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Temperature in a non-homogeneous bar and the multivariable Aleph-function

Research Article

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Abstract: Chaurasia [2] has used the Fox's H-function and the multivariable H-function defined by Srivastava et al. [8, 9] and has obtained a solution to a problem of heat conduction. The object of this paper is to employ Aleph-function and the multivariable Aleph-function in obtaining a solution of the partial differential equation

$$\frac{\partial\theta}{\partial t}=b\frac{\partial}{\partial x}\left[(1-x^2)\frac{\partial\theta}{\partial x}\right]$$

related to a problem of heat conduction. The result yields a number of particular cases on specialising the parameters and may prove to be useful in several interesting situations appearing in the literature on physics and mechanics. We shall see the particular cases concerning the Aleph-function of two variables, the I-function of two variables and the multivariable H-function.

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1. Introduction and preliminaries

As an exemple of the application of this multivariable Aleph-function in applied mathematics we shall consider the problem of determining a function $\theta(x,t)$ representing the temperature in a non-homogeneous bar with ends at $X=\pm1$ in which the thermal diffusivity is proportional to $(1-x^2)$ and if the lateral surface on the bar is insulated, it satisfies the partial differential equation of heat conduction, see Churchill [4].

$$\frac{\partial\theta}{\partial t}=b\frac{\partial}{\partial x}\left[(1-x^2)\frac{\partial\theta}{\partial x}\right] \tag{1}$$

where b is a constant, provided thermal coefficient is constant. The boundary conditions of the problem are that both ends of a bar at $u=\pm1$ are also insulated because the conductivity vanishes there ; and the initial conditions

$$\theta(x,0)=f(x), -1<x<1 \tag{2}$$

Here we shall consider

$$f(x)=(1-x)^{\sigma}\aleph^{M,N}_{P_i,Q_i,c_i;r'}\left(z(1-x)^h\ \middle|\ \begin{matrix}(a_j,A_j)_{1,\mathfrak{n}},[c_i(a_{ji},A_{ji})]_{\mathfrak{n}+1,p_i;r'}\\(b_j,B_j)_{1,m},[c_i(b_{ji},B_{ji})]_{m+1,q\_i;r'}\end{matrix}\right)\aleph^{0,\mathfrak{n}:V}_{U:W}\begin{pmatrix}z_1(1-x)^{k_1}\\ \vdots\\ z_r(1-x)^{k_r}\end{pmatrix} \tag{3}$$

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The multivariable Aleph-function is an extension of the multivariable I-function recently defined by C.K. Sharma and Ahmad [6], itself is a generalization of the multivariable H-function defined by Srivastava et al. [8]. The multivariable Aleph-function is defined by means of the multiple contour integral. We have

$$\aleph(z_1,\cdots,z_r)=\aleph^{0,\mathfrak{n}:m_1,n_1,\cdots,m_r,n_r}_{p_i,q_i,\tau_i;R:p_{i^{(1)}},q_{i^{(1)}},\tau_{i^{(1)}};R^{(1)};\cdots;p_{i^{(r)}},q_{i^{(r)}};\tau_{i^{(r)}};R^{(r)}}\left(\begin{array}{c} z_1 \\ \vdots \\ z_r \end{array}\middle| \begin{array}{l} [(a_j;\alpha_j^{(1)},\cdots,\alpha_j^{(r)})_{1,\mathfrak{n}}],[\tau_i(a_{ji};\alpha_{ji}^{(1)},\cdots,\alpha_{ji}^{(r)})_{\mathfrak{n}+1,p_i}]: \\ \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots,[\tau_i(b_{ji};\beta_{ji}^{(1)},\cdots,\beta_{ji}^{(r)})_{m+1,q_i}]: \end{array}\right.$$

$$\left.\begin{array}{l} [(c_j^{(1)}),\gamma_j^{(1)})_{1,n_1}],[\tau_{i^{(1)}}(c_{ji^{(1)}}^{(1)}\},\gamma_{ji^{(1)}}^{(1)})_{n_1+1,p_i^{(1)}}];\cdots;[(c_j^{(r)}),\gamma_j^{(r)})_{1,n_r}],[\tau_{i^{(r)}}(c_{ji^{(r)}}^{(r)},\gamma_{ji^{(r)}}^{(r)})_{n_r+1,p_i^{(r)}}] \\ [(d_j^{(1)}),\delta_j^{(1)})_{1,m_1}],[\tau_{i^{(1)}}(d_{ji^{(1)}}^{(1)},\delta_{ji^{(1)}}^{(1)})_{m_1+1,q_i^{(1)}}];\cdots;[(d_j^{(r)}),\delta_j^{(r)})_{1,m_r}],[\tau_{i^{(r)}}(d_{ji^{(r)}}^{(r)},\delta_{ji^{(r)}}^{(r)})_{m_r+1,q_i^{(r)}}] \end{array}\right)$$

