Int. J. Adv. Appl. Math. and Mech. 5(2) (2017) 1 – 8 (ISSN: 2347-2529)

Journal homepage: www.ijaamm.com

IJAAMM

International Journal of Advances in Applied Mathematics and Mechanics

Solving the problem of optimal control with free initial state under disturbance

Research Article

Kahina Louadj[a, c]*, Fethi Demim[b], Nait Abdesselam Aldjia[c], Mohamed Aidene[c], Abdelkrim Nemra[b]

[a] *Laboratoire d'Informatique, de Mathmatiques, et de Physique pour l'Agriculture et les Forts (LIMPAF). Universit de Bouira. Algria*

[b] *Laboratoire Robotique et Productique l'U.E.R Automatique, Ecole Militaire Polytechnique, Bordj El Bahri, Alger, Algria*

[c] *Laboratoire de Conception et Conduite de Systmes de Production (L2CSP) Tizi-Ouzou, Algeria*

Received 24 September 2017; accepted (in revised version) 15 November 2017

Abstract: The canonical optimal control problem with free initial condition for a linear time-dependent system in the class of discrete-time feasible controls is considered. The initial state of the optimized system is not known exactly, a priori information on the initial state is exhausted by inclusion $x_0 \in X_0$. Based on linear programming, method for synthesizing optimal feedback controls is presented. The results are illustrated by a forth-order problem.

MSC: 49Jxx • 58E25

Keywords: Linear programming • Adaptive method • Feedback control • Open-loop control

1. Introduction

Linear optimal control problems have been analyzed in the most detail in the literature on optimal processes [1, 2]. At the same time, despite numerous suggestions, no sufficiently efficient numerical solution methods completely adapted to these problems is available to this day. This is particularity true with regard to closed-loop solutions. Further analysis of linear dynamic control problems should not be surprising, because serious efforts are still being made to develop new efficient methods for solving even considerably simpler static problems in linear programming (LP) [3].

Here, we use bounded discrete-time controls defined on a quantized time axis as feasible controls for optimizing continuous dynamical systems on a finite time interval. In our view, this is a reasonable choice for dynamic problems whose solutions cannot be constructed and implemented without using discrete-time computing and actuating devices. However, the use of discrete-time controls eliminates some analytical difficulties, but hardly period is short and high accuracy is desired.

The idea of using LP methods for solving linear optimal control problems is quite natural. These methods have been implemented in various forms, we describe some implementation of the adaptive method presented in [1, 2, 4–7].

The aim of this paper is to study the canonical optimal control problem with free initial condition for a linear time-dependent system in the class of discrete-time feasible controls is considered. The initial sate of the optimized system is not known exactly, a priori information on the initial state is exhausted by inclusion $x_0 \in X_0$ [7]. During functioning of the system they are affected by disturbance which could not be taken into account in advance (before the beginning

* Corresponding author.

E-mail address(es): louadj_kahina@yahoo.fr (Kahina Louadj), demifethi@gmail.com (Fethi Demim), abdeslamaldja@yahoo.fr (Nait Abdesselam Aldjia)

of control process). The present paper aims at extending the problems of optimal control with free initial condition [8, 9]. Based on linear programming, method for synthesizing optimal feedback controls with the basic issues in open-loop and closed-loop optimization.

The work has the following structure. In section 2, the canonical optimal control problem is formulated with free initial condition. In section 3, we describe an approach in constructing a closed-loop solution to an optimal control problems, which relies on the numerical algorithm developed in the preceding sections. The results are illustrated with numerical example.

