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Thermoelastic interaction in a two-dimensional infinite space due to memory-dependent heat transfer

Research Article

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Abstract: The present study deals with a novel mathematical model of thermoelastic interaction in an infinite space introduced in the context of Taylor's series expansion involving memory-dependent derivative of the function for the Green-Naghdi model III (GNIII) heat conduction law, which is defined in an integral form of a common derivative with a kernel function on a slipping interval. The governing equations of this new model are applied to an infinite space which is subjected to finite linear opining mode I crack. The crack is subjected to prescribed temperature and stress distribution in the context of Green-Naghdi theory of generalized thermoelasticity. The analytical expressions of the thermophysical quantities are obtained in the physical domain employing the normal mode analysis. According to the graphical representations corresponding to the numerical results, conclusions about the new theory is constructed due to different choices of the kernel function and delay times. The results to the analogous problem corresponding to the case in absence of memory effect is also shown analytically. Excellent predictive capability is demonstrated due to the presence of memory dependent derivative also.

MSC: 74F05 • 74D05

Keywords: Memory dependent derivative • Caputo derivative • Green-Naghdi model • Mode-I crack • Normal mode analysis • Time delay

1. Introduction

The theory of generalized thermoelasticity has drawn attention of researchers due to its applications in various diverse fields such as engineering, nuclear reactor's design, high energy particle accelerators, etc. Actually, as is well known, the term 'generalized' usually refers to thermodynamic theories based on hyperbolic-type (wave-type) heat equations, so that a finite speed for propagation of thermal signal is admitted. Because of the experimental evidences in the support of finiteness of the heat propagation speed. Very recently, employing the generalized thermoelasticity theories, several remarkable studies have been reported [1–6]. Out of the modern theories, three models (models I, II and III) for generalized thermoelasticity of homogeneous and isotropic materials have been developed later by Green and Naghdi [7–9]. The linearized version of Model I reduces to the classical heat conduction theory (based on FourierÂŠs law) and those of Models II and III permit thermal waves to propagate with finite speed. According to the GN III model, the heat flux vector $\vec{q}$ is defined as

$$\dot{\vec{q}} = -K\vec{\nabla}\dot{T} - K^{\star}\vec{\nabla}T, \tag{1}$$

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where K is the thermal conductivity of the material and $K^\star$ is an additional material constant. During recent years several remarkable works employing the Green-Naghdi theories have been reported [10–14].

Diethelm [15] has developed the Caputo derivative as [16, 17]:

$$D_0^\zeta f(t) = \int_0^t K_\zeta(t-\xi) f^{(m)}(\xi) d\xi, \tag{2}$$

where

$$K_\zeta(t-\xi) = \frac{(t-\xi)^{m-\zeta-1}}{\Gamma(m-\zeta)}, \tag{3}$$

and $f^{(m)}$ indicates the usual $m-$th order derivative of the function. Differential equations of fractional order have been the focus of many studies due to their frequent appearance in various application in fluid mechanics, viscoelasticity, biology, physics, and engineering. The most important advantage of using fractional differential equations in these and other applications is their non-local property. It is well known that the integer order differential operator is a local operator but the fractional order differential operator is non-local [18–21].

From (2) and (3), it can be visualized that for any real number ζ, the kernel $K_\zeta(t-\xi)$ is a fixed function. But from the viewpoint of applications, different processes need different kernels to reflect their memory effects, so the kernel should be chosen freely. In fact, the memory effect of a real process basically occurs on a segment of time, i.e., on the delayed interval $[t-\tau, t]$ ($\tau(>0)$ indicates the time-delay). Enlightened by these, the novel concept of derivative was initiated as the "memory-dependent derivative" (MDD) to reflect the memory effect in a distinct manner. One may state that the definition of MDD is more intutionistic in realizing the physical significance and accordingly, the corresponding memory-dependent differential equations are more effective in real-world problems. Quite recently, introducing the concept of MDD, a few pioneering works can be reviewed from the following literatures [22–25].

Wang and Li [26] introduced a memory-dependent derivative, the first order of function f which is simply defined in an integral form of a common derivative with a kernel function on a slipping interval as follows

$$D_\tau f(t) = \frac{1}{\tau}\int_{t-\tau}^t K(t-\xi) f'(\xi) d\xi, \tag{4}$$

where τ (>0) is the delay time and $K(t-\xi)$ is the kernel function in which they can be chosen freely, such as $K(t-\xi)=1$, $(1-(t-\xi))^p$, where p is a natural number. The kernel function can be understood as the degree of the past effect on the present. In addition, if $K \equiv 1$,

$$D_\tau f(t) = \frac{1}{\tau}\int_{t-\tau}^t f'(\xi) d\xi = \frac{f(t)-f(t-\tau)}{\tau} \to f'(t), \tag{5}$$

so, the common derivative $\frac{d}{dt}$ can be seen as the limit of D_τ as $\tau \to 0$.
The right side of (5) can be understood as mean value of $f'(\xi)$ on the past interval $[t-\tau, t]$ with different weights. Generally, from the viewpoint of applications, the memory effect requires weight $0 \leqslant K(t-\xi) < 1$ for $\xi \in [t-\tau, t)$, so the magnitude of the memory dependent derivative is usually smaller than that of the common derivative $f'(t)$. The variational principles, reciprocal theorems and uniqueness of solutions due to memory dependence in a thermodiffusive medium have been proved by Karamany [27].

