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Heat conduction problem in an elliptical membrane and its associated thermal stresses

Research Article

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Abstract: The method of integral transformation technique is used to generate an exact solution of heat conduction equation in an elliptical membrane under thermal radiation type boundary conditions on both curved surfaces. The solution to conductivity equation is obtained by employing a new extended integral transform. The solution of stress components is achieved by using Goodier's and Airy's potential function, involving the Mathieu and modified functions and their derivatives. The numerical results obtained using these computational tools are found to be nearly accurate and can be utilised for practical purposes and for the better estimates of the thermal effect in the elliptical membrane. A conclusion also give emphasis for the better understanding of elliptic structures even taking circular object profile in consideration, and also better estimates the thermal effect of the thermoelastic problem.

MSC: 35B07 • 35G30 • 35K05 • 44A10

Keywords: Elliptical membrane • Temperature distribution • Thermal stresses • Elliptical co-ordinate • Integral transform

1. Introduction

The theoretical study of the heat conduction within a hollow elliptical structure has a great practical importance in a wide range of fields such as mechanical, aerospace, nuclear turbines, electronic gadgets and food engineering for the past few decades. Unfortunately, there are very few studies done with steady and transient state heat conduction problems in elliptical objects. Erdoğdu et al.[1, 2] investigated the heat conduction within an elliptical cylinder using a finite difference method. McLachlan [3] obtained a mathematical solution of the heat conduction problem for an elliptical cylinder in the form of an infinite Mathieu function series considering special case with neglecting surface resistance. Gupta [4] introduced a finite transform involving Mathieu functions and used for obtaining the solutions of boundary value problem involving elliptic cylinders. Choubey [5] also introduced a finite Mathieu transform whose kernel is given by Mathieu function to solve heat conduction in a hollow elliptic cylinder with radiation. Kirkpatic et al. [6] extended the McLachlan's solution with the involvement of numerical calculation. Sugano et al. [7] dealt with transient thermal stress in confocal hollow elliptical structures with both face surfaces insulated perfectly and obtained the analytical solution with couple-stresses. Sato [8] subsequently obtained heat conduction problem of an infinite elliptical cylinder during heating and cooling considering the effect of the surface resistance. Recently, El Dhaba [9] used boundary integral method to solve the problem of the plane, uncoupled linear thermoelasticity with heat sources for an infinite cylinder with elliptical cross section, subjected to a uniform pressure and a thermal radiation condition on its boundary. Very recently, Bhad et al. [13–16] and Dhakate et al. [17] has obtained few

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Nomenclature

ξ, η	Elliptical Coordinates
q	Parameter of Mathieu equation
$ce_n(\eta, q)$	Ordinary Mathieu function of first kind of order n
$Ce_n(\xi, q)$	Modified Mathieu function of second kind of order n
h	Interfocal distance [and $1/h^2 = c^2(\cosh\xi - \cos 2\eta)/2$]
k	Thermal conductivity
$Ce_n(k_j, \xi_i, q)$	Mathieu function defined in Eq. (11)
$Fey_n(k_j, \xi_o, q)$	Mathieu function defined in Eq. (11)
$S_{n,m}(k_1, k_2, \xi, \eta, q_{n,m})$	Mathieu function defined in Eq. (11)
$q_{n,m}$	Parametric roots of Eq. (13)
$\bar{f}(q_{n,m})$	Mathieu transform of $f(\xi, \eta)$
$\theta, (\xi, \eta, t)$	The temperature distribution at any time t
ϕ	The Airy's stress functions
$f(\eta, t)$	The heat supply available on curved surface
$2c$	Focal length, $= 2\sqrt{a_i^2 - b_i^2} = 2\sqrt{a_o^2 - b_o^2}$
ξ_i, ξ_o	$= \tanh^{-1}(b_i/a_i)$, $= \tanh^{-1}(b_o/a_o)$
$C_{n,m}$	$= \int_{\xi_i}^{\xi_o} \int_0^{2\pi} (\cosh 2\xi - \cos 2\eta)\, S_{n,m}^2(k_1, k_2, \xi, \eta, \pm q_{n,m})\, d\xi\, d\eta$

thermoelastic solution for elliptical objects using integral transform technique. The above reviews clearly suggest that, in contrast with the classical circular or rectangular structures case, nearly all investigators so far focused on thermoelastic problems in elliptical membranes either in the steady or unsteady state. However, there aren't many investigations done or studied to eliminate thermoelastic problems successfully.

