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Influence of slippage in heat and mass transfer for fractionalized MHD flows in porous medium

Research Article

Kashif Ali Abro*, Muhammad Anwar Solangi, Muzaffar Hussain Laghari

Department of Basic Sciences and Related Studies, Mehran University of Engineering Technology, Jamshoro, Pakistan

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Abstract: This paper is devoted to investigate the analytical expressions for mass concentration, temperature and velocity field for free convective flows of fractionalized magnetohydrodynamics viscous fluid in porous medium with slippage. The corresponding general solutions have been perused by employing integral transforms (discrete Laplace transform with inverses) satisfying initial and boundary conditions separately for concentration, temperature and velocity field of fluid. Expressions of concentration, temperature and velocity field have been presented in terms of newly defined generalized $M_q^p(z)$ function.Fractionalized solutions have been converted into ordinary solutionsby replacing integer order time derivative along with special cases. The impacts of mass diffusion, thermal radiation and pertinent rheological and fractionalized parameters have been depicted by graphical illustrations.

MSC: 80A20 • 76S05 • 94A11

Keywords: Magnetohydrodynamics • Slippage and porosity • Viscous fluid • Analytical solutions with generalized $M_q^p(z)$ function

1. Introduction

The mutual impacts of heat and mass transfer on magnetohydrodynamics flow of viscous fluid in porous medium have diverted researchers due its huge importance and significance in science and engineering.Flow of free convection happens not only due to concentration differences but also dueto temperature differences or both them.Due to moving platethe problems of free convective flow has wide applications for instance, engineering devices and material processing systems such as heat exchangers, hot rolling, paper drying, extrusion of plastics, welding, atmospheric boundary layer flows, solar collectors, nuclear and electronic equipment reactors etc. The engineers and scientists have enhanced the analysis for magnetohydrodynamics and porous media for free convection mass and heat transfer. Because,it implements diverse applications for instance extraction of crude oil, chemical catalytic reactors, moisture over agricultural fields, pollution of the environment, damage of crops due to freezing, distributions of temperature, thermal insulation, etc. Investigation of Newtonian fluid for mixed convection flow of nanofluid has been studied in [1] . Hayat et al. [2] reported vertical stretching cylinder for viscoelastic fluid in mixed convection flow.Characteristics of some nanofluid flows in mass and heat transfer for vertical plate have been considered by Turkyilmazoglu and Pop [3]. Rashidi et al. examined stretching vertical surface in porous medium and magnetohydrodynamics fluid flow for free convective mass and heat transfer [4]. In the environmental heat transfer processes flow of free convection are common due to their importance in thermal boundary conditions. Soundalgekar has investigated semi-infinite isothermal vertical plate to the flow of a viscous fluid using integral transforms approach [5]. The vertical plate with

* Corresponding author.
E-mail address: kashif.abro@faculty.muet.edu.pk (Kashif Ali Abro).

ramped wall temperature for natural convection flows with heat absorptionin the presence of thermal diffusion has studied by Seth and Ansari [6]. The accelerating vertical plate for free convection flow with constant heat flux and heat absorption in presence of magnetohydrodynamics is analyzed by Narahari and Debnath [7]. In the present literature, the fractional derivative approach is rarely applied for the exact solutions of constitutive models of fluid motion. The motivation of this investigation is on the analysis for magnetohydrodynamics, porosity, Slippage, effects of radiation and mass transfer for free convective flow fractional derivative approach.The properties of molecular mediums and viscous fluid are sometimes become inaccessible by ordinary differential equations but conversely fractional differential equations investigate these properties in efficient manners. Recently, several researcher and authors have made valuable contributionsfor various problems using fractional derivatives approach which are referenced therein [8–20]. In this paper, our motto is to analyze the effects of magnetohydrodynamics, porosity, Slippage, effects of radiation and mass transfer for viscous fluid. The solutions for concentration, temperature and velocity field have been obtained in terms of series form and presented in newly defined M-function satisfying initial and boundary conditions. The general solutions are particularized for similar solutions and reduced for different limiting cases. The interesting expressions have been discussed for concentration, temperature and velocity field under the influences of magnetohydrodynamics, porosity, Slippage, effects of radiation and mass transfer. Finally rheological parameter and fractional parameters have been illustrated by depicting graphs showing various differences and similarities.

2. Modeling of the problem

Let us assume fractionalized, incompressible electrically conducting,free convective viscous fluid embedded in a porous medium lying on infinite accelerating plate with slippage. Initially fluid and plate are in rest at concentration C_∞ and temperature T_∞ in the x-axis of the Cartesian coordinate system. At time $t = 0^+$ (the system is at rest), the plate start toaccelerate in its own plane with the velocity $w(0,t) = UH(t)\,t^p + \Upsilon \left.\frac{\partial w(y,t)}{\partial t}\right|_{y=0}$ simultaneously the concentration and the temperature of the plate are raised up to C_W and T_W, magnetic field acts perpendicular in uniform strength are considered to the plate in normal direction.The following governing partial differential equations have been obtained for such flow by employing the Boussinesq's approximation and above assumptions [21–23]:

$$\frac{\partial^\alpha C(y,t)}{\partial t^\alpha} - \frac{1}{s_c} C_{yy}(y,t) = 0, \tag{1}$$