Further, following the definition (4), the constitutive law for the heat flux in the context of memory dependent GN III model can be represented as

$$D_\tau \vec{q} = -K\nabla^2 \dot{T} - K^*\nabla^2 T, \tag{6}$$

where τ is the delay time. This is also a particular case of the heat conduction law due to memory-effect, which was recently proposed by Sur and Kanoria [28] as

$$\left(1 + \frac{\tau_q}{1!}D_{\tau_1} + \frac{\tau_q^2}{2!}D_{\tau_1}^2\right)\dot{q}_i = -K\left(1+\frac{\tau_T}{1!}D_{\tau_2}\right)\dot{T}_{,i} - K^\star\left(1+\frac{\tau_v}{1!}D_{\tau_3}\right)T_{,i},$$

where τ_1, τ_2 and τ_3 are the delay times due to 3P lag model.

The objective of the present contribution is to modify the conventional Fourier's law of heat conduction introducing a new Taylor's series expansion using MDD in the context of GN III model in an infinite space subjected to a finite linear opening mode I crack. The resulting non-dimensional equations are applied to a specific problem of the medium which is subjected to prescribed temperature and thermal stresses. Then, the analytical expression of the displacements, temperature and stresses have been found numerically for copper-like material. According to the graphical representations corresponding to the numerical results, conclusions about the new theory is constructed for different choices of the kernel functions. Excellent predictive capability is demonstrated due to the effect of delay time also. The validation of the results corresponding to the absence of memory effect is also shown analytically.

2. Formulation of the problem

We consider an infinite, homogeneous, isotropic thermoelastic space occupying the region G given by

$$G = \left\{(x, y, z) | -\infty < x < \infty,\ -\infty < y < \infty,\ -\infty < z < \infty\right\},$$

with a mode I crack subjected to prescribed temperature and stresses. The displacement vector $\vec{u}$ and the temperature T can be defined as

$$\begin{aligned} \vec{u} &= \ (u(x,y,t),\ v(x,y,t),\ 0), \\ T &= \ T(x,y,t). \end{aligned} \tag{7}$$

The constitutive relations are:

$$\sigma_{xx} = \left(\lambda + 2\mu\right)\frac{\partial u}{\partial x} + \lambda\frac{\partial v}{\partial y} - \gamma T, \tag{8}$$

$$\sigma_{yy} = \left(\lambda + 2\mu\right)\frac{\partial v}{\partial y} + \lambda\frac{\partial u}{\partial x} - \gamma T, \tag{9}$$

$$\sigma_{xy} = \mu\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right), \tag{10}$$

where λ, μ are Lame's constant, $\gamma = (3\lambda + 2\mu)\alpha_t$; α_t being the coefficient of linear thermal expansion of the material.

The equations of motion in absence of body forces are

$$(\lambda+\mu)\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 v}{\partial x\partial y}\right) + \mu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) - \gamma\frac{\partial T}{\partial x} = \rho\frac{\partial^2 u}{\partial t^2}, \tag{11}$$

$$(\lambda+\mu)\left(\frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 u}{\partial x\partial y}\right) + \mu\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right) - \gamma\frac{\partial T}{\partial y} = \rho\frac{\partial^2 v}{\partial t^2}, \tag{12}$$

where ρ is the density of the material.
The heat conduction equation in the context of memory dependent GN III model has the form

$$\frac{1}{\tau}\int_{t-\tau}^{t} K(t-\xi)\frac{\partial}{\partial \xi}\left[\rho c_v\frac{\partial T}{\partial \xi} + \gamma T_0\frac{\partial e}{\partial \xi}\right] d\xi = K\nabla^2\dot{T} + K^\star\nabla^2 T, \tag{13}$$

where, c_v is the specific heat T_0 is the reference temperature and the dilatation e is defined as

$$e = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}.$$

For convenience, we shall use the following non-dimensional variables

$$x' = \frac{\omega^\star}{C_2}x,\quad y' = \frac{\omega^\star}{C_2}y,\quad u' = \frac{\rho C_2\omega^\star}{\gamma T_0}u,\quad v' = \frac{\rho C_2\omega^\star}{\gamma T_0}v,\quad T' = \frac{T}{T_0},$$

$$\sigma'_{ij} = \frac{\sigma_{ij}}{\gamma T_0},\quad t' = \omega^\star t,\quad \xi' = \omega^\star\xi,\quad \omega^\star = \frac{\rho C_E C_2^2}{K},\quad C_2^2 = \frac{\mu}{\rho},$$

and after omitting primes, the above equations can be rewritten in nondimensional form as follows

$$\sigma_{xx} = a_2\frac{\partial u}{\partial x} + a_3\frac{\partial v}{\partial y} - T, \tag{14}$$

$$\sigma_{yy} = a_2\frac{\partial v}{\partial y} + a_3\frac{\partial u}{\partial x} - T, \tag{15}$$

$$\sigma_{xy} = \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right), \tag{16}$$

$$\nabla^2 u + \frac{(\mu+\lambda)}{\rho C_2^2}\frac{\partial e}{\partial x} - \frac{\partial T}{\partial x} = \frac{\partial^2 u}{\partial t^2}, \tag{17}$$

$$\nabla^2 v + \frac{(\mu+\lambda)}{\rho C_2^2}\frac{\partial e}{\partial y} - \frac{\partial T}{\partial y} = \frac{\partial^2 v}{\partial t^2}, \tag{18}$$

