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Free oscillations of water in a circular lake and the modified H-function of several variables with general class of polynomials and Srivastava-Daoust function

Research Article

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Abstract: Chaurasia et al. [1] have studied the free oscillations of water in a circular lake and the H-function of several variables, the Fox's H-function with a general class of polynomials. The object of this paper is to discuss the application of certain products involving the classes of polynomials and multivariable polynomials, the Srivastava-Daoust function and the modified multivariable H-function defined by Prasad and Singh [2] in obtaining a solution of the partial differential equation concerning to free oscillations of water in a circular lake. We shall see the particular cases.

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1. Introduction

The free oscillations of water in a lake is given by the following partial differential equation

$$x^2\frac{\partial^2\psi}{\partial x^2}+x\frac{\partial\psi}{\partial x}+\frac{\partial^2\psi}{\partial\theta^2}+\rho^2x^2\psi=0 \tag{1}$$

where ψ shows the depth of water surface from its position of equilibrium and $\rho=\frac{y}{\sqrt{\alpha_1\beta_1}}$ and with the following considerations that

1. In any vibrational mode ψ varies harmonically with line and ψ is small enough for its square to be neglected .
2. The lake is stationary in space.
3. There is no loss of energy.

We put the solution of (1) as follows (McLachlan [3], page 62)

$$\psi(x,\theta,t)=\sum_{\mu=0}^{\infty}R_\mu J_\mu(\beta x)\cos(\mu\theta-\mu\phi)\cos(\mu\rho t-\mu\xi) \tag{2}$$

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where $\theta = 0 = t$, let $\psi(x,0,0) = f(x)$ Letting

$$f(x) = x^{\lambda} S_N^M(zx^{2h}) S_{N_1,\cdots,N_u}^{M_1,\cdots,M_u}\begin{pmatrix} y_1 x^{2\rho_1} \\ \cdots \\ \cdots \\ y_u x^{2\rho_u}\end{pmatrix} F\begin{pmatrix} t_1 x^{2\sigma_1} \\ \cdots \\ \cdots \\ t_v x^{2\sigma_v}\end{pmatrix} H\begin{pmatrix} Z_1 x^{2\eta_1} \\ \cdots \\ \cdots \\ Z_s x^{2\eta_s}\end{pmatrix} \tag{3}$$

The generalized polynomials defined by Srivastava [4], is given in the following manner :

$$S_N^M(x) = \sum_{K=0}^{[N/M]} \frac{(-N)_{MK}}{K!} A[N,K] x^K \tag{4}$$

The class of multivariable polynomials defined by Srivastava [5], is given in the following manner:

$$S_{N_1,\cdots,N_u}^{M_1,\cdots,M_u}[y_1,\cdots,y_u] = \sum_{K_1=0}^{[N_1/M_1]} \cdots \sum_{K_u=0}^{[N_u/M_u]} \frac{(-N_1)_{M_1K_1}}{K_1!} \cdots \frac{(-N_u)_{M_uK_u}}{K_u!} A[N_1,K_1;\cdots;N_u,K_u] y_1^{K_1} \cdots y_u^{K_u} \tag{5}$$

The Srivastava-Daoust function is defined by (see [6]):

$$F_{\bar{C}:D^{(1)};\cdots,D^{(v)}}^{\bar{A}:B^{(1)};\cdots;B^{(v)}}\left(\begin{array}{c|c} x_1 & :[(b');\phi'];\cdots;[(b^{(v)});\phi^{(v)}] \\ \vdots & . \\ x_v & [(c);\psi',\cdots,\psi^{(v)}]:[(d');\delta'];\cdots;[(d^{(v)});\delta^{(v)}] \end{array}\right) = \sum_{r_1,\cdots,r_v=0}^{\infty} A' \frac{z_1^{r_1}\cdots z_v^{r_v}}{r_1!\cdots r_v!} \tag{6}$$

where

$$A' = \frac{\prod_{j=1}^{\bar{A}} (a_j)_{m_1\theta'_j+\cdots+m_v\theta_j^{(v)}} \prod_{j=1}^{B^{(1)}} (b'_j)_{m_1\phi'_j} \cdots \prod_{j=1}^{B^{(v)}} (b_j^{(v)})_{m_v\phi_j^{(v)}}}{\prod_{j=1}^{\bar{C}} (c_j)_{m_1\psi'_j+\cdots+m_v\psi_j^{(v)}} \prod_{j=1}^{D^{(1)}} (d'_j)_{m_1\delta'_j} \cdots \prod_{j=1}^{D^{(v)}} (d_j^{(v)})_{m_v\delta_j^{(v)}}} \tag{7}$$