Researchers have not considered any thermoelastic problem expressed in elliptical coordinates with boundary conditions of radiation type which satisfies the time-dependent heat conduction equation. It has been proved that ample cases of heat conduction in solids have led to various technical problems in mechanical applications. For instance, gas turbines blades, walls of I.C. engine, the outer surface of a space vehicle and other factors all depend on for their durability on rapid heat transfer from their surfaces. By considering a circle as a special case of an ellipse, it is shown that the temperature distribution in a circular membrane and its solution can be derived as a special case of the present mathematical solution for the elliptical membrane.

The success of this research mainly lies with the analytical procedures which present a much simpler approach for optimisation of the design regarding material usage and performance in engineering problem, particularly in the determination of thermoelastic behaviour in elliptical membrane engaged as the foundation of pressure vessels, furnaces, etc. In this paper, we have extended the integral transformation defined by one of the author in Ref. [5] involving ordinary and modified Mathieu functions of first and second kind of order n which is analogous to the finite Hankel transform. Integral transformation and its inversion formula are established. We do not claim to have obtained new integral transformation, but certainly, we have modified integral transform suiting to our boundary conditions and applied the transformation to determine the temperature distribution in a finite elliptical membrane occupying the space $D = \{(\xi, \eta, z) \in R^3 : \xi_i \le \xi \le \xi_o, 0 \le \eta \le 2\pi\}$.

2. Formulation of the problem

The thermoelastic issue of an elliptical membrane subjected to radiation type boundary conditions on the outside and inside surfaces can be rigorously analysed by introducing the elliptical coordinates (ξ, η), which are related to the rectangular coordinates (x, y) of the relation

$$x = c\cosh\xi\cos\eta, \quad y = c\sinh\xi\sin\eta \tag{1}$$

in which c is the semi-focal length as shown in Fig. 1. From the above equations, one obtains a group of confocal ellipses and hyperbolas with the common foci for various values of ξ and η, respectively.

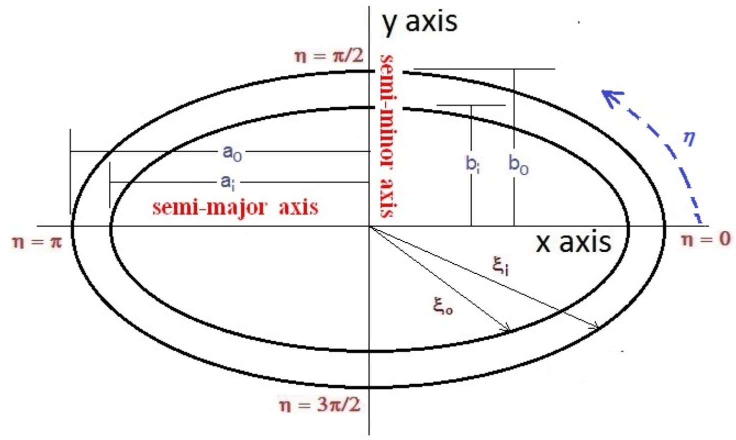

Fig. 1. Shows the geometry of the problem

2.1. Transient Heat Conduction Analysis

The governing equation of heat conduction with the initial condition and boundary conditions in elliptical cylindrical coordinates are given, respectively as

$$h^2\,(\theta_{,\xi\xi} + \theta_{,\eta\eta})\,(\xi,\eta,t) = (1/\kappa)\,\theta_{,t}\,(\xi,\eta,t) \tag{2}$$

$$\theta(\xi,\eta,t)\big|_{t=0} = \theta_0 \tag{3}$$

$$\theta(\xi,\eta,t) + k_1\,\theta(\xi,\eta,t)_{,\xi} = 0, \tag{4}$$

$$\theta(\xi,\eta,t) + k_2\,\theta(\xi,\eta,t)_{,\xi} = 0, \tag{5}$$

in which $\theta(\xi,\eta,t)$ is the temperature function, $k_i\,(i=1,2)$ are radiation coefficients,$\kappa = \lambda/\rho C$ represents thermal diffusivity in which λ being the thermal conductivity of the material, ρ is the density and C is the calorific capacity, assumed to be constant. The Eqs. (2) to (5) constitute the mathematical formulation for temperature change within the elliptical membrane.