$$\frac{1}{\tau}\int_{t-\tau}^{t} K(t-\xi)\left[\frac{\partial^2 T}{\partial \xi^2} + \epsilon_1 \frac{\partial e}{\partial \xi}\right] d\xi = \epsilon_3 \nabla^2 \dot{T} + \epsilon_2 \nabla^2 T, \tag{19}$$

where,

$$\epsilon_1 = \frac{\gamma^2 T_0}{\rho^2 C_E C_2^2}, \quad \epsilon_2 = \frac{K^\star}{\rho C_E C_2^2}, \quad \epsilon_3 = \frac{K\omega^\star}{\rho C_E C_2^2}, \quad a_2 = \frac{\lambda+2\mu}{\rho C_2^2} \quad \text{and} \quad a_3 = \frac{\lambda}{\rho C_2^2}. \tag{20}$$

We assume the scalar potential functions $\phi(x,y,t)$ and $\psi(x,y,t)$ defined by the relations in the non-dimensional form as follows

$$u = \frac{\partial \phi}{\partial x} + \frac{\partial \psi}{\partial y}, \quad v = \frac{\partial \phi}{\partial y} - \frac{\partial \psi}{\partial x}. \tag{21}$$

Using Eq. (21) in Eqs. (17)-(19), we arrive at

$$\left[\nabla^2 - a_0 \frac{\partial^2}{\partial t^2}\right]\phi - a_0 T = 0, \tag{22}$$

$$\left[\nabla^2 - \frac{\partial^2}{\partial t^2}\right]\psi = 0, \tag{23}$$

$$\frac{1}{\tau}\int_{t-\tau}^{t} K(t-\xi)\left[\frac{\partial^2 T}{\partial \xi^2} + \epsilon_1 \frac{\partial}{\partial \xi}\nabla^2\phi\right] d\xi = \epsilon_3 \nabla^2 \dot{T} + \epsilon_2 \nabla^2 T, \tag{24}$$

where,

$$C_1^2 = \frac{\lambda+2\mu}{\rho}, \quad a_0 = \frac{C_2^2}{C_1^2}, \quad e = \nabla^2\phi. \tag{25}$$

3. Normal mode analysis

The solution of the considered physical variable can be decomposed in terms of normal modes in the following form as follows

$$[\phi,\ \psi,\ T,\ \sigma_{ij}](x,y,t) = [\phi^\star,\ \psi^\star,\ T^\star,\ \sigma^\star_{ij}](x)\exp \omega t + iby, \tag{26}$$

where $\phi^\star(x)$, $\psi^\star(x)$, $T^\star(x)$, $\sigma^\star_{ij}(x)$ are the amplitudes of the functions, ω is the complex constant and b is the wave number in the y direction.

Using (26), the Eqs. (22) to (24) become,

$$(D^2 - A_1)\phi^\star - a_0 T^\star = 0. \tag{27}$$

$$(D^2 - A_1)\psi^\star = 0. \tag{28}$$

$$\frac{\omega^2\epsilon_1}{\tau}P(t)(D^2-b^2)\phi^\star = \left[\epsilon(D^2-b^2) - \frac{\omega^2}{\tau}P(t)\right]T^\star, \tag{29}$$

where

$$D = \frac{d}{dx}, \quad A_1 = b^2 + a_0\omega^2, \quad A_2 = b^2 + 1, \quad \epsilon = \epsilon_2 + \omega\epsilon_3, \quad P(t) = e^{-\omega t}\int_{t-\tau}^{t} K(t-\xi)e^{\omega\xi}d\xi.$$

Eliminating $T^\star$ from Eqs. (27) and (29) we get the following fourth order ordinary differential equation for $T^\star$ and $\phi^\star$ as

$$(D^4 - AD^2 + B)\{\phi^\star, T^\star\} = 0. \tag{30}$$

Eqn. (30) can be factored as

$$(D^2 - k_1^2)(D^2 - k_2^2)\phi^\star(x) = 0, \tag{31}$$

where, k_j^2 $(j = 1, 2)$ are roots of the equation

$$k^4 - Ak^2 + B = 0.$$

Also, the parameters A and B are defined as

$$A = b^2 + A_1 + \frac{\omega^2 P(t)(1 + a_0\epsilon_1)}{\epsilon},$$

$$B = b^2 A_1 + \frac{\omega^2 P(t)(1 + a_0\epsilon_1)}{\epsilon}.$$

We obtain also the uncoupled equation

$$(D^2 - A_2)\psi^\star = 0. \tag{32}$$

It can also be shown analytically that if we choose the kernel to be $K(t-\xi) = (1-(t-\xi))^p$, $p \in \mathbb{N}$, then in absence of memory effect (i.e., $\tau \to 0$), we arrive at

$$\begin{aligned}\lim_{\tau\to 0} A &= b^2 + A_1 + \tfrac{\omega^2(1+a_0\epsilon_1)}{\epsilon}\lim_{\tau\to 0}\frac{P(t)}{\tau}, \quad [\text{since}, \lim_{\tau\to 0}\frac{P(t)}{\tau} = 1]\\ &= b^2 + A_1 + \tfrac{\omega^2(1+a_0\epsilon_1)}{\epsilon}.\end{aligned}$$

Similarly,

$$\lim_{\tau\to 0} B = b^2 A_1 + \frac{\omega^2(A_1 + a_0 b^2\epsilon_1)}{\epsilon}.$$

So, due to the absence of memory effect, for different choices of p, the results agree with the results of existing literature [29].