The modified H-function defined by Prasad and Singh [2] generalizes the multivariable H-function defined by Srivastava and Panda [7, 8]. It is defined in term of multiple Mellin-Barnes type integral:

$$H(z_1,\cdots,z_r)$$

$$= H_{p,q:|R:p_1,q_1;\cdots,p_r,q_r}^{m,n:|R':m_1,n_1;\cdots;m_r,n_r}\left(\begin{array}{c|c} z_1 & (a_j;\alpha_j^{(1)},\cdots,\alpha_j^{(r)})_{1,p}:(e_j;u'_jg'_j,\cdots,u_j^{(r)}g_j^{(r)})_{1,R'}:(c_{j'};\gamma_{j'})_{1,p_1},\cdots,(c_j^{(r)};\gamma_j^{(r)})_{1,p_r} \\ \vdots & \\ z_r & (b_j;\beta_j^{(1)},\cdots,\beta_j^{(r)})_{1,q}:(l_j;U'_jf'_j,\cdots,U_j^{(r)}f_j^{(r)})_{1,R}:(d_{j'};\delta_{j'})_{1,q_1},\cdots,(d_j^{(r)};\delta_j^{(r)})_{1,q_r} \end{array}\right) \tag{8}$$

$$= \frac{1}{(2\pi\omega)^r} \int_{L_1} \cdots \int_{L_r} \phi(s_1,\cdots,s_r) \prod_{i=1}^{r} \theta_i(s_i) z_i^{s_i} ds_1 \cdots ds_r \tag{9}$$

where $\phi(s_1,\cdots,s_r), \theta_i(s_i), i = 1,\cdots,r$ are given by:

$$\phi(s_1,\cdots,s_r) = \frac{\prod_{j=1}^{m} \Gamma(b_j - \sum_{i=1}^{r} \beta_j^{(i)} s_i) \prod_{j=1}^{n} \Gamma\left(1 - a_j + \sum_{i=1}^{r} \alpha_j^{(i)} s_j\right)}{\prod_{j=n+1}^{p} \Gamma\left(a_j - \sum_{i=1}^{r} \alpha_j^{(i)} s_j\right) \prod_{j=m+1}^{q} \Gamma\left(1 - b_j + \sum_{i=1}^{r} \beta_j^{(i)} s_j\right)} \frac{\prod_{j=1}^{R'} \Gamma(e_j + \sum_{i=1}^{r} u_j^{(i)} g_j^{(i)} s_i)}{\prod_{j=1}^{R} \Gamma(l_j + \sum_{i=1}^{r} U_j^{(i)} f_j^{(i)} s_i)} \tag{10}$$

$$\phi_i(s_i) = \frac{\prod_{j=1}^{n_i} \Gamma\left(1 - c_j^{(i)} + \gamma_j^{(i)} s_i\right) \prod_{j=1}^{m_i} \Gamma\left(d_j^{(i)} - \delta_j^{(i)} s_i\right)}{\prod_{j=n_i+1}^{p_i} \Gamma\left(c_j^{(i)} - \gamma_j^{(i)} s_i\right) \prod_{j=m_i+1}^{q_i} \Gamma\left(1 - d_j^{(i)} + \delta_j^{(i)} s_i\right)} \tag{11}$$

The multiple integral (9) converges absolutely if

$$\left|\arg Z_k\right| < \frac{1}{2} U_i \pi \quad (i = 1,\cdots,r) \tag{12}$$

$$U_i=\sum_{j=1}^{m}\beta_j^{(i)}-\sum_{j=m+1}^{q}\beta_j^{(i)}+\sum_{j=1}^{n}\alpha_j^{(i)}-\sum_{j=n+1}^{p}\alpha_j^{(i)}+\sum_{j=1}^{m_i}\delta_j^{(i)}-\sum_{j=1+m_i}^{q_i}\delta_j^{(i)}$$
$$+\sum_{j=1}^{n_i}\gamma_j^{(i)}-\sum_{j=n_i+1}^{p_i}\gamma_j^{(i)}+\sum_{j=1}^{R'}g_j^{(i)}-\sum_{j=1}^{R}f_j^{(i)}>0+(i=1,\cdots,r) \quad (13)$$