$$\frac{\partial^\beta w(y,t)}{\partial t^\beta} - w_{yy}(y,t) - G_m C(y,t) - G_r T(y,t) + B w(y,t) + \Phi w(y,t) = 0, \tag{2}$$

$$\frac{\partial^\gamma T(y,t)}{\partial t^\gamma} - \frac{1}{p_r} T_{yy}(y,t) + \frac{F}{p_r} T(y,t) = 0. \tag{3}$$

where $C(y,t), w(y,t), T(y,t), s_c, G_m, G_r, B, \Phi, F$ and p_r concentration of fluid, velocity profile, temperature field, Schmidth number, mass Grashof number, thermal Grashof number, magnetic field, porosity, thermal radiation parameter and Prandtl number respectively. The corresponding initial, boundary and natural conditions are

$$C(y,0) = w(y,0) = T(y,0) = 0; y \geq 0, \tag{4}$$

$$C(0,t) = t, T(0,t) = 1, \tag{5a}$$

$$w(0,t) = UH(t)\,t^p + \Upsilon \left.\frac{\partial w(y,t)}{\partial t}\right|_{y=0} t \geq 0, \tag{5b}$$

$$C(y,t) = w(y,t) = T(y,t) = 0; \quad y \to \infty, \tag{6}$$

Also α, β and γ are fractional parameters $0 \leq \alpha, \beta, \gamma < 1$, ans $\frac{\partial^\alpha}{\partial t^\alpha}, \frac{\partial^\beta}{\partial t^\beta}$ and $\frac{\partial^\gamma}{\partial t^\gamma}$ are the fractional differential operators so called Caputo fractional operator D_t^m defined by [24]

$$\frac{\partial^m f(t)}{\partial t^m} = \begin{cases} \frac{1}{\Gamma(1-m)} \int_0^t \frac{f'(q)}{(t-q)^m} dq, \\ \frac{df(t)}{dt}, \quad m = 1 \end{cases} \tag{7}$$

where, $0 < m < 1$ and $\Gamma(.)$ is the Gamma function.

3. Solution of the problem

3.1. Computation of concentration

In order to solve governing eq. (1), we applying the Laplace transform to Eq. (1), and taking into account the initial and boundary conditions (4) and (5a), we obtain

$$\bar{C}(y,s) = \frac{Exp\{-\sqrt{s_c s^{\alpha}}\}\,y}{s^2}, \tag{8}$$

where $\bar{C}(y,s)$ be the Laplace transform of $C(y,t)$, For finding $C(y,t)$, we write an equivalent form of Eq. (8) in series form,

$$\bar{C}(y,s) = \sum_{k=0}^{\infty} \frac{\left(-y\sqrt{s_c}\right)^k}{k!} \frac{1}{s^{2-\frac{k\alpha}{2}}}, \tag{9}$$

In order to obtain $C(y,t)$, we apply the discrete inverse Laplace transform on Eq. (9), we find that

$$C(y,t) = \sum_{k=0}^{\infty} \frac{\left(-y\sqrt{s_c}\right)^k}{k!} \frac{t^{1-\frac{k\alpha}{2}}}{\Gamma\left(2-\frac{k\alpha}{2}\right)}, \tag{10}$$

Writing an expression of concentration of fluid in terms of newly defined M-function $M_q^p(z)$,

$$C(y,t) = M_1^0\left[-y\sqrt{s_c}\left|\begin{array}{c}(0,0)\\(2,-\frac{\alpha}{2})\end{array}\right.\right], \tag{11}$$

In which property of M- function $M_q^p(z)$ is

$$t^{b_q-1}\sum_{n}^{\infty} \frac{z^n \prod_{j=1}^{p}\Gamma\left(a_j+A_j n\right)}{n!\prod_{j=1}^{q}\Gamma\left(b_j+B_j n\right)} == M_q^p\left[z\left|\begin{array}{c}(a_1,A_1),(a_2,A_2),\ldots,(a_p,A_p)\\(b_1,B_1),(b_2,B_2),\ldots,(b_q,B_q)\end{array}\right.\right] \tag{12}$$

3.2. Computation of temperature f1ield

In order to solve governing Eq. (3), weapplying the Laplace transform to Eq. (3), and taking into account the initial and boundary conditions (4) and (5b), we obtain

$$\bar{T}(y,s) = \frac{Exp\{-\sqrt{F+p_r s^{\gamma}}\}\,y}{s}, \tag{13}$$

Rewriting Eq. (13) into suitable form

$$\bar{T}(y,s) = \frac{1}{q} + \sum_{m=1}^{\infty} \frac{\left(-y\sqrt{p_r}\right)^m}{m!} \sum_{k=0}^{\infty}\left(\frac{-F}{p_r}\right)^k \times \frac{\Gamma\left(\frac{m}{2}+1\right)}{k!\Gamma\left(\frac{m}{2}-k+1\right)} \frac{1}{s^{\gamma k-\frac{m\gamma}{2}+1}}, \tag{14}$$

Employing discrete inverse Laplace transform to Eq. (14), we get

$$T(y,t) = 1 + \sum_{m=1}^{\infty} \frac{\left(-y\sqrt{p_r}\right)^m}{m!} \sum_{k=0}^{\infty}\left(\frac{-F}{p_r}\right)^k \times \frac{\Gamma\left(\frac{m}{2}+1\right) t^{\gamma k-\frac{\gamma m}{2}}}{k!\Gamma\left(\frac{m}{2}-k+1\right)\Gamma\left(\gamma k-\frac{\gamma m}{2}+1\right)}, \tag{15}$$