The solution of the Eqs. (30)-(32) which are bounded as $x \to \infty$, has the form

$$\phi^\star(x) = \sum_{j=1}^{2} M_j e^{-k_j x}, \tag{33}$$

$$T^\star(x) = \sum_{j=1}^{2} M_j' e^{-k_j x}, \tag{34}$$

$$\psi^\star(x) = M_3 e^{-k_3 x}. \tag{35}$$

Where M_j, M_j' and M_3 are some parameters dependent on b and ω and k_3 is the positive root of the characteristic equation of Eqn. (32). Using Eqs. (33) and (34) into Eqn. (27), we have

$$M_j' = h_j M_j, \; j = 1, 2$$

$$M_3 = \sqrt{A_2}.$$

Therefore, the temperature takes the form

$$T^\star = \sum_{j=1}^{2} h_j M_j e^{-k_j x}, \tag{36}$$

where

$$h_j = \frac{(k_j^2 - A_1)}{a_0}. \ j = 1, 2$$

Using Eqn. (21) and Eqs. (14)-(16) with non dimensional boundary conditions and using Eqs. (33), (35) and (36), we obtain the expressions for the displacement components, the stresses, and the temperature distribution as follows

$$u^\star(x) = -k_1 M_1 e^{-k_1 x} - k_2 M_2 e^{-k_2 x} + ibM_3 e^{-k_3 x}, \tag{37}$$

$$v^\star(x) = ibM_1 e^{-k_1 x} + ibM_2 e^{-k_2 x} + k_3 e^{-k_3 x}, \tag{38}$$

$$\sigma^\star_{yy}(x) = s_1 M_1 e^{-k_1 x} + s_2 M_2 e^{-k_2 x} + s_3 M_3 e^{-k_3 x}, \tag{39}$$

$$\sigma^\star_{xx}(x) = s_4 M_1 e^{-k_1 x} + s_5 M_2 e^{-k_2 x} - s_3 M_3 e^{-k_3 x}, \tag{40}$$

$$\sigma^\star_{xy}(x) = r_1 M_1 e^{-k_1 x} + r_2 M_2 e^{-k_2 x} + r_3 M_3 e^{-k_3 x}, \tag{41}$$

$$T^\star(x) = h_1 M_1 e^{-k_1 x} + h_2 M_2 e^{-k_2 x}, \tag{42}$$

where

$$\begin{aligned} s_1 &= -a_2 b^2 + a_3 k_1^2 - h_1, \\ s_2 &= -a_2 b^2 + a_3 k_2^2 - h_2, \\ s_3 &= ibk_3(a_2 - a_3), \\ s_4 &= a_2 k_1^2 - a_3 b^2 - h_1, \\ s_5 &= a_2 k_2^2 - a_3 b^2 - h_2, \\ r_1 &= -2ibk_1, \\ r_2 &= -2ibk_2, \\ r_3 &= -(a^2 + k_3^2). \end{aligned} \tag{43}$$

4. Applications

The plane boundary subjects to an instantaneous normal point force. So, the boundary conditions on the plane $x = 0$ is given by

(1) Mechanical boundary condition

The mechanical boundary condition is that the surface of the half-space obeys

$$\sigma_{yy}(0, y, t) = -p(0, y, t). \tag{44}$$

In terms of normal modes, the boundary condition can be written as

$$\sigma_{yy}(0, y, t) = -p^\star e^{\omega t + iby}.$$

$$\sigma_{xy}(0, y, t) = 0. \tag{45}$$

(2) Thermal boundary condition

The surface of the half-space subjects to a time-dependent thermal shock as follows

$$T(0, y, t) = f(0, y, t). \tag{46}$$

In terms of normal modes, the thermal boundary condition can be written as

$$T(0, y, t) = f^\star e^{\omega t + iby}.$$

Substituting the expressions of the boundary conditions in terms of normal modes to Eqs. (44)-(46), we get

$$\sum_{j=1}^{3} s_j M_j = -p^\star, \tag{47}$$

$$\sum_{j=1}^{2} h_j M_j = f^\star, \tag{48}$$

$$\sum_{j=1}^{3} r_j M_j = 0. \tag{49}$$

Invoking the boundary conditions (47)-(49) at the surface $x = 0$ of the plate, we obtain a system of three equations. After applying the inverse of matrix method, we have the values of the three parameters M_j, $(j = 1, 2, 3)$ as follows

$$\begin{bmatrix} s_1 & s_2 & s_3 \\ r_1 & r_2 & r_3 \\ h_1 & h_2 & 0 \end{bmatrix} \begin{bmatrix} M_1 \\ M_2 \\ M_3 \end{bmatrix} = \begin{bmatrix} -p^* \\ 0 \\ f^* \end{bmatrix}. \tag{50}$$