$$=\frac{1}{(2\pi\omega)^r}\int_{L_1}\cdots\int_{L_r}\psi(s_1,\cdots,s_r)\prod_{k=1}^{r}\theta_k(s_k)z_k^{s_k}\,ds_1\cdots ds_r \tag{4}$$

with $\omega=\sqrt{-1}$.

$$\psi(s_1,\cdots,s_r)=\frac{\prod_{j=1}^{\mathfrak{n}}\Gamma\left(1-a_j+\sum_{k=1}^{r}\alpha_j^{(k)}s_k\right)}{\sum_{i=1}^{R}\left[\tau_i\prod_{j=n+1}^{p_i}\Gamma\left(a_{ji}-\sum_{k=1}^{r}\alpha_{ji}^{(k)}s_k\right)\prod_{j=1}^{q_i}\Gamma\left(1-b_{ji}+\sum_{k=1}^{r}\beta_{ji}^{(k)}s_k\right)\right]} \tag{5}$$

and

$$\theta_k(s_k)=\frac{\prod_{j=1}^{m_k}\Gamma\left(d_j^{(k)}-\delta_j^{(k)}s_k\right)\prod_{j=1}^{n_k}\Gamma\left(1-c_j^{(k)}+\gamma_j^{(k)}s_k\right)}{\sum_{i^{(k)}=1}^{R^{(k)}}\left[\tau_{i^{(k)}}\prod_{j=m_k+1}^{q_{i^{(k)}}}\Gamma\left(1-d_{ji^{(k)}}^{(k)}+\delta_{ji^{(k)}}^{(k)}s_k\right)\prod_{j=n_k+1}^{p_{i^{(k)}}}\Gamma\left(c_{ji^{(k)}}^{(k)}-\gamma_{ji^{(k)}}^{(k)}s_k\right)\right]} \tag{6}$$

For more details, see Ayant [1]. The condition for absolute convergence of multiple Mellin-Barnes type contour can be obtained by extension of the corresponding conditions for multivariable H-function given by as,

$$\left|arg z_k\right|<\frac{1}{2}A_i^{(k)}\pi,\text{where}$$

$$A_i^{(k)}=\sum_{j=1}^{\mathfrak{n}}\alpha_j^{(k)}-\tau_i\sum_{j=\mathfrak{n}+1}^{p_i}\alpha_{ji}^{(k)}-\tau_i\sum_{j=1}^{q_i}\beta_{ji}^{(k)}+\sum_{j=1}^{n_k}\gamma_j^{(k)}-\tau_{i^{(k)}}\sum_{j=n_k+1}^{p_{i^{(k)}}}\gamma_{ji^{(k)}}^{(k)}$$
$$+\sum_{j=1}^{m_k}\delta_j^{(k)}-\tau_{i^{(k)}}\sum_{j=m_k+1}^{q_{i^{(k)}}}\delta_{ji^{(k)}}^{(k)}>0,\ k=1,\cdots,r,i=1,\cdots,R,i^{(k)}=1,\cdots,R^{(k)} \tag{7}$$

The complex numbers z_i are not zero.Throughout this document , we assume the existence and absolute convergence conditions of the multivariable Aleph-function.

We may establish the the asymptotic expansion in the following convenient form:

$$\aleph(z_1,\cdots,z_r)=0\left(|z_1|^{\alpha_1},\cdots,|z_r|^{\alpha_r}\right),\max\left(|z_1,|\cdots,|z_r|\right)\to 0$$
$$\aleph(z_1,\cdots,z_r)=0\left(|z_1|^{\beta_1},\cdots,|z_r|^{\beta_r}\right),\max\left(|z_1,|\cdots,|z_r|\right)\to\infty$$

where

$$k=1,\cdots,r:\alpha_k=\min[Re(d_j^{(k)}/\delta_j^{(k)})],j=1,\cdots,m_k$$

and

$$\beta_k=\max[Re((c_j^{(k)}-1)/\gamma_j^{(k)})],j=1,\cdots,n_k$$

For convenience, we will use the following notations in this paper.