2.2. Displacement and thermal stress analysis

Following Gosh [10] and Jeffery [11], the displacements are given by

$$\left.\begin{aligned}(2\mu)\,u/h &= -\phi(\xi,\eta,t)_{,\xi} + P(\xi,\eta,t)_{,\eta}/h^2,\\ (2\mu)\,v/h &= -\phi(\xi,\eta,t)_{,\eta} + P(\xi,\eta,t)_{,\xi}/h^2\end{aligned}\right\} \tag{6}$$

in which (u,v) are displacements in the directions normal to the curves (ξ,η) constant, P satisfies the equations

$$\left.\begin{aligned}&\nabla^2 P = 0,\\ &(\lambda+\mu)\,[(h^{-2}P_{,\eta})_{,\xi} + (h^{-2}P_{,\xi})_{,\eta}] = (\lambda+2\mu)[(\phi_{,\xi\xi} + \phi_{,\eta\eta})]\end{aligned}\right\} \tag{7}$$

and stress function in Eq. (6) satisfies the following equation of the fourth order

$$h^2\nabla^2 h^2\nabla^2\phi = -h^2\nabla^2\theta \tag{8}$$

The components of the stresses are represented as

$$\left.\begin{aligned}\sigma_{\xi\xi} &= h^2\phi_{,\eta\eta} + (e^2h^4/2)\,\sinh 2\xi\,\phi_{,\xi} - (e^2h^4/2)\,\sin 2\eta\,\phi_{,\eta},\\ \sigma_{\eta\eta} &= h^2\phi_{,\xi\xi} - (e^2h^4/2)\,\sinh 2\xi\,\phi_{,\xi} + (e^2h^4/2)\,\sin 2\eta\,\phi_{,\eta},\\ \sigma_{\xi\eta} &= -h^2\phi_{,\xi\eta} + (e^2h^4/2)\,\sin 2\eta\,\phi_{,\xi} + (e^2h^4/2)\,\sinh 2\xi\,\phi_{,\eta}\end{aligned}\right\} \tag{9}$$

For traction free surface the stress functions

$$\sigma_{\xi\xi} = \sigma_{\xi\eta} = 0 \text{ at } \xi = \xi_i, \xi_o \tag{10}$$

The set of Eqs. (8) to (10) constitute mathematical formulation for displacement and thermal stresses developed within solid due to temperature change.

3. Solution for the Problem

3.1. Transient heat conduction analysis

To solve the fundamental differential equation, we firstly introduce the extended integral transformation of order n and m over the variable ξ and η as

$$\bar{f}(\pm q_{n,m}) = \int_{\xi_i}^{\xi_o} \int_0^{2\pi} f(\xi,\eta)(\cosh 2\xi - \cos 2\eta)\, S_{n,m}(k_1,k_2,\xi,\eta,\pm q_{n,m})\, d\xi\, d\eta \tag{11}$$

in which the kernel can be given as

$$\begin{aligned} S_{n,m}(k_1,k_2,\xi,\eta,\pm q_{n,m}) = {} & Ce_n(\xi,\pm q_{n,m})\, ce_n(\eta,\pm q_{n,m})\, [Fey_n(k_1,\xi_i,\pm q_{n,m}) \\ & + Fey_n(k_2,\xi_o,\pm q_{n,m})] + Fey_n(\xi,\pm q_{n,m})\, ce_n(\eta,\pm q_{n,m}) \\ & \times [Ce_n(k_1,\xi_i,\pm q_{n,m}) + Ce_n(k_2,\xi_o,\pm q_{n,m})] \end{aligned}$$

Inversion theorem of (11) is

$$f(\xi,\eta) = \sum_{n=0}^{\infty}\sum_{m=1}^{\infty} \bar{f}(\pm q_{n,m}) S_{n,m}(k_1,k_2,\xi,\eta,\pm q_{n,m}) / C_{n,m} \tag{12}$$

in which $\pm q$ are the roots of the transcendental equation

$$Ce_n(k_1,\xi_i,\eta,\pm q)\, Fey_n(k_2,\xi_o,\eta,\pm q) - Ce_n(k_2,\xi_0,\eta,\pm q)\, Fey_n(k_1,\xi_i,\eta,\pm q) = 0 \tag{13}$$

and

$$C_{n,m} = \int_{\xi_i}^{\xi_o} \int_0^{2\pi} (\cosh 2\xi - \cos 2\eta)\, S^2_{n,m}(k_1,k_2,\xi,\eta,\pm q_{n,m}) d\xi\, d\eta \tag{14}$$