Eq. (15) can also be written in terms of newly defined M-function $M_q^p(z)$,

$$T(y,t) = 1 + \sum_{m=1}^{\infty} \frac{\left(-y\sqrt{p_r}\right)^m}{m!} \times M_2^1\left[-\frac{F}{p_r}\left|\begin{array}{c}(\frac{m}{2}+1,0)\\(\frac{m}{2}+1,-1)(-\frac{\gamma m}{2}+1,\gamma)\end{array}\right.\right], \tag{16}$$

3.3. Computation of velocity profile

For computing the solution of velocity profile, we apply Laplace transform to Eq. (2) and keeping in mind Eqs. (4) and (5), we get

$$s^{\beta}\bar{w}(y,s)-\bar{w}_{yy}(y,s)-G_m\bar{C}(y,s)-G_r\bar{T}(y,s)+B\bar{w}(y,s)+\Phi\bar{w}(y,s)=0, \tag{17}$$

Eq. (17) has to justify boundary conditions

$$\bar{u}(0,s)=\frac{Up!}{s^{p+1}}+\Upsilon\left.\frac{\partial\bar{w}(y,s)}{\partial t}\right|_{y=0}, \tag{18}$$

Employing Eqs. (8), (13) and (18) in Eq. (17), we find that

$$\bar{w}(y,s)=\frac{Up!Exp\left\{-\sqrt{s^{\beta}-\Phi-\mathrm{B}}\right\}y}{s^{p+1}\left(1+\Upsilon\sqrt{s^{\beta}-\Phi-\mathrm{B}}\right)}+\frac{G_rExp\{-\sqrt{F+p_rs^{\gamma}}\}y}{s\left(s^{\beta}-\Phi-\mathrm{B}\right)}+\frac{G_mExp\{-\sqrt{s_cs^{\alpha}}\}y}{s^2\left(s^{\beta}-\Phi-\mathrm{B}\right)}, \tag{19}$$

Now expanding Eq. (19) in terms of series form, we find analytical expression

$$\bar{w}(y,s)=\frac{Up!}{s^{p+1}}+Up!\sum_{l=0}^{\infty}\frac{(\Phi+B)^l\Gamma\left(1+\frac{k}{2}\right)}{l!\Gamma\left(1+\frac{k}{2}-l\right)}\times M_3^2\left[(\Phi)\left|\begin{matrix}\left(1+\frac{m}{2},0\right)(1+k,0)\\ \left(1+\frac{m}{2}-j,0\right)(1+k,-1)\left(\gamma k-\frac{m\gamma}{2}+1,\beta\right)\end{matrix}\right.\right]$$
$$+\sum_{m=0}^{\infty}G_m\left(\frac{-y\sqrt{S_c}}{m!}\right)^m\sum_{k=0}^{\infty}(-1)^k\times M_2^1\left[(\Phi)\left|\begin{matrix}(1+k,0)\\(1+k,-1)\left(\beta k-\frac{m\alpha}{2}+2,\beta\right)\end{matrix}\right.\right]. \tag{20}$$

Finally applying discrete inverse Laplace transform to Eq. (20), we find velocity profile in terms of newly defined M-function as

$$w(y,t)=UH(t)\,t^p+Up!\sum_{k=1}^{\infty}(-\Upsilon)^k\times M_2^1\left[(\Phi+\mathrm{B})\left|\begin{matrix}\left(1+\frac{k}{2},0\right)\\ \left(1+\frac{k}{2},-1\right)\left(\frac{\beta k}{2}+p+1,\beta\right)\end{matrix}\right.\right]+Up!\sum_{m=1}^{\infty}\left(\frac{-y}{m!}\right)^m\sum_{k=0}^{\infty}(-\Upsilon)^k\sum_{j=0}^{\infty}\frac{(\Phi+\mathrm{B})^j}{j!}$$
$$\times M_3^2\left[(\Phi+\mathrm{B})\left|\begin{matrix}\left(1+\frac{k}{2},0\right),\left(1+\frac{k}{2},0\right)\\ \left(1+\frac{k}{2},-1\right),\left(1+\frac{k}{2}-j,0\right),(\beta j-\beta k+p+1,\beta)\end{matrix}\right.\right]\times\sum_{k=0}^{\infty}G_r(-1)^k\sum_{m=0}^{\infty}\left(\frac{-y\sqrt{Fp_r}}{m!}\right)^m\sum_{j=0}^{\infty}\frac{1}{j!}\left(\frac{-1}{F}\right)^j$$
$$+\sum_{m=0}^{\infty}G_m\left(\frac{-y\sqrt{S_c}}{m!}\right)^m\sum_{k=0}^{\infty}(-1)^k\times M_2^1\left[(\Phi+\mathrm{B})\left|\begin{matrix}(1+k,0)\\(1+k,-1)\left(\beta k-\frac{m\alpha}{2}+2,\beta\right)\end{matrix}\right.\right] \tag{21}$$