$$V=m_1,n_1;\cdots;m_r,n_r \tag{8}$$

$$W=p_{i^{(1)}},q_{i^{(1)}},\tau_{i^{(1)}};R^{(1)},\cdots,p_{i^{(r)}},q_{i^{(r)}},\tau_{i^{(r)}};R^{(r)} \tag{9}$$

$$A=\{(a_j;\alpha_j^{(1)},\cdots,\alpha_j^{(r)})_{1,n}\},\{\tau_i(a_{ji};\alpha_{ji}^{(1)},\cdots,\alpha_{ji}^{(r)})_{n+1,p_i}\},\{(c_j^{(1)};\gamma_j^{(1)})_{1,n_1}\}$$
$$\tau_{i^{(1)}}(c_{ji^{(1)}}^{(1)};\gamma_{ji^{(1)}}^{(1)})_{n_1+1,p_{i^{(1)}}},\cdots,\{(c_j^{(r)};\gamma_j^{(r)})_{1,n_r}\},\tau_{i^{(r)}}(c_{ji^{(r)}}^{(r)};\gamma_{ji^{(r)}}^{(r)})_{n_r+1,p_{i^{(r)}}} \quad (10)$$

$$B=\{\tau_i(b_{ji};\beta_{ji}^{(1)},\cdots,\beta_{ji}^{(r)})_{m+1,q_i}\},\{(d_j^{(1)};\delta_j^{(1)})_{1,m_1}\},\tau_{i^{(1)}}(d_{ji^{(1)}}^{(1)};\delta_{ji^{(1)}}^{(1)})_{m_1+1,q_{i^{(1)}}},\cdots,$$
$$\{(d_j^{(r)};\delta_j^{(r)})_{1,m_r}\},\tau_{i^{(r)}}(d_{ji^{(r)}}^{(r)};\delta_{ji^{(r)}}^{(r)})_{m_r+1,q_{i^{(r)}}} \quad (11)$$

The contracted form concerning the multivariable Aleph-function writes:

$$\aleph(z_1,\cdots,z_r)=\left(\begin{array}{c|c} z_1 & A \\ \vdots & \vdots \\ z_r & B \end{array}\right) \quad (12)$$

The Aleph- function , introduced by Südland et al. [10], however the notation and complete definition is presented here in the following manner in terms of the Mellin-Barnes type integral.

$$\aleph(z)=\aleph_{P_i,Q_i,c_i;r'}^{M,N}\left(z \left| \begin{array}{c}(a_j,A_j)_{1,\mathfrak{n}},[c_i(a_{ji},A_{ji})]_{\mathfrak{n}+1,p_i;r'}\\(b_j,B_j)_{1,m},[c_i(b_{ji},B_{ji})]_{m+1,q_i;r'}\end{array}\right.\right)=\frac{1}{2\pi\omega}\int_L \Omega_{P_i,Q_i,c_i;r'}^{M,N}(s)z^s ds \quad (13)$$

for all z different to 0 and

$$\Omega_{P_i,Q_i,c_i;r'}^{M,N}(s)=\frac{\prod_{j=1}^{M}\Gamma(b_j-B_js)\prod_{j=1}^{N}\Gamma(1-a_j+A_js)}{\sum_{i=1}^{r'}c_i\prod_{j=N+1}^{P_i}\Gamma(a_{ji}-A_{ji}s)\prod_{j=M+1}^{Q_i}\Gamma(1-b_{ji}+B_{ji}s)} \quad (14)$$

With

$$|\arg z|<\frac{1}{2}\pi\Omega \ \text{where}\Omega=\sum_{j=1}^{M}\beta_j+\sum_{j=1}^{N}\alpha_j-c_i\left(\sum_{j=M+1}^{Q_i}\beta_{ji}+\sum_{j=N+1}^{P_i}\alpha_{ji}\right)>0, i=1,\cdots,r$$

For convergence conditions and other details of Aleph-function, see Südland et al. [10]. The serie representation of Aleph-function is given by Chaurasia et al. [3].

$$\aleph_{P_i,Q_i,c_i;r'}^{M,N}(z)=\sum_{G=1}^{M}\sum_{g=0}^{\infty}\frac{(-)^g\Omega_{P_i,Q_i,c_i,r'}^{M,N}(s)}{B_G g!}z^s \quad (15)$$

with $s=\eta_{G,g}=\dfrac{b_G+g}{B_G}, P_i<Q_i, |z|<1 \text{and} \Omega_{P_i,Q_i,c_i;r'}^{M,N}(s)$ is given by (14).