5. Numerical results and discussions

In order to illustrate our theoretical results obtained in the preceding section numerically and to compare these under GN theory for different choices of kernel function and variable delay times, we now present the results in the form of their graphical representations. In the numerical computations, we have taken a copper crystal. For the computation, Matlab Software package is used as a tool. Since, ω is the complex constant then we choose $\omega = \omega_0 + i\zeta$.The material constants are given by

$$\rho = 8954\,\text{kg m}^{-1}, \quad \lambda = 7.76 \times 10^{10}\,\text{N/m}^2, \quad \mu = 3.86 \times 10^{10}\,\text{N/m}^2, \quad \alpha_t = 1.78 \times 10^{-5}\,\text{K}^{-1},$$

$$K = 0.6 \times 10^{-2}\,\text{cal/cm s}^\circ\text{C}, \quad K^\star = 0.9 \times 10^{-2}\,\text{cal/cm s}^\circ\text{C}, \quad T_0 = 293\,\text{K}, \quad f^\star = 0.5,$$

$$C_E = 383.1\,\text{J kg}^{-1}\text{K}^{-1}, \quad p^\star = 4, \quad b = 2, \quad \omega_0 = 2, \quad \zeta = 1.$$

In order to study the variation of the thermophysical quantities for different times ($t = 0.2, 0.4, 0.6$) and for delay time $\tau = 0.2$, Figs. 1-4 have been plotted. In these figures, the kernel has been taken to be $K(t-\xi) = (1-(t-\xi))^5$, with $p^\star = 4$.

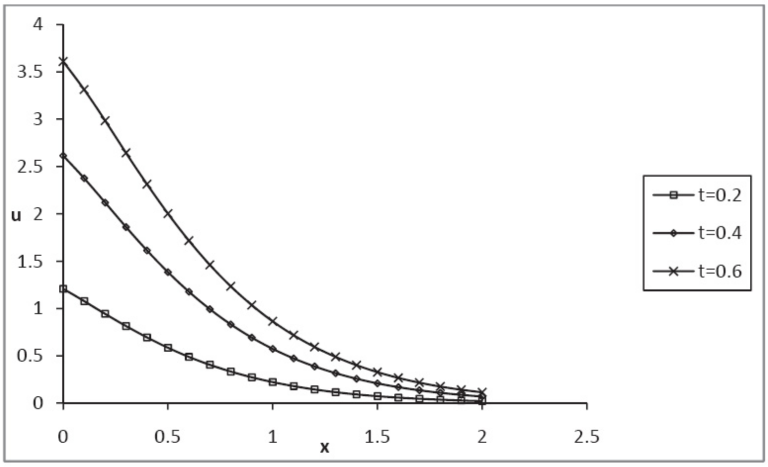

Fig. 1. Variation of u vs x for different t when $K = (1-(t-\xi))^5, p^* = 4, \tau = 0.2$

Figs. 1 and 2 depict the variation of the displacement components u and v against the distance x for delay time $\tau = 0.2$. From the figures, it is observed that the displacements u and v attain their maximum magnitudes on the plane $x = 0$ and the magnitudes diminishes as we move far from the boundary. Also, it is seen that with the increase of time t, magnitude of the displacements u and v also increases.

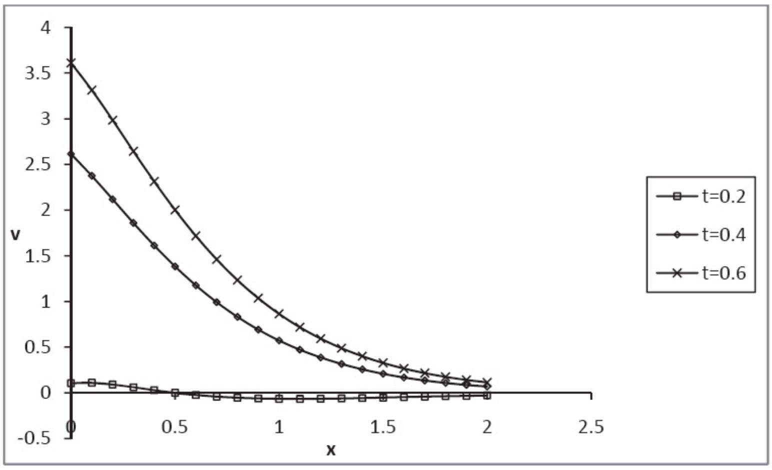

Fig. 2. Variation of v vs x for different t when $K = (1-(t-\xi))^5, p^* = 4, \tau = 0.2$

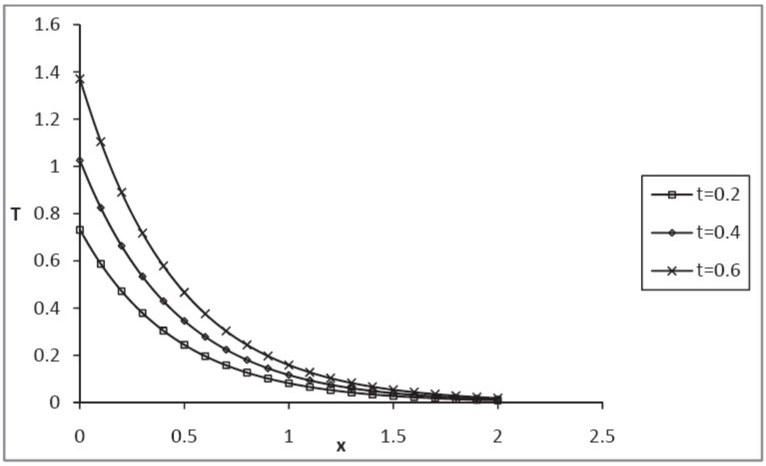

Fig. 3. Variation of T vs x for different t when $K = (1-(t-\xi))^5, p^* = 4, \tau = 0.2$

Fig. 3 depicts the variation of the temperature T against the distance x for different times ($t = 0.2$, 0.4, 0.6) where the kernel function is taken to be $K(t-\xi) = (1-(t-\xi))^5$ while $\tau = 0.2$. As seen from the figure, on the plane of application of the thermal loading, (i.e., $x = 0$), T satisfies the thermal boundary condition as laid down in eqn. (46). Also, with the increase of time t, T also increases in magnitude.