2. Statement of the problem

Let us consider the optimal control problem for a linear system at the time interval $T = [0, t^*]$:

$$c'x(t^*) \to max \tag{1}$$

$$\dot{x} = Ax + bu, x(t_0) = z \in X_0 = \{z \in \Re^n, \quad Gz = \gamma, \quad d_* \le z \le d^*\}, \tag{2}$$

$$Hx(t^*) = g, \tag{3}$$

$$|u(t)| \le 1, \quad t \in T = [t_0, t^*]. \tag{4}$$

Here $x \in \Re^n$ is a state of control system (2); $u(.) = (u(t), t \in t), T = [0, t^*]$, is a piece wise continuous function; $A \in \Re^{n\times n}$; $b, c \in \Re^n$; $g \in \Re^{m\times n}, rankH = m \le n; f_*, f^*$ are scalars; $d_* = (d_{*j}, j \in J), d^* = d^*(J) = (d^*_j, j \in J)$ are $n-$vectors; $G \in \Re^{l\times n}, rankG = l \le n, \gamma \in \Re^l, I = \{1, ..., m\}, J = \{1, ..., n\}, L = \{1, ..., l\}$ are sets of indices. The solution given by Cauchy formula, replace it, in the problem (1) – (4), we obtain the following equivalent formulation of the problem [8, 9]:

$$\tilde{c}'z + \int_0^{t^*} c(t)u(t)dt \longrightarrow max, \tag{5}$$

$$D(I, J)z + \int_0^{t^*} \varphi(t)u(t)dt = g, \tag{6}$$

$$G(L, J)z = \gamma, \; d_* \le z \le d^*, \tag{7}$$

$$f_* \le u(t) \le f^*, t \in T, \tag{8}$$

where $\tilde{c}' = c'F(t^*)$, $c(t) = c'F(t^*)F^{-1}(t)b$, $D(I, J) = HF(t^*)$, $\varphi(t) = HF(t^*)F^{-1}(t)b$.

3. Essentials definitions

Definition 3.1.
A pair $v = (z, u(.))$ formed of an $n-$vector z and a piecewise continuous function $u(.)$ is called a generalized control.

Definition 3.2.
A generalized control $v = (z, u(.))$ is said to be an admissible control if it satisfied the constraints (2) – (4). We denote

$$J(v) = c'x(t^*) = \tilde{c}'z + \int_0^{t^*} c(t)u(t)dt.$$

Definition 3.3.
An admissible control $v^0 = (z^0, u^0(.))$ is said to be an optimal open-loop control if a control criterion reaches its maximal value

$$J(v^0) = \max_v J(v).$$

Definition 3.4.
For a given $\varepsilon \ge 0$, an $\varepsilon-$optimal control $v^\varepsilon = (z^\varepsilon, u^\varepsilon(.))$ is defined by the inequality

$$J(v^0) - J(v^\varepsilon) \le \varepsilon.$$

The problem (1) – (4) is solved in [8, 9], and we used the same problem with disturbance. Define it in the following section.

4. Synthesis of a closed-loop solution to the canonical optimal control problem

In order to introduce the concept of optimal feedback under the assumption that, in the course of control, accurate values of the state $x^*(t), t \in T_h$ of the object of control will be available, we embed problem (1) – (4) into a family of problems

$$c'x(t^*) \to max \tag{9}$$

$$\dot{x} = Ax + bu,\ x(\tau) = \lambda, \tag{10}$$

$$Hx(t^*) = g,\ |u(t)| \leq 1,\ t \in T(\tau) = [\tau, t^*]. \tag{11}$$

depending on the scalar $\tau \in T_h$ and $n-$dimensional vector λ.

Let $u^0(t|\tau,\lambda), t \in T(\tau)$ be an optimal open-loop control of problem (9) – (11) for the position (τ,λ); X_τ is the set of all initial states $\lambda \in \Re^n$ for which solution to problem (9) – (11) exists.
The function

$$u^0(\tau,x) = u^0(\tau|\tau,x),\ x \in X_\tau,\ \tau \in T_h \tag{12}$$

is called the optimal control of the type of discrete feedback of problem (1) – (4).