2. Main integral

The integral to be establish here is

$$\int_{-1}^{1}(1-x)^{\sigma}(1+x)^{v}P_w^{(u,v)}(x)\aleph_{P_i,Q_i,c_i;r'}^{M,N}\left(\begin{array}{c|c} z_1(1-x)^{k_1} & A \\ \vdots & \vdots \\ z_r(1-x)^{k_r} & B \end{array}\right)dx=$$
$$\sum_{G=1}^{M}\sum_{g=0}^{\infty}\frac{(-)^g\Omega_{P_i,Q_i,c_i,r'}^{M,N}(\eta_{G,g})(-)^w 2^{v+\sigma+1+h\eta_{G,g}}\Gamma(1+v+w)}{w!B_G g!}z^{\eta_{G,g}}\aleph_{p_i+2,q_i+2,\tau_i;R:W}^{0,\mathfrak{n}+2:V}\left(\begin{array}{c|c} 2^{k_1}z_1 & \\ \vdots & \\ 2^{k_r}z_r & \end{array}\right.$$
$$\left.\begin{array}{c}(-\sigma-h\eta_{G,g}:k_1,\cdots,k_r),(\mu-\sigma-h\eta_{G,g}:k_1,\cdots,k_r),A\\ \cdots \\ (-1-v-\sigma-w-h\eta_{G,g}:k_1,\cdots,k_r),(\mu+w-\sigma-h\eta_{G,g}:k_1,\cdots,k_r),B\end{array}\right) \quad (16)$$

where

$$Re(v)>-1;\ Re\left[\sigma+h\min_{1\leq j\leq m_i}\frac{b_j}{B_j}+\sum_{i=1}^{r}k_i\min_{1\leq j\leq m_i}\frac{d_j^{(i)}}{\delta_j^{(i)}}\right]>-1; \quad h>0, k_i>0, i=1,\cdots,r$$

$$|\arg z_k|<\frac{1}{2}A_i^{(k)}\pi, \quad k=1,\cdots,r$$

where $A_i^{(k)}$ is given (7).

$$|\arg z|<\frac{1}{2}\pi\Omega \quad \text{where } \Omega=\sum_{j=1}^{M}\beta_j+\sum_{j=1}^{N}\alpha_j-c_i\left(\sum_{j=M+1}^{Q_i}\beta_{ji}+\sum_{j=N+1}^{P_i}\alpha_{ji}\right)>0$$

Proof. We first replace the multivariable Aleph-function on the L.H.S of (17) by its Mellin-barnes contour integral (4), the Aleph-function in series using (15), Now we interchange the order of summation and integration (which is permissible under the conditions stated). Evaluate the inner integral with the help of result [[9], page 175). Interpreting the resulting Mellin-Barnes contour integral as an Aleph-function of r variables, we arrive at the desired result. □

3. Solution of the problem

The solution of the problem to be obtained is

$$\theta(x,t) = 2^u \sum_{N=0}^{\infty}\sum_{G=1}^{M}\sum_{g=0}^{\infty} A_N e^{-\lambda N(N+1)t} P_N^{(u,v)}(x) z^{\eta_{G,g}} \aleph^{0,\mathfrak{n}+2:V}_{p_i+2,q_i+2,\tau_i;R:W}\left(\begin{array}{c|}2^{k_1}z_1\\ \vdots\\ 2^{k_r}z_r\end{array}\begin{array}{c}(-\sigma-h\eta_{G,g}:k_1,\cdots,k_r),(1-\mu-\sigma-h\eta_{G,g}:k_1,\cdots,k_r),A\\ \cdots\\ (-1-v-\sigma-u-N-h\eta_{G,g}:k_1,\cdots,k_r),(N-\sigma-h\eta_{G,g}:k_1,\cdots,k_r),B\end{array}\right) \tag{17}$$

where

$$A_N = \frac{(-)^g \Omega^{M,N}_{P_i,Q_i,c_i,r'}(\eta_{G,g})(-)^N 2^{u+\eta_{G,g}}\Gamma(2N+u+v+1)\Gamma(N+u+v+1)}{\Gamma(N+u+1)B_G g!}$$

provided that

$$Re(v) > -1;\quad Re\left[\sigma + h\min_{1\leq j\leq M}\frac{b_j}{B_j} + \sum_{i=1}^{r} k_i \min_{1\leq j\leq M}\frac{d_j^{(i)}}{\delta_j^{(i)}}\right] > -1; h>0,\quad k_i>0, i=1,\cdots,r$$