Fig. 4 is now plotted to study the variation of the stress component σ_{xy} against the distance x for delay time $\tau = 0.2$ and $p^\star = 4$. In this figure, the kernel is taken to be $K(t-\xi) = (1-(t-\xi))^5$. As seen from the figure, σ_{xy} vanishes on the bounding plane $x = 0$ satisfying the mechanical boundary condition of the problem as laid down in eqn. (45). Also, it is observed from the figure that the magnitude of σ_{xy} decreases to attain the maximum magnitude near $x = 0.3$ and then the magnitude also increases as we move far from the boundary. Also, it is seen that for $t = 0.2$, the magnitude of σ_{xy} is compressive in nature in $0 < x < 0.5$ and after that it shows an expansive behavior. Further, for $t = 0.4$ and $t = 0.6$, the similar qualitative behavior is seen though, σ_{xy} is now compressive in $0 < x < 1$ and $0 < x < 1.5$ respectively, and after that, the stress shows an expansive behavior. Here also, increase of time has a tendency to increase the magnitude of σ_{xy}.

Figs. 4 and 6 are now plotted to analyze the graphical representation of σ_{xy} and σ_{yy} for the kernel function $K(t-\xi) = (1-(t-\xi))^5$ for the delay time $\tau = 0.2$ when $t = 0.2$ is taken. These figures represent the effect of pressure ($p^\star$) on the stress component. Fig. 5 shows the variation of stress σ_{xy} against the distance x for $p^\star = 1$, 4 and 10 respectively for the same set of parameters as mentioned above. From the figure, it is seen that σ_{xy} satisfies the boundary condition of the problem. Also, the oscillatory nature in the propagation of σ_{xy} is found near the plane of application of the thermal loading. Also, the stress σ_{xy} is compressive in the interval $0 < x < 0.5$ and the peak increases with the increase of the load. Further, σ_{xy} show an expansive nature after $x = 0.5$. The magnitude of the profile of σ_{xy} is increasing in nature as $p^\star$ increases.

Fig. 6 is plotted to study the variation of σ_{yy} against the distance x for different values of $p^\star$ for the same set of parameters as mentioned earlier. In the figure, on the plane $x = 0$, σ_{yy} satisfies the mechanical boundary condition

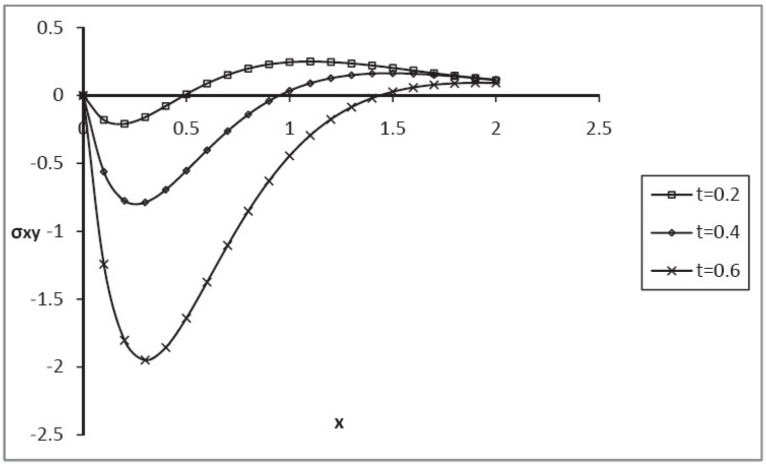

Fig. 4. Variation of σ_{xy} vs x for different t when $K = (1-(t-\xi))^5, p^* = 4, \tau = 0.2$

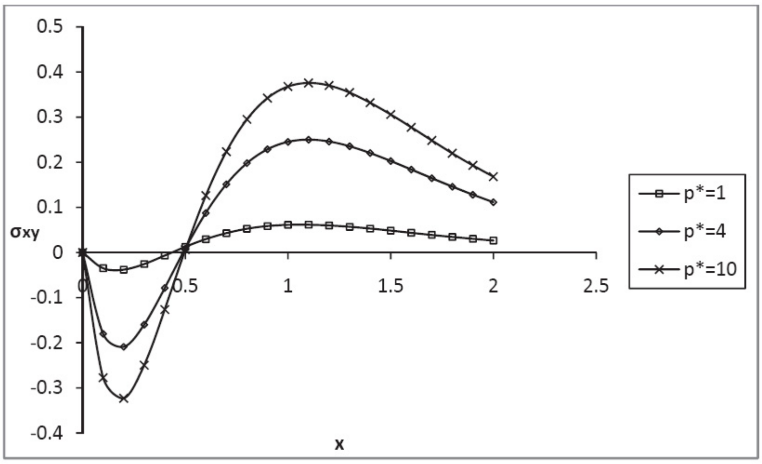

Fig. 5. Variation of σ_{xy} vs x for different p^* when $K = (1-(t-\xi))^5, \tau = 0.2$

given in eqn. (44) which also represents another partial correctness of the numerical codes prepared in the problem. Further, with the increase of the magnitude of $p^\star$, magnitude of σ_{yy} increases. Also, as we move far from the boundary, magnitude of σ_{yy} diminishes and approach to zero value.