We apply feedback control (12) to the control system of problem (1) – (4) and analyze the behavior of the resulting closed-loop system under constantly acting piece wise continuous disturbances $w(t),\ t \in T$:

$$\dot{x} = Ax + bu(t) + w(t), x(t_*) = z \in X_0 = \{z \in \mathrm{R}^n, Gz = \gamma, d_* \leq z \leq d^*\},$$

$$u(t) = u^0(\tau, x(\tau)),\ t \in [\tau, \tau+h[,\ \tau = t_* + kh,\ k = 0,1,...,N-1.$$

The introduction of disturbances into system (9) – (11) of fundamental importance in the constructive theory of closed-loop solutions, because the idea of feedback control is that the resulting closed-loop system is capable of rejecting unknown disturbances.

Suppose that an initial state $x(t_*) = z^*$, that correspond to a trajectory $x^*(t),\ t \in T$ of the closed-loop system

$$\dot{x} = Ax^*(t) + bu^0(t, x^*(t)) + w^*(t),$$
$$x(t_*) = z^* \in X_0 = \{z \in \mathrm{R}^n, Gz = \gamma, d_* \leq z \leq d^*\}$$

By $w^*(t),\ t \in T$, we denote an unknown bounded perturbation occurring in the control process, where z^* is the actual initial state.

The method can be used to the construction of an optimal controller which computes quickly the current values of the optimal feedback. Assume that the optimal controller has operated at $t_*, t_* + h, ..., \tau$ and produced control signals $u^*(t_*), u^*(t_* + h), ..., u^*(\tau)$ respectively. denote by $x^*(\tau + h)$ the state of the control system at $\tau + h$ resulting from the action of these signals and a disturbance $w^*(t), t \in [t_*, \tau + h[$.
By the relation (12), the open-loop solution $u^0(t|\tau+h, x^*(\tau+h)), t \in T(\tau+h)$ to problem (9) – (11) at $(\tau+h, x^*(\tau+h))$ is required for the controller to calculate the realization $u^*(\tau+h)$ of feedback control (12). By assumption, the optimal controller has constructed an open-loop solution $u^0(t|\tau, x^*(\tau)), t \in T(\tau)$ to problem (9) – (11) with $\lambda = x^*(\tau)$ at the preceding point τ. A part of the solution $u^0(t|\tau, x^*(\tau)), t \in T(\tau+h)$ is the optimal open-loop control for the position $(\tau+h, x^0(\tau+h))$ resulting from the action of $u^*(\tau) = u^0(\tau|\tau, x^*(\tau))$ in the absence of any disturbance $(w(t) = 0, t \in [\tau, \tau+h[)$. The actual state $x^*(\tau+h)$ is related to the ideal state $x^*(\tau+h)$ is related to the ideal state $x^0(\tau+h)$ as follows:

$$x^*(\tau+h) = x^0(\tau+h) + \int_\tau^{\tau+h} F(\tau+h)F^{-1}(s)w^*(s)ds.$$

For small $h > 0$ and bounded $w^*(t), t \in [\tau, \tau+h[$, the vectors $x^*(\tau+h)$ and $x^0(\tau+h)$ are nearly identical.

Therefore, the controller correct an optimal control $u^0(t|\tau+h, x^*(\tau+h)), t \in T(\tau+h)$, by the known control $u^0(t|\tau, x^*(\tau)), t \in T(\tau+h)$.

Given the optimal support $S_B^0(\tau) = \{J_B^0(\tau), T_B^0(\tau)\}$ of problem (9) – (11) at position, $(\tau, x^*(\tau))$, with $J_B^0(\tau) = \overline{J}_B$ along the iteration, find an optimal support $T_B^0(\tau+h)$ and an optimal control $u^*(\tau+h) = u^0(\tau+h|\tau+h, x^*(\tau+h))$ in the problem

$$\dot{x} = Ax + bu,\ x(\tau+h) = x^*(\tau+h), x(t_*) = z^* \in \widehat{X}_0(\tau), \tag{13}$$