$|\arg z_k| < \frac{1}{2}A_i^{(k)}\pi$where$A_i^{(k)}$ are given in (7)

$$|arg z| < \frac{1}{2}\pi\Omega \quad \text{where}, \Omega = \sum_{j=1}^{M}\beta_j + \sum_{j=1}^{N}\alpha_j - c_i\left(\sum_{j=M+1}^{Q_i}\beta_{ji} + \sum_{j=N+1}^{P_i}\alpha_{ji}\right) > 0$$

Proof. The solution of the problem can be written as Churchill [4].

$$\theta(x,t) = \sum_{N=0}^{\infty} R_N e^{-\lambda N(N+1)} P_N^{(u,v)}(x) \tag{18}$$

□

If $t=0$ in (18), then by vertu of (3)

$$f(x) = (1-x)^\sigma \aleph^{M,N}_{P_i,Q_i,c_i;r'}\left(z(1-x)^h \left| \begin{array}{c}(a_j,A_j)_{1,\mathfrak{n}},[c_i(a_{ji},A_{ji})]_{\mathfrak{n}+1,p_i;r'}\\ (b_j,B_j)_{1,m},[c_i(b_{ji},B_{ji})]_{m+1,q_i;r'}\end{array}\right.\right)\aleph^{0,\mathfrak{n}:V}_{U:W}\left(\begin{array}{c}z_1(1-x)^{k_1}\\ \vdots\\ z_r(1-x)^{k_r}\end{array}\right)$$
$$= \sum_{N=0}^{\infty} R_N P_N^{(u,v)}(x) \text{with} -1\leq x\leq 1 \tag{19}$$

The Eq. (3) is valid since $f(x)$ is continuous in the closed interval $-1\leqslant x\leqslant 1$ and has a piece wise continuous derivative there, then $u>-1, v>-1$, the Jacobi series (19) associated with $f(x)$ converges uniformly to $f(x)$ in $-1+\epsilon\leqslant x\leqslant 1-\epsilon, 0<\epsilon<1$.

Now multiply both sides of (19) by $(1-x)^u(1+x)^v P_w^{(u,v)}(x), u>-1, v>-1$ and integrating from -1 to 1

$$(1-x)^{\sigma+u}(1+x)^v P_w^{(u,v)}(x)\aleph^{M,N}_{P_i,Q_i,c_i;r'}\left(z(1-x)^h\right)\aleph^{0,\mathfrak{n}:V}_{U:W}\left(\begin{array}{c|c}z_1(1-x)^{k_1} & A\\ \vdots & \vdots\\ z_r(1-x)^{k_r} & B\end{array}\right)dx$$
$$= \sum_{N=0}^{\infty} R_N \int_{-1}^{1}(1-x)^v(1+x)^v P_N^{(u,v)}(x)P_w^{(u,v)}(x)dx \tag{20}$$

Now using (16) and the orthogonality property of Jacobi polynomials, we have

$$R_N = \frac{(-)^N 2^u \Gamma(2N+u+v+1)\Gamma(N+u+v+1)}{\Gamma(N+u+1)} \sum_{G=1}^{M}\sum_{g=0}^{\infty} \frac{(-)^g \Omega^{M,N}_{P_i,Q_i,c_i,r'}(\eta_{G,g}) 2^{\eta_{G,g}}}{w! B_G g!} z^{\eta_{G,g}}$$

$$\aleph^{0,\mathfrak{n}+2:V}_{p_i+2,q_i+2,\tau_i;R:W}\left(\begin{array}{c} 2^{k_1} z_1 \\ \vdots \\ 2^{k_r} z_r \end{array}\middle| \begin{array}{c} (-\sigma - h\eta_{G,g} : k_1,\cdots,k_r),(1-\mu-\sigma-h\eta_{G,g} : k_1,\cdots,k_r), A \\ \cdots\cdots \\ (-1-v-\sigma-u-N-h\eta_{G,g} : k_1,\cdots,k_r),(N-\sigma-h\eta_{G,g} : k_1,\cdots,k_r), B \end{array}\right) \quad (21)$$

with the help of (18) and (21), the solution is obtained.