Performing above integral transformation under the conditions (4) and (5), we obtain

$$\overline{\theta},_t (n,m,t) + \alpha^2_{n,m} \overline{\theta}(n,m,t) = 0 \tag{15}$$

in which $\bar{\theta}(n,m,t)$ is the transformed function of $\theta(\xi,\eta,t)$ and $\alpha^2_{n,m} = 2\kappa\, q_{n,m}\, h^2$. On solving (15) under initial boundary condition given in Eq. (3), one obtain

$$\overline{\theta}(n,m,t) = \theta_0 \exp(-\alpha_{n,m}\, t) \tag{16}$$

On applying inversion theorems defined in (12), one obtain the expression for temperature as

$$\theta(\xi,\eta,\ t) = \theta_0 \sum_{n=0}^{\infty}\sum_{m=1}^{\infty} S_{n,m}(k_1,k_2,\xi,\eta,q_{n,m}) \exp(-\alpha_{n,m}\, t)/C_{n,m} \tag{17}$$

The function given in Eq. (17) represents the temperature at every instant and at all points of elliptical membrane under the influence of radiation.

3.2. Thermoelastic solution

Assuming Airy's stress function $\phi(\xi,\eta,t)$ which satisfies condition (8) as,

$$\phi(\xi,\eta,t) = \theta_0 \sum_{n=0}^{\infty}\sum_{m=1}^{\infty} \left(\frac{\xi + X_{n,m}\xi^2 + Y_{n,m}\eta^2}{C_{n,m}\,(\omega - \alpha_{n,m})} \right) S_{n,m}(k_1,k_2,\xi,\eta,q_{n,m}) \exp(-\alpha_{n,m}\, t) \tag{18}$$

Arbitrary functions $X_{n,m}$ and $Y_{n,m}$ are determined using Eq. (18) and (9) in Eq. (10). Thus Eq. (18) becomes

$$\phi(\xi,\eta,t) = \sum_{n=0}^{\infty}\sum_{m=1}^{\infty} \left\{ \frac{(-\omega+\alpha_{n,m}) \exp[t(-\omega+\alpha_{n,m})]\, C_1 S_{n,m}(\xi,\eta) + C_2 S_{n,m}(\xi_0,\eta)}{2C_{n,m}(-\omega+\alpha_{n,m})^2 \xi_0 (2A_1 + C_3 S_{n,m}[\xi_0,\eta] + C_4) + 4\eta C_5 S_{n,m}[\xi_0,\eta]} \right\} \times \exp(-\alpha_{n,m}\, t) \tag{19}$$

Now using Eq. (19) in (9), one obtains the resulting equations of stresses (i.e. $\sigma_{\xi\xi}$, $\sigma_{\eta\eta}$ and $\sigma_{\xi\eta}$) which are rather lengthy, and consequently are omitted here for the sake of brevity, but considered during graphical discussion described in below section.

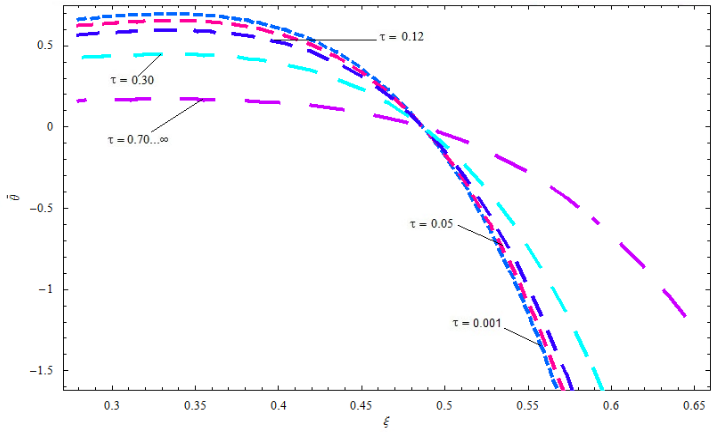

Fig. 2. Temperature distribution versus ξ at η=90^0 for different values of time

4. Numerical Results, Discussion and Remarks

For the sake of simplicity of calculation, we introduce the following dimensionless values

$$\left.\begin{array}{l}\bar{b}_o = b_o/a_o,\ \overline{b_i} = b_i/a_i,\ e = c/a_o,\ \bar{h}^2 = h^2 a_o^2,\ \tau = \kappa\, t/a_o^2,\\ \overline{\theta}(\xi,\eta,\ t) = \theta(\xi,\eta,\ t)/\theta_k,\ (\overline{\theta}_i,\overline{\theta}_o) = (\theta_i,\theta_o)/\theta_k\ (k = i, o),\\ \bar{\phi}(\xi,\eta,t) = \phi(\xi,\eta,t)/E\alpha_t\theta_k a_o^2,\ \bar{\sigma}_{ij} = \sigma_{ij}/E\alpha_t\theta_k\ (i,j = \xi,\eta)\end{array}\right\} \tag{20}$$