In this paper, we shall note

$$X=m_1,n_1;\cdots;m_r,n_r:Y=p_1,q_1;\cdots;p_r,q_r \quad (14)$$

$$\mathbb{A}=(a_j;\alpha_j^{(1)},\cdots,\alpha_j^{(r)})_{1,p}:(e_j;u'{}_jg'{}_j,\cdots,u_j^{(r)}g_j^{(r)})_{1,R'}:(c_{j'};\gamma_{j'})_{1,p_1},\cdots,(c_j^{(r)};\gamma_j^{(r)})_{1,p_r} \quad (15)$$

$$\mathbb{B}=(b_j;\beta_j^{(1)},\cdots,\beta_j^{(r)})_{1,q}:(l_j;U'{}_jf'{}_j,\cdots,U_j^{(r)}f_j^{(r)})_{1,R}:(d_{j'};\delta_{j'})_{1,q_1},\cdots,(d_j^{(r)};\delta_j^{(r)})_{1,q_r} \quad (16)$$

$$B'=\frac{(-N_1)_{M_1K_1}}{K_1!}\cdots\frac{(-N_u)_{M_uK_u}}{K_u!}A[N_1,K_1;\cdots;N_u,K_u] \quad (17)$$

$$F=F^{\bar{A}:B^{(1)};\cdots;B^{(v)}}_{\bar{C}:D^{(1)};\cdots;D^{(v)}} \quad (18)$$

(19)

2. Main integral

Lemma (Erdelyi et al. [9] page 49, Eq. (19))

$$\int_0^\infty x^{\lambda-1}J_v(\beta x)dx=\frac{2^{\lambda-1}\beta^{-\lambda}\Gamma\left(\frac{\lambda+v}{2}\right)}{\Gamma\left(1+\frac{v-\lambda}{2}\right)} \quad (20)$$

where $-Re(v)<Re(\lambda)<\frac{3}{2}$
We have the following integral.

Theorem 2.1.

$$\int_0^\infty x^{\lambda-1}J_v(\beta x)S_N^M(zx^{2h})S_{N_1,\cdots,N_u}^{M_1,\cdots,M_u}\begin{pmatrix}y_1x^{2\rho_1}\\ \cdots\\ \cdots\\ y_ux^{2\rho_u}\end{pmatrix}F\begin{pmatrix}t_1x^{2\sigma_1}\\ \cdots\\ \cdots\\ t_vx^{2\sigma_v}\end{pmatrix}H\begin{pmatrix}Z_1x^{2\eta_1}\\ \cdots\\ \cdots\\ Z_rx^{2\eta_r}\end{pmatrix}dx$$

$$=\frac{2^{\lambda-1}}{\beta^\lambda}\sum_{K=0}^{[N/M]}\sum_{K_1=0}^{[N_1/M_1]}\cdots\sum_{K_u=0}^{[N_u/M_u]}\sum_{r_1,\cdots,r_v=0}^{\infty}\frac{(-N)_{MK}}{K!}A[N,K]z^KA'B'\frac{y_1^{K_1}\cdots y_u^{K_u}t_1^{r_1}\cdots t_v^{r_v}4^{Kh+\sum_{i=1}^{u}\rho_iK_i+\sum_{i=1}^{v}\sigma_ir_i}}{r_1!\cdots r_v!\beta^{2(Kh+\sum_{i=1}^{u}\rho_iK_i+\sum_{i=1}^{v}\sigma_ir_i)}}$$
$$H^{m,n+1:|R':X}_{p+1,q+1:|R:Y}\left(\begin{array}{c|l}Z_1\beta^{-2h_1} & (1-\frac{v}{2}-\frac{\lambda}{2}-Kh-\sum_{i=1}^{u}\rho_iK_i-\sum_{i=1}^{v}\sigma_ir_i:2\eta_1,\cdots,2\eta_r),\mathbb{A}\\ \cdots & \cdots\\ \cdots & \cdots\\ Z_r\beta^{-2h_r} & (-\frac{v}{2}+\frac{\lambda}{2}-Kh-\sum_{i=1}^{u}\rho_iK_i-\sum_{i=1}^{v}\sigma_ir_i:2\eta_1,\cdots,2\eta_r),\mathbb{B}\end{array}\right) \quad (21)$$