4. Limiting case

4.1. Velocity field with slip effect without magnetic field $B\to 0$

The absence of magnetic field i.e. $B\to 0$ into Eq. (19) the solution is

$$\bar{w}(y,s)=\frac{Up!Exp\left\{-\sqrt{s^{\beta}-\Phi}\right\}y}{s^{p+1}\left(1+\Upsilon\sqrt{s^{\beta}-\Phi}\right)}+\frac{G_rExp\{-\sqrt{F+p_rs^{\gamma}}\}y}{s\left(s^{\beta}-\Phi\right)}+\frac{G_mExp\{-\sqrt{s_cs^{\alpha}}\}y}{s^2\left(s^{\beta}-\Phi\right)}, \tag{22}$$

Expanding and writing Eq. (22) in terms of series form and M-function, we have final following expression.

$$w(y,t)=UH(t)\,t^p+Up!\sum_{k=1}^{\infty}(-\Upsilon)^k\times M_2^1\left[(\Phi)\left|\begin{matrix}\left(1+\frac{k}{2},0\right)\\ \left(1+\frac{k}{2},-1\right)\left(\frac{\beta k}{2}+p+1,\beta\right)\end{matrix}\right.\right]$$
$$+Up!\sum_{m=1}^{\infty}\left(\frac{-y}{m!}\right)^m\sum_{k=0}^{\infty}(-\Upsilon)^k\sum_{j=0}^{\infty}\frac{(\Phi+\mathrm{B})^j}{j!}\times M_3^2\left[(\Phi)\left|\begin{matrix}\left(1+\frac{k}{2},0\right)\left(1+\frac{k}{2},0\right)\\ \left(1+\frac{k}{2},-1\right)\left(1+\frac{k}{2}-j,0\right)(\beta j-\beta k+p+1,\beta)\end{matrix}\right.\right]$$
$$\times\sum_{k=0}^{\infty}G_r(-1)^k\sum_{m=0}^{\infty}\left(\frac{-y\sqrt{Fp_r}}{m!}\right)^m\sum_{j=0}^{\infty}\frac{1}{j!}\left(\frac{-1}{F}\right)^j\times M_3^2\left[(\Phi)\left|\begin{matrix}\left(1+\frac{m}{2},0\right)(1+k,0)\\ \left(1+\frac{m}{2}-j,0\right)(1+k,-1)\left(\gamma k-\frac{m\gamma}{2}+1,\beta\right)\end{matrix}\right.\right]$$
$$+\sum_{m=0}^{\infty}G_m\left(\frac{-y\sqrt{S_c}}{m!}\right)^m\sum_{k=0}^{\infty}(-1)^k\times M_2^1\left[(\Phi)\left|\begin{matrix}(1+k,0)\\(1+k,-1)\left(\beta k-\frac{m\alpha}{2}+2,\beta\right)\end{matrix}\right.\right] \tag{23}$$

4.2. Velocity field with slip effect without porous effect $\Phi \to 0$

The absence of porosity i.e. $\phi \to 0$ into Eq. (19) the solution is

$$\bar{w}(y,s) = \frac{Up! Exp\left\{-\sqrt{s^{\beta}-B}\right\} y}{s^{p+1}\left(1+\Upsilon\sqrt{s^{\beta}-B}\right)} + \frac{G_r Exp\{-\sqrt{F+p_r s^{\gamma}}\} y}{s\left(s^{\beta}-B\right)} + \frac{G_m Exp\{-\sqrt{s_c s^{\alpha}}\} y}{s^2\left(s^{\beta}-B\right)}, \tag{24}$$

Expanding and writing Eq. (24) in terms of series form and M-function, we have final following expression

$$\begin{aligned} w(y,t) &= UH(t)\, t^p + Up! \sum_{k=1}^{\infty} (-\Upsilon)^k \times M_2^1 \left[(B) \middle| \begin{matrix} \left(1+\frac{k}{2},0\right) \\ \left(1+\frac{k}{2},-1\right)\left(\frac{\beta k}{2}+p+1,\beta\right) \end{matrix} \right] \\ &+ Up! \sum_{m=1}^{\infty} \left(\frac{-y}{m!}\right)^m \sum_{k=0}^{\infty} (-\Upsilon)^k \sum_{j=0}^{\infty} \frac{(\Phi+B)^j}{j!} \times M_3^2 \left[(B) \middle| \begin{matrix} \left(1+\frac{k}{2},0\right)\left(1+\frac{k}{2},0\right) \\ \left(1+\frac{k}{2},-1\right)\left(1+\frac{k}{2}-j,0\right)\left(\beta j-\beta k+p+1,\beta\right) \end{matrix} \right] \\ &\times \sum_{k=0}^{\infty} G_r (-1)^k \sum_{m=0}^{\infty} \left(\frac{-y\sqrt{Fp_r}}{m!}\right)^m \sum_{j=0}^{\infty} \frac{1}{j!}\left(\frac{-1}{F}\right)^j \times M_3^2 \left[(B) \middle| \begin{matrix} \left(1+\frac{m}{2},0\right)(1+k,0) \\ \left(1+\frac{m}{2}-j,0\right)(1+k,-1)\left(\gamma k-\frac{m\gamma}{2}+1,\beta\right) \end{matrix} \right] \\ &+ \sum_{m=0}^{\infty} G_m \left(\frac{-y\sqrt{S_c}}{m!}\right)^m \sum_{k=0}^{\infty} (-1)^k \times M_2^1 \left[(B) \middle| \begin{matrix} (1+k,0) \\ (1+k,-1)\left(\beta k-\frac{m\alpha}{2}+2,\beta\right) \end{matrix} \right] \end{aligned} \tag{25}$$