Here E stands for Young's modulus, α_t for Thermal expansion coefficient, respectively. Substituting the values in Eqs. (17), (19) and (20), we obtained the expressions for the temperature, displacement and stresses respectively for our numerical discussion. The numerical computations have been carried out for Aluminum metal with parameter a=2.65 cm, b=3.22 cm, h=2.00 cm, Modulus of Elasticity E= 6.9 × 10^{6}N/cm^2, Shear modulus G= 2.7 × 10^{6}N/cm^2, Poisson ratio υ = 0.281, Thermal expansion coefficient α_t= 25.5× 10^{6}cm/cm-^{0}C, Thermal diffusivity κ= 0.86 cm^2/sec, Thermal conductivity λ = 0.48 calsec^{-1}/cm ^{0}C with $q_{n,m}$ =0.0986, 0.3947, 0.8882, 1.5791, 2.4674, 3.5530, 4.8361, 6.3165, 7.9943, 9.8696, 11.9422, 14.2122, 16.6796, 19.3444, 22.2066, 25.2661, 28.5231, 31.9775, 35.6292, 39.4784 are the positive & real roots of the transcendental equation (A). The foregoing analysis is performed by setting the radiation coefficients constants, k_i = 0.86 (i = 1, 2) so as to obtain considerable mathematical simplicities. In order to examine the influence of uniform heating on the membrane, we performed the numerical calculation for time τ = 0.001, 0.05, 0.12, 0.30, 0.70..∞ and numerical calculations are depicted in the following figures with the help of MATHEMATICA software. The theoretical analysis on the heat conduction & its thermal stress in a confocal hollow elliptical plate without internal heat source subjected to non-axisymmetric heating on internal and outer elliptical boundaries was investigated by integral transform by Sugano et al. [7], where kernel was expressed in the form of Mathieu and modified Mathieu functions. The thermoelastic effects on the temperature, displacements, and thermal stresses without internal heat source are fully discussed in research paper [7]. For the sake of brevity, discussion of these effects is omitted here and graphical illustration is investigated for thermoelastic responses for an elliptical membrane considering interior heat generation. Figs. 2-6 illustrate the numerical results of dimensionless temperature, displacement and stresses of elliptical membrane under thermal boundary condition that are subjected to arbitrary initial temperature on the upper and lower face at zero temperature and boundary conditions of radiation type on the outside and inside surfaces, with independent radiation constants in radial direction at η=90^{0}for different values of time. As shown in Fig. 2, the temperature falls as the time proceeds along radial direction and is greatest in a steady & initial state. From Fig. 2, it can be seen that the temperature change on the heated surface decreases when the radius of plate increases. Fig. 3 shows the variation of displacement in the radial direction. It can be seen from Fig. 3 that the displacement increases when the radius increases. The variation of normal stresses$\bar{\sigma}_{\xi\xi}$,$\bar{\sigma}_{\eta\eta}$,and $\bar{\sigma}_{\xi\eta}$is shown in Figs. 4, 5 6, respectively. From Fig. 4, the large compressive stress occurs on the inner heated surface and the tensile stress occurs on the inner surface which drops along the radial direction. From Fig. 5, the compressive stress occurs on the outer edge of the ellipse and the absolute value rises as the time proceeds. From Fig. 6, the maximum tensile stress occurs during uniform heating inside the core of the membrane which follows assumed traction free property. Figs. 6 to 9 shows dimensionless temperature, displacement and thermal stresses along angular direction. Fig. 7 shows the time variation of temperature distribution along angular direction of the membrane. The temperature increases with time, and the maximum value of temperature magnitude occurs at higher steady state with available internal heat energy. The aforementioned results agree with the results [7]. The distribution of the dimensionless temperature gradient at each time decreases in the unheated area of the outer ellipse boundary tending below zero in one direction. Fig. 8 shows the thermal displacement in η direction of the membrane. It is noted from this graph that the values