provided,

$$\min\{h,\rho_i,\sigma_j,\eta_k\}>0, i=1,\cdots,s, j=1,\cdots,v,\quad k=1,\cdots,r$$

$$1+\sum_{j=1}^{\bar{C}}\psi_j^{(i)}+\sum_{j=1}^{D^{(i)}}\delta_j^{(i)}-\sum_{j=1}^{\bar{A}}\theta_j^{(i)}-\sum_{j=1}^{B^{(i)}}\phi_j^{(i)}>0;\quad i=1,\cdots,v$$

$$Re\left[\lambda+v+\sum_{i=1}^{u}K_i\rho_i+\sum_{i=1}^{v}r_i\sigma_i+\sum_{i=1}^{r}\eta_i\min_{1\leqslant j\leqslant m_i}\frac{d_j^{(i)}}{\delta_j^{(i)}}\right]>0$$

$\left|\arg Z_k\right|<\frac{1}{2}\pi U_i, U_i$ *is defined by (1.13)*

and

$$Re\left[\lambda-\sum_{i=1}^{u}K_i\rho_i-\sum_{i=1}^{v}r_i\sigma_i-\sum_{i=1}^{r}\eta_i\max_{1\leqslant j\leqslant n_i}\frac{1-c_j^{(i)}}{\gamma_j^{(i)}}\right]<\frac{3}{2}.$$

Proof. To prove (21), first expressing a class of polynomials $S_N^M(.)$ defined by Srivastava [4], a class of multivariable polynomials defined by Srivastava [5] $S_{N_1,\cdots,N_s}^{M_1,\cdots,M_s}[.]$ and the Srivastava-Daoust function [6] $F[.]$ in series with the help of (4), (5) and (6) respectively and we interchange the order of summations and x -integral (which is permissible under the conditions stated). Expressing the modified multivariable H-function of defined by Prasad and Singh [2] in Mellin-contour integral with the help of (9) and interchange the order of integrations which is justifiable due to absolute convergence of the integral involved in the process. Now collecting the power of x and evaluating the inner x-integral with the help of the Lemma. Interpreting the Mellin-Barnes contour integral in modified multivariable H-function, we obtain the desired result (21). □

3. Application in free oscillations of water in a circular lake

The solution of the problem posed is given by the following solution.

$$\psi(x,\theta,t)=\left(\frac{2}{\beta}\right)^{\lambda}\sum_{\mu=0}^{\infty}\sum_{K=0}^{[N/M]}\sum_{K_1=0}^{[N_1/M_1]}\cdots\sum_{K_u=0}^{[N_u/M_u]}\sum_{r_1,\cdots,r_v=0}^{\infty}\frac{(-N)_{MK}}{K!}$$
$$A[N,K]z^K A'B'\frac{y_1^{K_1}\cdots y_u^{K_u}t_1^{r_1}\cdots t_v^{r_v}4^{Kh+\sum_{i=1}^{u}\rho_iK_i+\sum_{i=1}^{v}\sigma_ir_i}}{r_1!\cdots r_v!\beta^{2(Kh+\sum_{i=1}^{u}\rho_iK_i+\sum_{i=1}^{v}\sigma_ir_i)}}\mu J_\mu(\beta x)\frac{\cos(\mu\theta-\mu\phi)\cos(\mu\rho t-\mu\xi)}{\cos(\mu\theta)\cos(\mu\xi)}$$
$$H_{p+1,q+1:|R:Y}^{m,n+1:|R':X}\left(\begin{array}{c|l} Z_1\beta^{-2h_1} & (1-\frac{v}{2}-\frac{\lambda}{2}-Kh-\sum_{i=1}^{u}\rho_iK_i-\sum_{i=1}^{v}\sigma_ir_i:2\eta_1,\cdots,2\eta_r),\mathbb{A}\\ \cdots & \cdots\\ \cdots & \cdots\\ Z_r\beta^{-2h_r} & (-\frac{v}{2}+\frac{\lambda}{2}-Kh-\sum_{i=1}^{u}\rho_iK_i-\sum_{i=1}^{v}\sigma_ir_i:2\eta_1,\cdots,2\eta_r),\mathbb{B}\end{array}\right) \tag{22}$$

Under the same conditions and notations needed for (21).