4.3. Velocity field with slip effect without radiative heat flux $F \to 0$

Letting $F \to 0$ into Eq. (19) the solution is

$$\begin{aligned} w(y,t) &= UH(t)\, t^p + Up! \sum_{k=1}^{\infty} (-\Upsilon)^k \times M_2^1 \left[(B) \middle| \begin{matrix} \left(1+\frac{k}{2},0\right) \\ \left(1+\frac{k}{2},-1\right)\left(\frac{\beta k}{2}+p+1,\beta\right) \end{matrix} \right] \\ &+ Up! \sum_{m=1}^{\infty} \left(\frac{-y}{m!}\right)^m \sum_{k=0}^{\infty} (-\Upsilon)^k \sum_{j=0}^{\infty} \frac{(\Phi+B)^j}{j!} \times M_3^2 \left[(B) \middle| \begin{matrix} \left(1+\frac{k}{2},0\right)\left(1+\frac{k}{2},0\right) \\ \left(1+\frac{k}{2},-1\right)\left(1+\frac{k}{2}-j,0\right)\left(\beta j-\beta k+p+1,\beta\right) \end{matrix} \right] \\ &\times M_3^2 \left[(B) \middle| \begin{matrix} \left(1+\frac{k}{2},0\right)\left(1+\frac{k}{2},0\right) \\ \left(1+\frac{k}{2},-1\right)\left(1+\frac{k}{2}-j,0\right)\left(\beta j-\beta k+p+1,\beta\right) \end{matrix} \right] \times M_2^1 \left[(B) \middle| \begin{matrix} (1+k,0) \\ (1+k,-1)\left(\beta k-\frac{m\alpha}{2}+2,\beta\right) \end{matrix} \right], \end{aligned} \tag{26}$$

4.4. Concentration, temperature and velocity field for ordinary differential operator when $\alpha \to 1, \beta \to 1, \gamma \to 1$

Setting $\alpha \to 1, \beta \to 1, \gamma \to 1$ into Eqs. (11), (16) and (21) the solutions are

$$C(y,t) = M_1^0 \left[-y\sqrt{S_c} \middle| \begin{matrix} (0,0) \\ \left(2,-\frac{1}{2}\right) \end{matrix} \right], \tag{27}$$

$$T(y,t) = 1 + \sum_{m=1}^{\infty} \frac{\left(-y\sqrt{p_r}\right)^m}{m!} \times M_2^1 \left[-\frac{F}{p_r} \middle| \begin{matrix} \left(\frac{m}{2}+1,0\right) \\ \left(\frac{m}{2}+1,-1\right)\left(-\frac{m}{2}+1,1\right) \end{matrix} \right], \tag{28}$$

$$\begin{aligned} w(y,t) &= UH(t)\, t^p + Up! \sum_{k=1}^{\infty} (-\Upsilon)^k \times M_2^1 \left[(\Phi+B) \middle| \begin{matrix} \left(1+\frac{k}{2},0\right) \\ \left(1+\frac{k}{2},-1\right)\left(\frac{k}{2}+p+1,1\right) \end{matrix} \right] \\ &+ Up! \sum_{m=1}^{\infty} \left(\frac{-y}{m!}\right)^m \sum_{k=0}^{\infty} (-\Upsilon)^k \sum_{j=0}^{\infty} \frac{(\Phi+B)^j}{j!} \times M_3^2 \left[(\Phi+B) \middle| \begin{matrix} \left(1+\frac{k}{2},0\right)\left(1+\frac{k}{2},0\right) \\ \left(1+\frac{k}{2},-1\right)\left(1+\frac{k}{2}-j,0\right)\left(j-k+p+1,1\right) \end{matrix} \right] \\ &\times \sum_{k=0}^{\infty} G_r (-1)^k \sum_{m=0}^{\infty} \left(\frac{-y\sqrt{Fp_r}}{m!}\right)^m \sum_{j=0}^{\infty} \frac{1}{j!}\left(\frac{-1}{F}\right)^j \times \sum_{k=0}^{\infty} G_r (-1)^k \sum_{m=0}^{\infty} \left(\frac{-y\sqrt{Fp_r}}{m!}\right)^m \sum_{=0}^{j\infty} \frac{1}{j!}\left(\frac{-1}{F}\right)^j \\ &\times M_3^2 \left[(\Phi+B) \middle| \begin{matrix} \left(1+\frac{m}{2},0\right)(1+k,0) \\ \left(1+\frac{m}{2}-j,0\right)(1+k,-1)\left(k-\frac{m}{2}+1,1\right) \end{matrix} \right] \\ &+ \sum_{m=0}^{\infty} G_m \left(\frac{-y\sqrt{S_c}}{m!}\right)^m \sum_{k=0}^{\infty} (-1)^k \times M_2^1 \left[(\Phi+B) \middle| \begin{matrix} (1+k,0) \\ (1+k,-1)\left(k-\frac{m}{2}+2,1\right) \end{matrix} \right], \end{aligned} \tag{29}$$

5. Concluding remarks

The motto of this analysis is to investigate the exact solution for mass concentration, temperature and velocity field for free convective flows of fractionalized magnetohydrodynamics viscous fluid in porous medium with slippage. The governing partial differential equations have been solved by using transformation technique (Laplace) and considered for time fractionalized differential equations of order $\alpha, \beta, \gamma \in (0, 1]$. The general solutions for mass concentration, temperature and velocity field are expressed in terms of series form and written in terms of newly defined generalized $M_q^p(z)$ function. The solutions have been particularized as the special casesin four models namely

(i) when $\Upsilon \to 0$ general solutions are considered without slippage,

(ii) when $\Phi \to 0$ general solutions are considered without porous effects,

(iii) when $B \to 0$ general solutions are considered without magnetic field and

(iv) when $\alpha \to 1, \beta \to 1, \gamma \to 1$ general solutions are considered without fractionalized form i.e. in terms of ordinary differential equations.