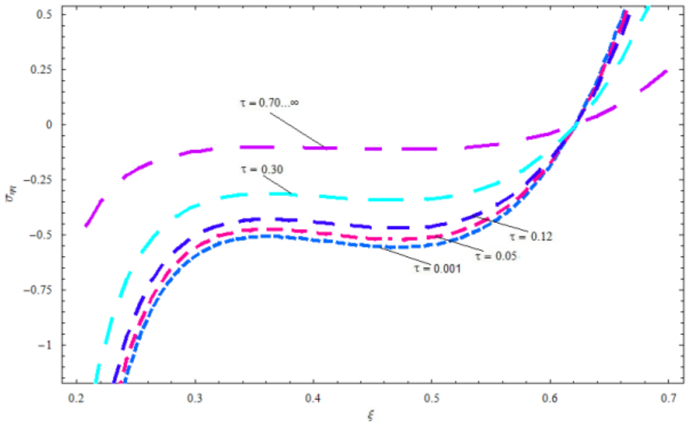

Fig. 3. $\bar{\sigma}_{\eta\eta}$ versus ξ at η= 90^0 for different values of time

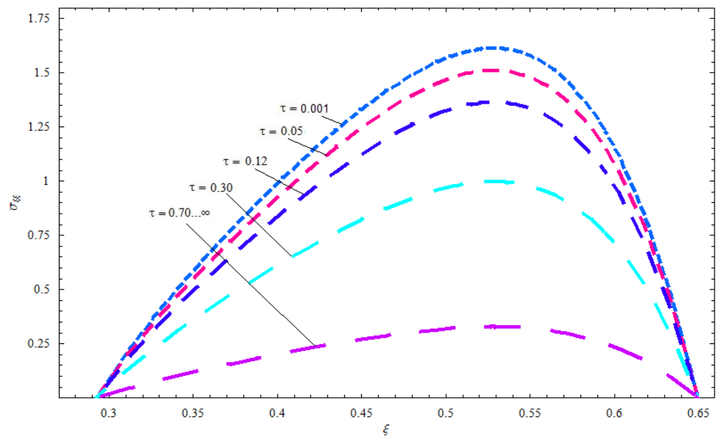

Fig. 4. $\bar{\sigma}_{\xi\xi}$ versus ξ at η= 90^0 for different values of time

of thermal displacement decreases over time and their maximum values are located beyond the neighborhood from central part. The stress distributions are shown from Fig. 89. It is observed that the stress patterns from elliptical inner hole to mid core part which follows the similar pattern of the applied mechanical boundary conditions. The radial stress $\bar{\sigma}_{\xi\xi}$, circumferential stress $\bar{\sigma}_{\eta\eta}$ and shear stress $\bar{\sigma}_{\xi\eta}$ at inner surface are nearly zero due to the assumed traction free boundary conditions. It is noted that maximum tensile stress occurs near the outer surface and the compressive stress occurs inside the membrane and its absolute value increases with time.

5. Transition to annular-circular plate

When the elliptical membrane degenerates into an annular circular membrane with the thickness $h \to 0$, internal radius ξ_i, and external radius $\xi_o \to \infty$, occupying the space $D' = \{(x, y, z) \in R^3 : a \le (x^2 + y^2)^{1/2} \le b, z = \ell\}$, where $r = (x^2 + y^2)^{1/2}$ in such a way that $h\ \exp(\xi)/2 \to r, h\ \exp(\xi_i)/2 \to a$, and $h\ \exp(\xi_0)/2 \to b$ [3] and taking θ independent of η. For that we take,

$$n = 0, q \to 0, e \to 0, \cosh 2\xi\, d\xi \to 2rh^2 dr, A_2^{(0)} \to 0, A_0^{(0)} \to 1/\sqrt{2}, \tag{21}$$

$$\lambda_{0,m}^2 \to \alpha_m^2, ce_0(\eta, q_{0,m}) \to 1/\sqrt{2}, ce_0(\xi, q_{0,m}) \to J_0(\alpha_m r), Fey_0(\xi_0, q_{0,m}) \to Y_0(\alpha_m r), \tag{22}$$