Proof. Let

$$f(x)=\sum_{\mu=0}^{\infty}R_\mu J_\mu(\beta x)\cos(\mu\phi)\cos(\mu\xi) \tag{23}$$

Multiplying both sides of (22) by $x^{-1}J_v(\beta x)$, integrate with respect to x from 0 to ∞ and use (21) and the orthogonal property of the Bessel functions. We thus obtain

$$R_v=\left(\frac{2}{\beta}\right)^{\lambda}\sum_{K=0}^{[N/M]}\sum_{K_1=0}^{[N_1/M_1]}\cdots\sum_{K_u=0}^{[N_u/M_u]}\sum_{r_1,\cdots,r_v=0}^{\infty}\frac{(-N)_{MK}}{K!}A[N,K]z^K A'B'$$
$$\frac{y_1^{K_1}\cdots y_u^{K_u}t_1^{r_1}\cdots t_v^{r_v}4^{Kh+\sum_{i=1}^{u}\rho_iK_i+\sum_{i=1}^{v}\sigma_ir_i}}{r_1!\cdots r_v!\beta^{2(Kh+\sum_{i=1}^{u}\rho_iK_i+\sum_{i=1}^{v}\sigma_ir_i)}}\frac{1}{\cos(v\theta)\cos(v\xi)}$$
$$H_{p+1,q+1:|R:Y}^{m,n+1:|R':X}\left(\begin{array}{c|l} Z_1\beta^{-2h_1} & (1-\frac{v}{2}-\frac{\lambda}{2}-Kh-\sum_{i=1}^{u}\rho_iK_i-\sum_{i=1}^{v}\sigma_ir_i:2\eta_1,\cdots,2\eta_r),\mathbb{A}\\ \cdots & \cdots\\ \cdots & \cdots\\ Z_r\beta^{-2h_r} & (-\frac{v}{2}+\frac{\lambda}{2}-Kh-\sum_{i=1}^{u}\rho_iK_i-\sum_{i=1}^{v}\sigma_ir_i:2\eta_1,\cdots,2\eta_r),\mathbb{B}\end{array}\right) \tag{24}$$

Substituting the value of R_v from (24) in (2), we obtain the desired solution (22). □

4. Special case

(1) By applying our result given in (21) and (22) to the case of Hermite polynomials ([10], page 106, Eq.(5.54)) and ([11], page 158) and by setting

$$S_N^2 x)\rightarrow x^{n/2}H_n\left(\frac{1}{2\sqrt{x}}\right)$$

In which case $m=2, A_{N,K}=(-)^K$ we have the following interesting consequencies of the main results.

$$\int_0^\infty x^{\lambda-1}J_v(\beta x)H_N\left\{\frac{1}{2\sqrt{zx^{2h}}}\right\}S_{N_1,\cdots,N_u}^{M_1,\cdots,M_u}\begin{pmatrix} y_1x^{2\rho_1}\\ \cdots\\ \cdots\\ y_ux^{2\rho_u}\end{pmatrix}F\begin{pmatrix} t_1x^{2\sigma_1}\\ \cdots\\ \cdots\\ t_vx^{2\sigma_v}\end{pmatrix}H\begin{pmatrix} Z_1x^{2\eta_1}\\ \cdots\\ \cdots\\ Z_rx^{2\eta_r}\end{pmatrix}dx$$