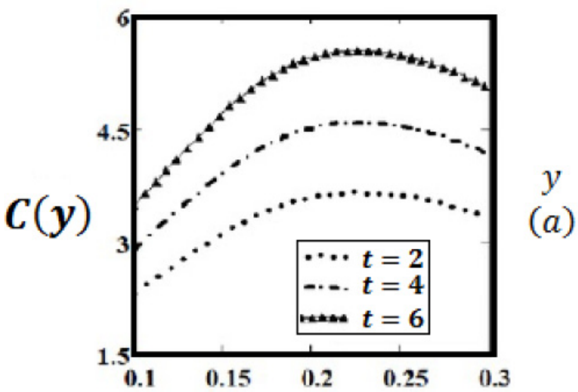

Fig. 1. Concentration profiles for different values of t, at $S_c = 0.2$ and $\alpha = 0.2$.

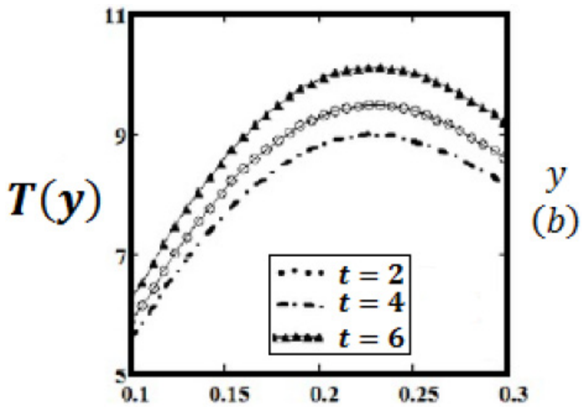

Fig. 2. Temperature profiles for different values of t, at $P_r = 0.5, F = 1$ and $\gamma = 0.2$.

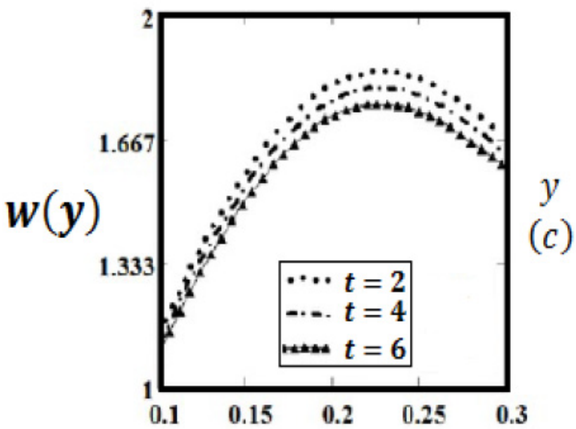

Fig. 3. Velocity profiles for different values of t, at $G_r = 0.5, G_m = 0.25, B = 2, F = 0.7, \Phi = 3.5$ and $\alpha = 0.2$.

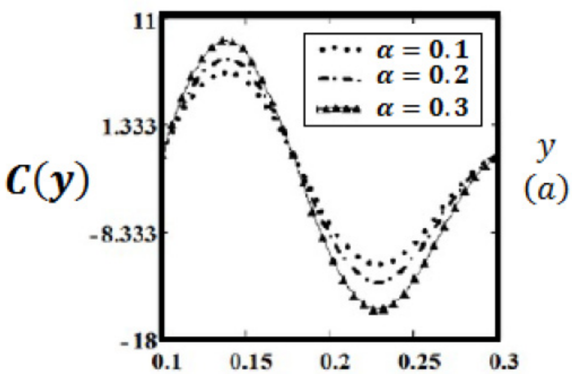

Fig. 4. Concentration profiles for different values of α at $S_c = 0.2$ and $t = 2s$.

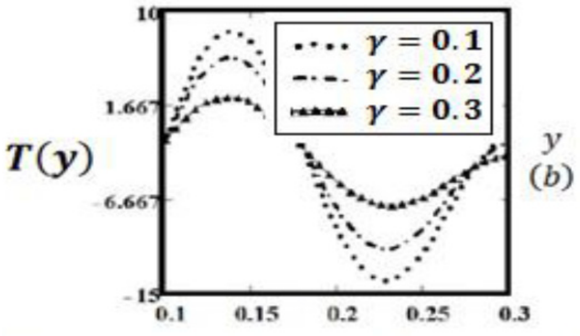

Fig. 5. Temperature profiles for different values of γ at $P_r = 3, F = 2.5$ and $t = 2s$.