4. Particular cases

a) If $r = 2$, the multivariable Aleph-function reduces to Aleph-function of two variables defined by Sharma [5] and we have

$$\theta(x,t) = 2^u \sum_{N=0}^{\infty}\sum_{G=1}^{M}\sum_{g=0}^{\infty} A_N e^{-\lambda N(N+1)t} P_N^{(u,v)}(x) z^{\eta_{G,g}} \aleph^{0,\mathfrak{n}+2:V}_{p_i+2,q_i+2,\tau_i;R:W}\left(\begin{array}{c} 2^{k_1} z_1 \\ \vdots \\ 2^{k_2} z_2 \end{array}\middle| \begin{array}{c} (-\sigma - h\eta_{G,g} : k_1,k_2),(1-\mu-\sigma-h\eta_{G,g} : k_1,k_2), A \\ \cdots \\ (-1-v-\sigma-u-N-h\eta_{G,g} : k_1,k_2),(N-\sigma-h\eta_{G,g} : k_1,k_2), B \end{array}\right) \quad (22)$$

where

$$A_N = \frac{(-)^g \Omega^{M,N}_{P_i,Q_i,c_i,r'}(\eta_{G,g})(-)^N 2^{u+\eta_{G,g}} \Gamma(2N+u+v+1)\Gamma(N+u+v+1)}{\Gamma(N+u+1) B_G g!} \quad (23)$$

under the same notations and conditions that (17) with $r = 2$.

b) If $r = 2$ and $\tau_i, \tau_{i'}, \tau_{i''} \to 1$ the multivariable Aleph-function reduces to I-function of two variables defined by Sharma et al. [7] and we have

$$\theta(x,t) = 2^u \sum_{N=0}^{\infty}\sum_{G=1}^{M}\sum_{g=0}^{\infty} A_N e^{-\lambda N(N+1)t} P_N^{(u,v)}(x) z^{\eta_{G,g}} I^{0,\mathfrak{n}+2:V}_{p_i+2,q_i+2,;R:W}\left(\begin{array}{c} 2^{k_1} z_1 \\ \vdots \\ 2^{k_2} z_2 \end{array}\middle| \begin{array}{c} (-\sigma - h\eta_{G,g} : k_1,k_2),(1-\mu-\sigma-h\eta_{G,g} : k_1,k_2), A \\ \cdots \\ (-1-v-\sigma-u-N-h\eta_{G,g} : k_1,k_2),(N-\sigma-h\eta_{G,g} : k_1,k_2), B \end{array}\right) \quad (24)$$

Where A_N is defined by (23), under the same conditions and notations that (17) with $r = 2$ and $\tau_i, \tau_{i'}, \tau_{i''} \to 1$.

c) The multivariable Aleph-function reduces to multivariable H-function and we have the following formula,

$$\theta(x,t) = 2^u \sum_{N=0}^{\infty}\sum_{G=1}^{M}\sum_{g=0}^{\infty} A_N e^{-\lambda N(N+1)t} P_N^{(u,v)}(x) z^{\eta_{G,g}} H^{0,\mathfrak{n}+2:V}_{p+2,q+2:W}\left(\begin{array}{c} 2^{k_1} z_1 \\ \vdots \\ 2^{k_r} z_r \end{array}\middle| \begin{array}{c} (-\sigma - h\eta_{G,g} : k_1,\cdots,k_r),(1-\mu-\sigma-h\eta_{G,g} : k_1,\cdots,k_r), A \\ \cdots \\ (-1-v-\sigma-u-N-h\eta_{G,g} : k_1,\cdots,k_r),(N-\sigma-h\eta_{G,g} : k_1,\cdots,k_r), B \end{array}\right) \quad (25)$$

where

$$A_N = \frac{(-)^g \Omega^{M,N}_{P_i,Q_i,c_i,r'}(\eta_{G,g})(-)^N 2^{u+\eta_{G,g}} \Gamma(2N+u+v+1)\Gamma(N+u+v+1)}{\Gamma(N+u+1) B_G g!}$$

under the same conditions and notations that (17).

5. Conclusion

Specializing the parameters of the Aleph-function and the multivariable Aleph-function, we can obtain a large number of results involving various special functions of one and several variables useful in Mathematics analysis, Applied Mathematics, Physics and Mechanics. The result derived in this paper is of general character and may prove to be useful in several interesting situations appearing in the literature of sciences.

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