$$=\frac{2^{\lambda-1}}{\beta^\lambda}\sum_{K=0}^{[N/2]}\sum_{K_1=0}^{[N_1/M_1]}\cdots\sum_{K_u=0}^{[N_u/M_u]}\sum_{r_1,\cdots,r_v=0}^{\infty}\frac{(-N)_{2K}}{K!}\$-z)^K A'B'\frac{y_1^{K_1}\cdots y_u^{K_u}t_1^{r_1}\cdots t_v^{r_v}4^{Kh+\sum_{i=1}^u\rho_iK_i+\sum_{i=1}^v\sigma_ir_i}}{r_1!\cdots r_v!\beta^{2(Kh+\sum_{i=1}^u\rho_iK_i+\sum_{i=1}^v\sigma_ir_i)}}$$

$$H_{p+1,q+1:|R:Y}^{m,n+1:|R':X}\left(\begin{array}{c|l} Z_1\beta^{-2h_1} & (1-\frac{v}{2}-\frac{\lambda}{2}-Kh-\sum_{i=1}^u\rho_iK_i-\sum_{i=1}^v\sigma_ir_i:2\eta_1,\cdots,2\eta_r),\mathbb{A}\\ \cdots & \cdots\\ \cdots & \cdots\\ Z_r\beta^{-2h_r} & (-\frac{v}{2}+\frac{\lambda}{2}-Kh-\sum_{i=1}^u\rho_iK_i-\sum_{i=1}^v\sigma_ir_i:2\eta_1,\cdots,2\eta_r),\mathbb{B}\end{array}\right) \tag{25}$$

under the same notations and conditions that (21) and

$$\psi(x,\theta,t)=\left(\frac{2}{\beta}\right)^\lambda\sum_{\mu=0}^{\infty}\sum_{K=0}^{[N/2]}\sum_{K_1=0}^{[N_1/M_1]}\cdots\sum_{K_u=0}^{[N_u/M_u]}\sum_{r_1,\cdots,r_v=0}^{\infty}\frac{(-N)_{2K}}{K!}(-z)^K$$

$$A'B'\frac{y_1^{K_1}\cdots y_u^{K_u}t_1^{r_1}\cdots t_v^{r_v}4^{Kh+\sum_{i=1}^u\rho_iK_i+\sum_{i=1}^v\sigma_ir_i}}{r_1!\cdots r_v!\beta^{2(Kh+\sum_{i=1}^u\rho_iK_i+\sum_{i=1}^v\sigma_ir_i)}}\mu J_\mu(\beta x)\frac{\cos(\mu\theta-\mu\phi)\cos(\mu\rho t-\mu\xi)}{\cos(\mu\theta)\cos(\mu\xi)}$$

$$H_{p+1,q+1:|R:Y}^{m,n+1:|R':X}\left(\begin{array}{c|l} Z_1\beta^{-2h_1} & (1-\frac{v}{2}-\frac{\lambda}{2}-Kh-\sum_{i=1}^u\rho_iK_i-\sum_{i=1}^v\sigma_ir_i:2\eta_1,\cdots,2\eta_r),\mathbb{A}\\ \cdots & \cdots\\ \cdots & \cdots\\ Z_r\beta^{-2h_r} & (-\frac{v}{2}+\frac{\lambda}{2}-Kh-\sum_{i=1}^u\rho_iK_i-\sum_{i=1}^v\sigma_ir_i:2\eta_1,\cdots,2\eta_r),\mathbb{B}\end{array}\right) \tag{26}$$

under the same notations and conditions that (21).

(2) For the Laguerre polynomials ([10], page 101, Eq.(15.1.6)) and ([11] , page 159) and by setting

$$S_N^1(x)\to L_N^{\alpha'}(x)$$

In which case $m=1, A_{N,K}=\binom{N+\alpha'}{N}\frac{1}{(\alpha'+1)_K}$ we have the following interesting consequencies of the main results.

