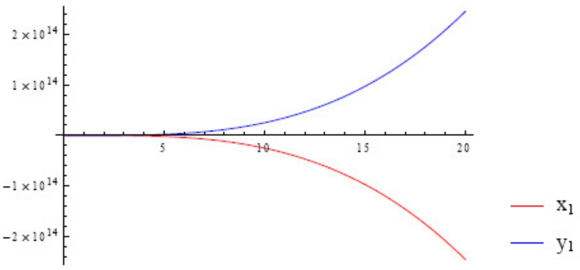

Fig. 31.

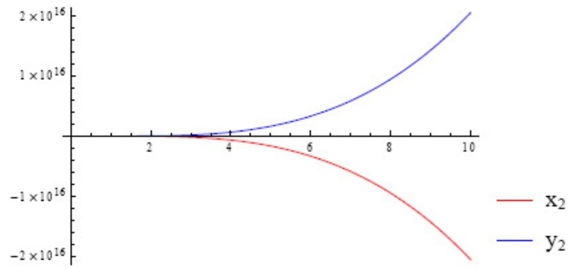

Fig. 32.

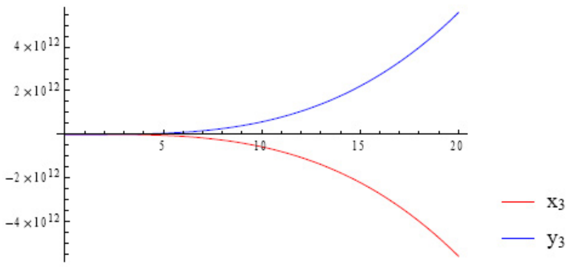

Fig. 33.

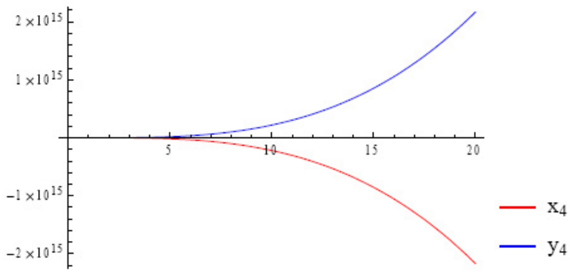

Fig. 34.

Case I

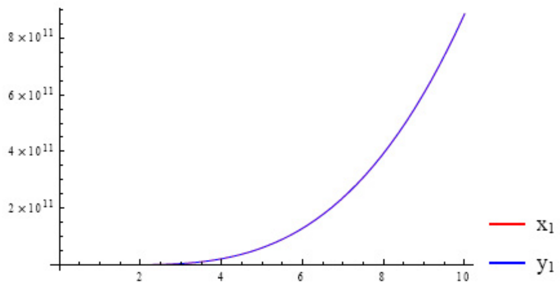

Fig. 35.

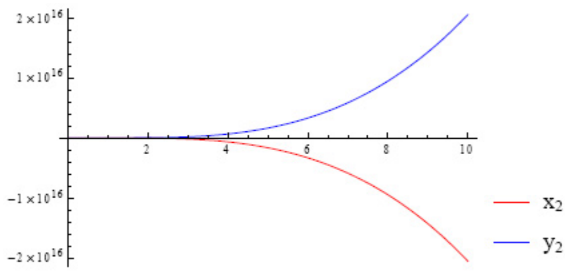

Fig. 36.

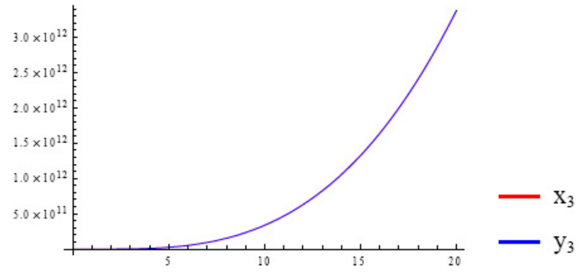

Fig. 37.

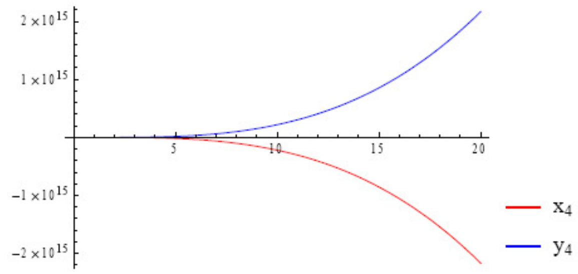

Fig. 38.

Case I

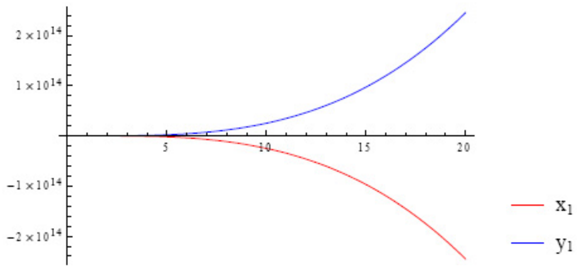

Fig. 39.

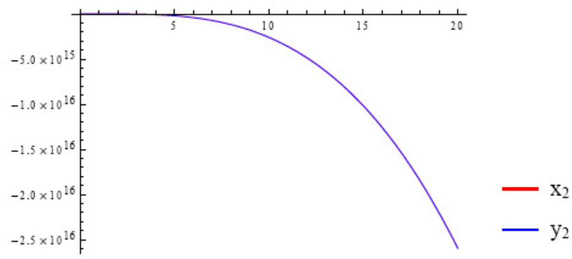

Fig. 40.

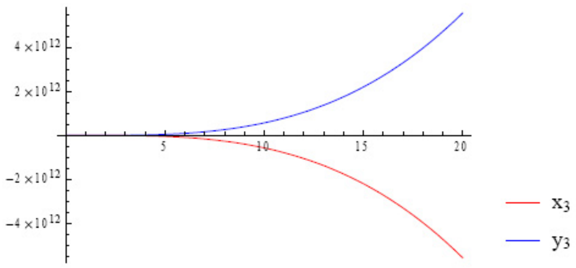

Fig. 41.

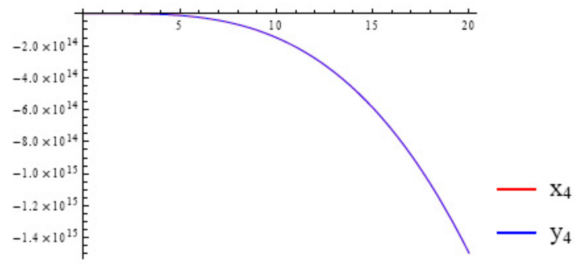

Fig. 42.

7. Conclusion

An investigation on complete synchronization anti synchronization and hybrid synchronization in the planar restricted three problem by taking into consideration the small primary is ellipsoid and bigger primary an oblate spheroid , via active control technique based on Lyapunov stability theory and Routh-Hurwitz criteria have been made. Here two systems (master and slave) are compete synchronized and start with deferent initial conditions. This problem may be treated as the design of control laws for chaotic slave system using the known information of the master system so as to ensure that the controlled receiver synchronizes with master system. Hence the slave chaotic system completely traces the dynamics of the master system in the course of time. The results were validated by numerical simulations using Mathematica.

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