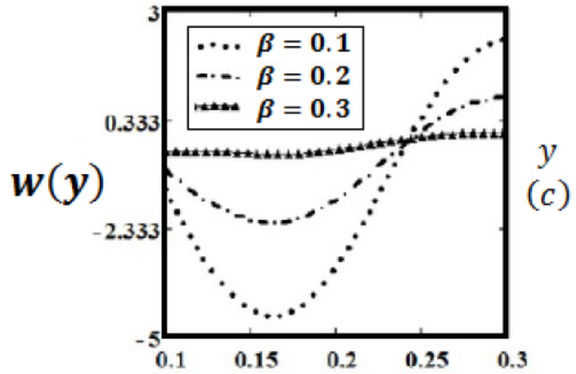

Fig. 6. Velocity profiles for different values of β at $G_r = 1.3, G_m = 1.11, B = 0.2, F = 2, \Phi = 0.5$ and $t = 2S$.

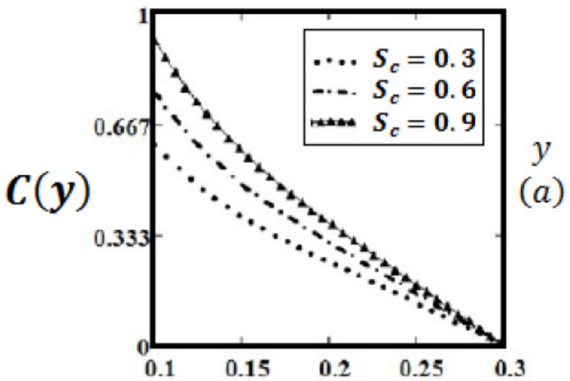

Fig. 7. Concentration profiles for different values of S_c at $\alpha = 0.6$ and $t = 2s$.

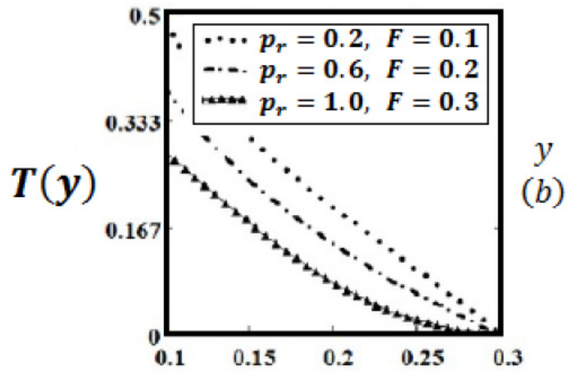

Fig. 8. Temperature profiles for different values of P_r, F at $\gamma = 2.5$ and $t = 2s$.

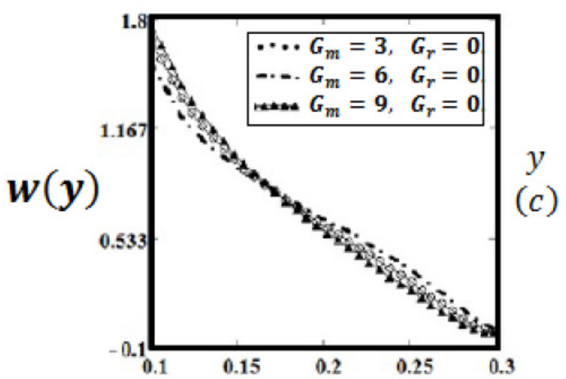

Fig. 9. Velocity profiles for different values of G_r and G_m at $\beta = 0.2, B = 0.2, F = 2, \Phi = 0.5$ and $t = 2S$.

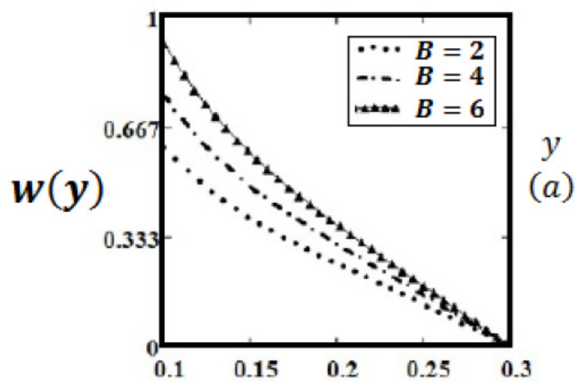

Fig. 10. Velocity profiles for different values of B at $G_r = 0.1, G_m = 1.25, \beta = 0.2, F = 2, \Phi = 0.5$ and $t = 2S$.

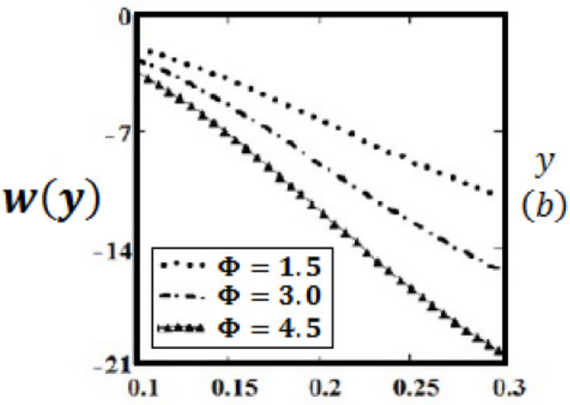

Fig. 11. Velocity profiles for different values of Φ at $G_r = 0.1, G_m = 1.25, \beta = 0.2, F = 2, B = 0.5$ and $t = 2S$.