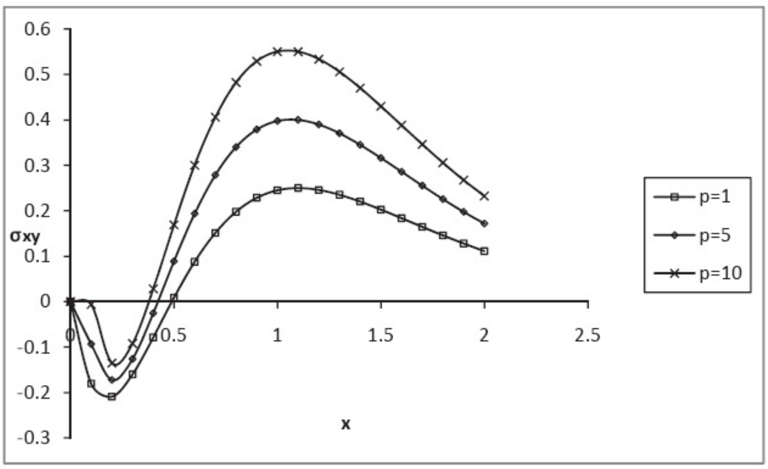

Fig. 7. Variation of σ_{xy} vs x for different K when $p^* = 4, \tau = 0.2$

Fig. 7 is plotted to study the variation of σ_{xy} for $p^\star = 4$, time $t = 0.2$ and the delay time is taken to be $\tau = 0.2$. This

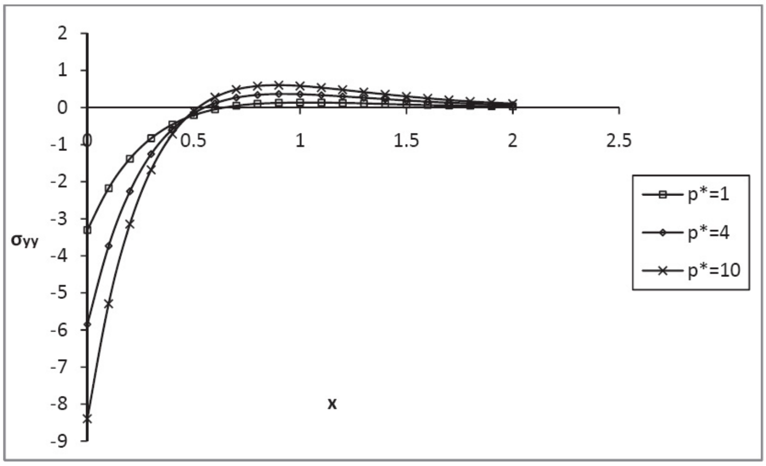

Fig. 6. Variation of σ_{yy} vs x for different p^* when $K = (1-(t-\xi))^5, \tau = 0.2$

figure is plotted for different choices of $K(t-\xi) = (1-(t-\xi))^p$ for $p = 1,\ 5,\ 10$ respectively. It is seen from the figure that on the plane $x = 0$, σ_{xy} vanishes satisfying the mechanical boundary condition. Also, the stress is compressive in nature in $0 < x < 0.2$ and after $x = 0.2$ the stress shows an expansive behavior. The magnitude of σ_{xy} increases and the maximum magnitude of σ_{xy} is found near $x = 1.1$. The profile of σ_{xy} is larger for $K(t-\xi) = (1-(t-\xi))^{10}$ compared to $(1-(t-\xi))^5$ which is again larger compared to $(1-(t-\xi))$.

6. Conclusions

Relative to the fractional derivative, here we have brought a fourth concept of "memory dependent derivative" which is directly defined by the integral of a common derivative. Here $D_\tau \to \frac{d}{dt}$ as $\tau \to 0$, with $K(t-\xi) = 1$, where τ is the delay time reflecting the past effects on the present state. The kernel function for fractional type is fixed, yet that of the memory-dependent type can be chosen freely according to the necessity of applications. Though the figures are self-explanatory in exhibiting the different peculiarities which occur in the propagation of waves, yet the following remarks may be added.

1. Significant differences in the propagation of thermophysical quantities are observed for different choices of kernel function. To study the memory-effect, it is more advantageous to deal with different kernel functions.
2. The magnitude of the thermophysical quantities increases with the increase of time. Also, the increment of applied pressure on the bounding plane also represent the increment of the magnitude of the profile of the thermophysical quantities.
3. In absence of memory-effect, for different choices of the kernel, our results agree with the existing results analytically [29].

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References

[1] A. Sur, M. Kanoria, Fibre-reinforced magneto-thermoelastic rotating medium with fractional heat conduction, Procedia Engineering 127 (2015) 605-612.

[2] A. Sur, M. Kanoria, Thermoelastic interaction in a viscoelastic functionally graded half-space under three phase lag model, European Journal of Computational Mechanics 23(5-6) (2014) 179-198.

[3] P. Das, M. Kanoria, Study of finite thermal wavs in a magneto-thermo-elastic roatating medium, Journal of Thermal Stresses 37 (2014) 405-428.