$$\int_0^\infty x^{\lambda-1}J_v(\beta x)L_N^{\alpha'}(zx^{2h})S_{N_1,\cdots,N_u}^{M_1,\cdots,M_u}\begin{pmatrix} y_1x^{2\rho_1}\\ \cdots\\ \cdots\\ y_ux^{2\rho_u}\end{pmatrix}F\begin{pmatrix} t_1x^{2\sigma_1}\\ \cdots\\ \cdots\\ t_vx^{2\sigma_v}\end{pmatrix}H\begin{pmatrix} Z_1x^{2\eta_1}\\ \cdots\\ \cdots\\ Z_rx^{2\eta_r}\end{pmatrix}dx$$

$$=\frac{2^{\lambda-1}}{\beta^\lambda}\sum_{K=0}^{N}\sum_{K_1=0}^{[N_1/M_1]}\cdots\sum_{K_u=0}^{[N_u/M_u]}\sum_{r_1,\cdots,r_v=0}^{\infty}\binom{N+\alpha'}{N-K}\frac{(-z)^K}{K!}A'B'\frac{y_1^{K_1}\cdots y_u^{K_u}t_1^{r_1}\cdots t_v^{r_v}4^{Kh+\sum_{i=1}^u\rho_iK_i+\sum_{i=1}^v\sigma_ir_i}}{r_1!\cdots r_v!\beta^{2(Kh+\sum_{i=1}^u\rho_iK_i+\sum_{i=1}^v\sigma_ir_i)}}$$

$$H_{p+1,q+1:|R:Y}^{m,n+1:|R':X}\left(\begin{array}{c|l} Z_1\beta^{-2h_1} & (1-\frac{v}{2}-\frac{\lambda}{2}-Kh-\sum_{i=1}^u\rho_iK_i-\sum_{i=1}^v\sigma_ir_i:2\eta_1,\cdots,2\eta_r),\mathbb{A}\\ \cdots & \cdots\\ \cdots & \cdots\\ Z_r\beta^{-2h_r} & (-\frac{v}{2}+\frac{\lambda}{2}-Kh-\sum_{i=1}^u\rho_iK_i-\sum_{i=1}^v\sigma_ir_i:2\eta_1,\cdots,2\eta_r),\mathbb{B}\end{array}\right) \tag{27}$$

under the same notations and conditions that (21) and

$$\psi(x,\theta,t)=\left(\frac{2}{\beta}\right)^{\lambda}\sum_{\mu=0}^{\infty}\sum_{K=0}^{N}\sum_{K_1=0}^{[N_1/M_1]}\cdots\sum_{K_u=0}^{[N_u/M_u]}\sum_{r_1,\cdots,r_v=0}^{\infty}\binom{N+\alpha'}{N-K}\frac{(-z)^K}{K!}$$

$$A'B'\frac{y_1^{K_1}\cdots y_u^{K_u}t_1^{r_1}\cdots t_v^{r_v}4^{Kh+\sum_{i=1}^{u}\rho_iK_i+\sum_{i=1}^{v}\sigma_ir_i}}{r_1!\cdots r_v!\beta^{2(Kh+\sum_{i=1}^{u}\rho_iK_i+\sum_{i=1}^{v}\sigma_ir_i)}}\mu J_\mu(\beta x)\frac{\cos(\mu\theta-\mu\phi)\cos(\mu\rho t-\mu\xi)}{\cos(\mu\theta)\cos(\mu\xi)}$$

$$H^{m,n+1:|R':X}_{p+1,q+1:|R:Y}\left(\begin{array}{c|c} Z_1\beta^{-2h_1} & (1-\frac{v}{2}-\frac{\lambda}{2}-Kh-\sum_{i=1}^{u}\rho_iK_i-\sum_{i=1}^{v}\sigma_ir_i:2\eta_1,\cdots,2\eta_r),\mathbb{A} \\ \cdots & \cdots \\ \cdots & \cdots \\ Z_r\beta^{-2h_r} & (-\frac{v}{2}+\frac{\lambda}{2}-Kh-\sum_{i=1}^{u}\rho_iK_i-\sum_{i=1}^{v}\sigma_ir_i:2\eta_1,\cdots,2\eta_r),\mathbb{B} \end{array}\right) \quad (28)$$

under the same notations and conditions that (21).

5. Conclusion

Similarly, specializing the following coefficients $A[N,K]$, $A[N_1,K_1;\cdots;N_u,K_u]$ and and parameters of modified multivariable H-function defined by Prasad and Singh [2], and Srivastava-Daoust function, we can obtain large number of results involving various special functions of one and several variables useful in Mathematics analysis, Applied Mathematics, Physics and Mechanics in particular the problem concerning the free oscillations of water in a circular lake.

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