$\alpha_m (= \alpha_{0,m})$are the roots of$J_0(k_1, \alpha_a)\, Y_0(k_2, \alpha_b) - J_0(k_2, \alpha_b)\, Y_0(k_2, \alpha_a) = 0$ Where,

$$\left.\begin{aligned} J_0(k_j, \alpha_i r) &= J_0(\alpha_i r) + k_j J_0'(\alpha_i r) \\ Y_0(k_j, \alpha_i r) &= Y_0(\alpha_i r) + k_j\, Y_0'(\alpha_i r) \end{aligned}\right\} i = 1, 2 \tag{23}$$

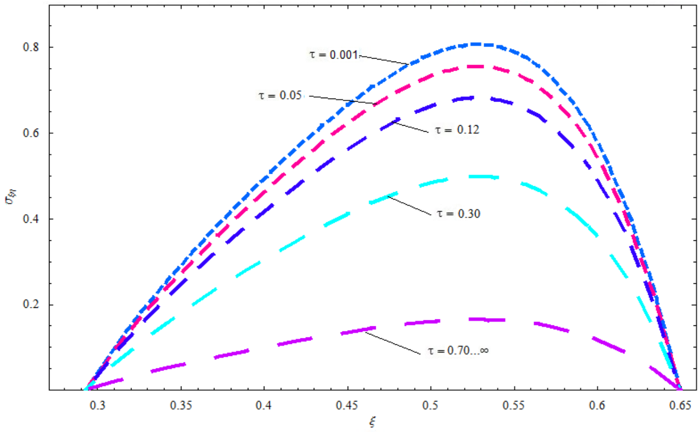

Fig. 5. $\bar{\sigma}_{\xi\eta}$ versus ξ at η= 90^0 for different values of time

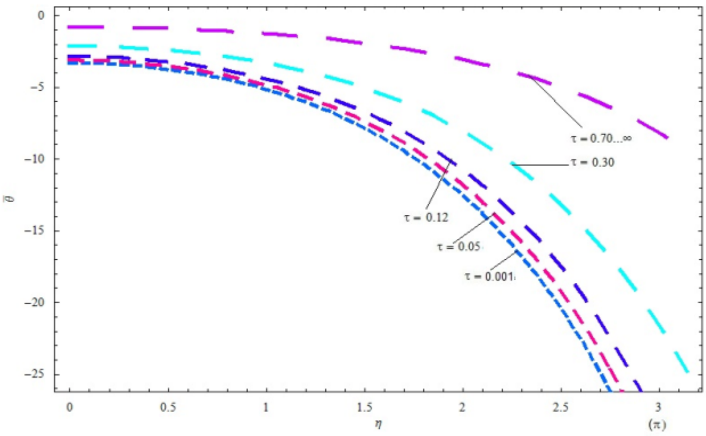

Fig. 6. Temperature distribution versus η at ξ= 0.45 for different values of time

$$\left.\begin{aligned} &Ce_0(k_1,\xi,\eta,q_{0,m}) \to Ce_0(k_1,r\alpha_m) \\ &Fey_0(k_1,\xi,\eta,q_{0,m}) \to Fey_0(k_1,r\alpha_m) \\ &S_{0,m}(k_1,k_2,\xi,\eta,q_{0,m}) \to S_{0,m}(k_1,k_2,r\alpha_m) \quad (= S_m(k_1,k_2,r\alpha_m)) \end{aligned}\right\} \tag{24}$$

Eq. (17) degenerates into temperature distribution in hollow circular membrane

$$\theta\,(r,\ z,\ t) \ = \ \sum_{m=1}^{\infty} \frac{1}{C_m}\left(1+\frac{\exp\left(\alpha_m-\omega\right)t}{\alpha_m-\omega}\right) S_m(k_1,k_2,r\alpha_m)\,\exp(-\alpha_m\,t) \tag{25}$$

Where,

$$C_m = \int_a^b r\,S_m^2(k_1,k_2,r\alpha_m)\,dr,$$

and kernel as

$$\begin{aligned} S_m(k_1,k_2,r\alpha_m) = {} & J_0(r\alpha_m)\,[Y_0(k_1,a\alpha_m)+Y_0(k_2,b\alpha_m)] \\ & - Y_0(r\alpha_m)\,[J_0(k_1,a\alpha_m)+J_0(k_2,b\alpha_m)] \end{aligned}$$

The aforementioned results agree with the results [12].