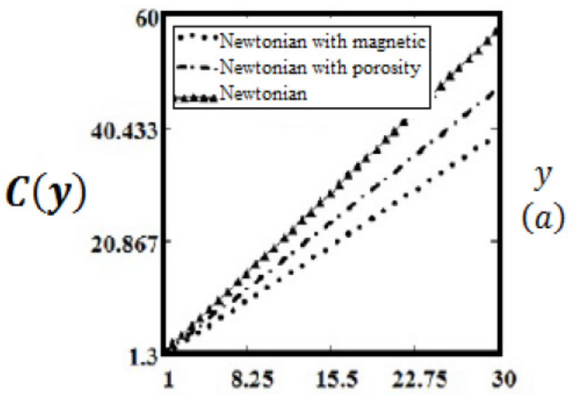

Fig. 12. Comparision of concentration profile for Newtonian with megnatic field, Newtonian with porosity and Newtonian.

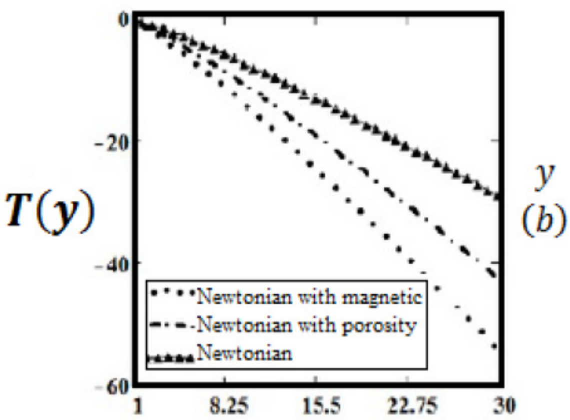

Fig. 13. Comparision of temperature profile for Newtonian with megnatic field, Newtonian with porosity and Newtonian.

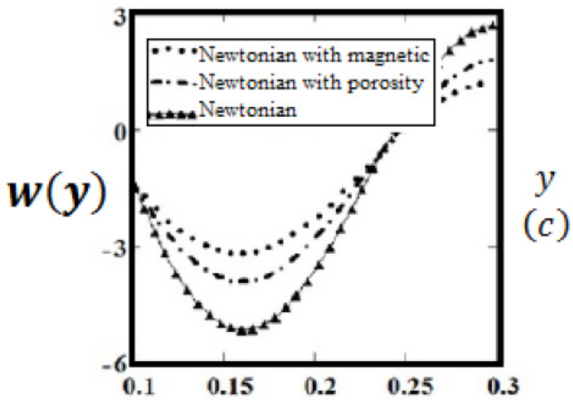

Fig. 14. Comparision of velocity profile for Newtonian with megnatic field, Newtonian with porosity and Newtonian.

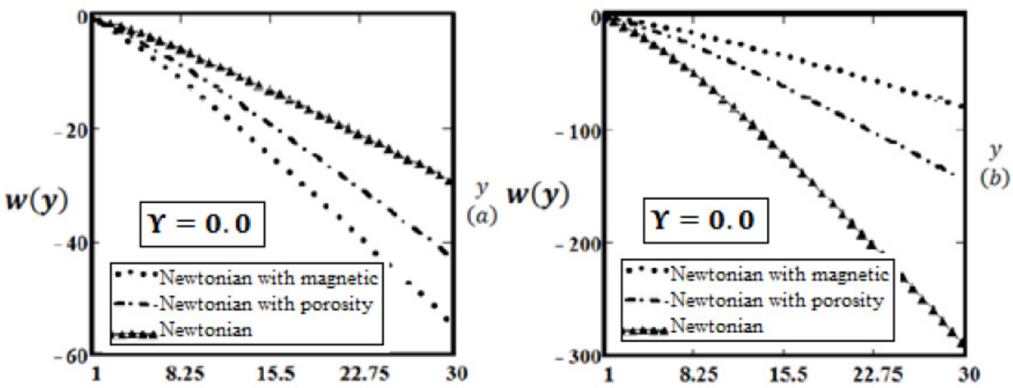

Fig. 15. Comparision of velocity profiles with slip and without slip assumptions for Newtonian with megnatic field, Newtonian with porosity and Newtonian fluids.

In continuation, these solutions are predicted for the influences of fractionalized and rheological parameters along with few non-dimensional numbers. The effects on the fluid motion are shown in Figs. 1-3 in which as time increases the mass concentration has scattering behavior on fluid as compared to velocity field and temperature distribution. Figs. 4-5 are drawn to illustrate the influence of fractional parameters in which mass concentration and temperature distribution has oscillating effects on fluid motion but velocity field has converse oscillations as expected. Fig. 7 is depicted to show the impacts of Schmidth number, as Schmidth number increases the mass concentration is decreasing function, on contrary Fig. 8 as thermal radiation parameter and Prandtl number increase the temperature distribution have increasing behavior on fluid flow.The illustrations of mass Grashof number and thermal Grashof numberhave been analyzed in Fig. 9 by fixing rheological and fractionalized parameters. It is observed in Fig. 10 that the velocity field has slowdown the fluid flow due to increasing magnetic field, furthermore reciprocal behavior is noted due to permeability and porosity in Fig. 11. Figs. 12-14 have been drawn for comparison among mass concentration, temperature distribution and velocity profile for three models i-e, Newtonian with magnetic field, Newtonian with porous and Newtonian in which temperature distribution and mass concentration show reverse behavior and velocity profile has oscillating behavior under no slip assumptions. Fig. 15 represents the profile of velocity field under the existence of slip effects and no slip effects having opposite behavior respectively. Same can be done for mass concentration and temperature distribution.

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