[4] P. Das, A. Kar, M. Kanoria, Analysis of Magneto-Thermoelastic Response in a Transversely Isotropic Hollow Cylinder Under Thermal Shock with Three-Phase-Lag Effect, Journal of Thermal Stresses 36 (2013) 239-258.

[5] A. Sur, M. Kanoria, Modelling of fibre-reinforced magneto-thermoelastic plate with heat sources, Procedia Engineering 173 (2016) 875-882.

[6] A. Sur, M. Kanoria, Propagation of thermal waves in a functionally graded thick plate, Mathematics and Mechanics of Solids doi: 10.1177/1081286515609652 (2015).

[7] A. E. Green, P. M. Naghdi, A re-examination of the basic results of thermomechanics, Proceedings of Royal Society London, Series A 432 (1991) 171-194.

[8] A. E. Green, P. M. Nagdhi, On undamped heat waves in an elastic solid, Journal of Thermal Stresses 15 (1992) 252-264.

[9] A. E. Green, P. M. Nagdhi, Thermoelasticity without energy dissipation, Journal of Elasticity 31 (1993) 189-208.

[10] S. H. Mallik, M. Kanoria, Generalized thermoelastic functionally graded infinite solid with a peridically varying heat source, International Journal of Solids and Structures 44 (2007) 7633-7645.

[11] A. Sur, M. Kanoria, Analysis of thermoelastic response in a functionally graded infinite space subjected to a Mode-I crack, International Journal of Advances in Applied Mathematics and Mechanics 3(2) (2015) 33-44.

[12] S. Y. Atwa, Generalized magneto-thermoelasticity with two temperature and initial stress under Green-Naghdi theory, Applied Mathematical Modelling (2014) Doi: 10.1016/j.apm.2014.04.023.

[13] N. Sarkar, Transient Disturbance in a Three-Dimensional Thermo-elastic Half-Space Under Green-Naghdi Theory, Vietnam Journal of Mathematics 41 (2013) 269-288.

[14] B. Singh, Wave Propagation in a Green-Naghdi Thermoelastic Solid with Diffusion, International Journal of Thermophysics 34 (2013) 553-566.

[15] K. Diethelm, The analysis of fractional differential equations: An application-oriented exposition using differential operators of Caputo type, Springer-Verlag, Berlin, Heidelberg (2010).

[16] M. Caputo, Linear models of dissipation whose Q is almost frequency independent II Geophys, Journal of Royal Astronomical Society 3 (1967) 529-539.

[17] M. Caputo, F. Mainardi, Linear model of dissipation in anelastic solids. Rivis ta. el. Nuovo. cimento. 1 (1971) 161-198.

[18] A. Sur, M. Kanoria, Fractional order two-temperature thermoelasticity with finite wave speed, Acta Mechanica 223 (2012) 2685-2701.

[19] A. Sur, M. Kanoria, Fractional order generalized thermoelastic functionally graded solid with variable material properties, Journal of Solid Mechanics 6(1) (2014) 54-69.

[20] A. Sur, M. Kanoria, Fractional order heat conduction with finite wave speed in a thermo-visco-elastic spherical shell, Latin American Journal of Solids and Structures 11(7) (2014) 1132-1162.

[21] S. Chakravorty, S. Ghosh, A. Sur, Thermo-viscoelastic interaction in a three-dimensional problem subjected to fractional heat conduction, Procedia Engineering 173 (2016) 875-882.

[22] Y.J. Yu, W. Hu, X. G. Tian, A novel generalized thermoelasticity model based on memory-dependent derivative, International Journal of Engineering Science 81 (2014) 123-134.

[23] M. A. Ezzat, A. S. El-Karamany, A. A. El-Bary, Electro-thermoelasticity theory with memory-dependent derivative heat transfer, International Journal of Engineering Science 99 (2016) 22-38.

[24] K. Lotfy, N. Sarkar, Memory-dependent derivatives for photothermal semiconducting medium in generalized thermoelasticity with two-temperature, Mechanics of Time-dependent Materials (2017) DOI 10.1007/s1104-017-9340-5.

[25] M. A. Ezzat, A. S. El-Karamany, A. A. El-Bary, Modeling of memory-dependent derivative in generalized thermoelasticity, The European Physics Journal Plus 131 (2016) 372.

[26] J. L.Wang, H. F. Li, Surpassing the fractional derivative: concept of the memory dependent derivative, Computers & Mathematics with Applications 62 (2011) 1562-1567.

[27] A. S. El-Karamany, M. A. Ezzat, Thermoelastic diffusion with memory-dependent derivative, Journal of Thermal Stresses 39(9) (2016) 1035-1050.

[28] A. Sur, M. Kanoria, Modeling of memory-dependent derivative in a fibre-reinforced plate, Thin Walled Structures doi: 10.1016/j.tws.2017.05.005 (2017).

[29] M. I. A. Othman, S. Y. Atwa, 2-D problem of a Mode-I crack for a generalized thermoelasticity under Green-Naghdi theory, Meccanica 48 (2013) 1543-1551.

[30] K.J Chauhan, D.M Patel, The similarity solutions of concentration dependent diffusion equation, International Journal of Advances in Applied Mathematics and Mechanics, 1(2) (2013) 80-85.

[31] R. Patra, S. P. Barik, P.K Chaudhuri, An internal crack problem in an infinite transversely isotropic elastic layer, International Journal of Advances in Applied Mathematics and Mechanics 3(1) (2015) 62-70.