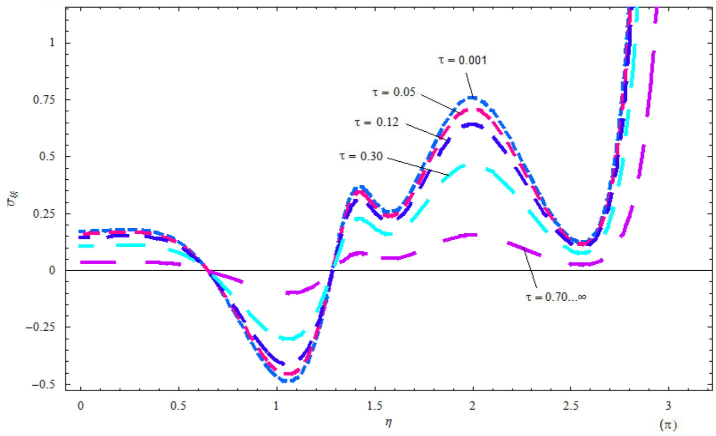

Fig. 7. $\bar{\sigma}_{\xi\xi}$ versus η at ξ= 0.45 for different values of time

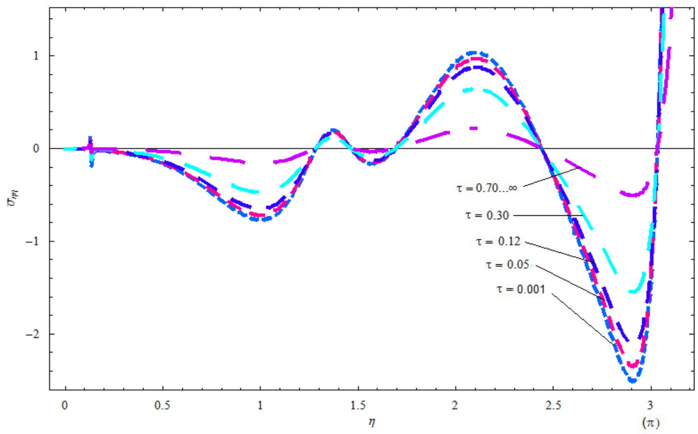

Fig. 8. $\bar{\sigma}_{\eta\eta}$ versus η at ξ= 0.45 for different values of time

6. Conclusion

The proposed analytical solution of transient plane thermal stress problem of the confocal elliptical region was handled in elliptical coordinate system. To author's knowledge there have been no reports of solution so far in which sources are generated according to the linear function of the temperature in mediums in the form of elliptical membrane of finite height with boundaries conditions of the radiation type. The analysis of non-stationary two-dimensional equation of heat conduction is investigated with the integral transformation method as when there are conditions of radiation type contour acting on the object under consideration. With proposed integral transformation method, it is possible to apply widely to analysis stationary as well as non-stationary temperatures. Also by using the Airy's stress function induced by Sugano [7], we have proposed an exact solution theoretically and illustrated graphically for better understanding. The following results were obtained to carry away during our research are:

1. The advantage of this method is its generality and its mathematical power to handle different types of mechanical and thermal boundary conditions.

2. The maximum tensile stress is shifting from central core to outer region may be due to heat, stress, concentration under considered temperature field.

3. Finally, the maximum tensile stress occurs in the circular hole on the major axis compared to elliptical hole indicates that the distribution of weak heating. It may be due to insufficient penetration of heat through elliptical inner surface. The aforementioned integral transform will also be extended to other elliptical objects having finite height with conditions of radiation type contour during further research work.

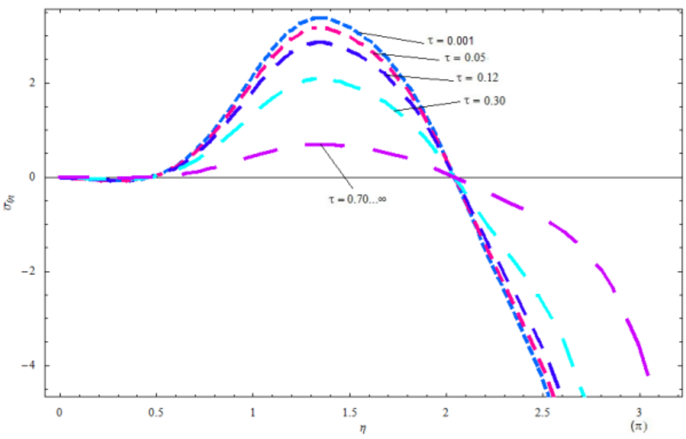

Fig. 9. $\bar{\sigma}_{\xi\eta}$ versus η at ξ= 0.7 for different values of time

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