$$Hx(t^*) = g,\ |u(t)| \leq 1,\ t \in T(\tau + h) = [\tau + h, t^*]. \tag{14}$$

Under this following notations:
$T_h(\tau) = \{\tau, \tau + h, .., t^* - h\}$, $T_*(t_*) = \{t_*, t_* + h, .., \tau - h\}$, $\underline{T_{n0}(\tau)}$ is the set of nonsupport zeros at $(\tau, x^*(\tau))$, $T_H(\tau) = T_B(\tau) \cup T_{n0}(\tau) \cup \{\tau, t^*\}$, $|T_H(\tau)| = k^*(\tau) + 1$, and $T_k(\tau)$ $(k = \overline{0, k^*(\tau)}$ are the intervals where the cocontrol $\Delta_h^{\tau}(t)$ $(t \in T_h(\tau))$ has a definite sign.

Assume that the following data have been obtained by solving problem (9) – (11) at $(\tau, x^*(\tau))$ and stored: an optimal support $T_B^0(\tau)$; the set $T_{n0}(\tau)$; the support matrix $P_B(\tau)$ and vector $d_h(\tau); \varrho(t)$ and $\psi_c(t)$ at $t \in T_H(\tau) \setminus t^*; u^*(\tau)$; the vector of potentials $\nu(\tau)$; the vector $y^*(\tau) = \varrho(\tau)x^*$.

At τ, the optimal controller solved a problem that had the following form as represented in LP terms:

$$\tilde{c}'z + \sum_{t \in T_h} q(t)u(t) \to max, \tag{15}$$

$$y^*(\tau) + D(I, J)z + \sum_{t \in T_h(\tau)} d(t)u(t) = g, \tag{16}$$

$$G(L, J)z = \gamma, d_* \leq z \leq d^*, \tag{17}$$

$$|u(t)| \leq 1, t \in T_h(\tau). \tag{18}$$

The problem to be solved at $\tau + h$ is formulated as

$$\tilde{c}'z + \sum_{t \in T_h} q(t)u(t) \to max, \tag{19}$$

$$y^*(\tau) + \int_{\tau}^{\tau+h} \varrho(t)w^*(t)dt + D(I, J)z + \sum_{t \in T_h(\tau)} d(t)u(t) = g, \tag{20}$$

$$G(L, J)z = \gamma, d_* \leq z \leq d^*, \tag{21}$$

$$u^*(\tau) \leq u(\tau) \leq u(\tau + h),\ |u(t)| \leq 1, t \in T_h(\tau + h). \tag{22}$$

Let us put

$$g(\tau) = g - y^*(\tau),$$

We rewrite (15) – (18) that

$$\tilde{c}'z + \sum_{t \in T_h} q(t)u(t) \to max, \tag{23}$$

$$D(I, J)z + \sum_{t \in T_h(\tau)} d(t)u(t) = \overline{g}(\tau + h), \tag{24}$$

$$G(L, J)z = \gamma, d_* \leq z \leq d^*, \tag{25}$$

$$u^*(\tau) \leq u(\tau) \leq u(\tau + h),\ |u(t)| \leq 1, t \in T_h(\tau + h), \tag{26}$$

where

$$\overline{g}(\tau + h) = g(\tau) + \Delta g(\tau), \Delta g(\tau) = -\int_{\tau}^{\tau+h} \varrho(t)w^*(t)dt. \tag{27}$$

$$x^*(\tau+h) = F(\tau+h)z + F(\tau+h)F^{-1}(\tau)x^*(\tau)$$
$$+\int_{\tau}^{\tau+h} F(\tau+h)F^{-1}(t)bu^*(\tau)dt + \int_{\tau}^{\tau+h} F(\tau+h)F^{-1}(t)w^*(\tau)dt,$$

then

$$\int_{\tau}^{\tau+h} F^{-1}(t)w^*(\tau)dt = F(\tau+h)z + F^{-1}(\tau+h)x^*(\tau+h)$$
$$-F^{-1}(\tau)x^*(\tau) - \int_{\tau}^{\tau+h} F^{-1}(t)bu^*(\tau)dt.$$

By identification with (23), we obtain

$$\overline{g}(\tau) = y^*(\tau) + D(I,J)z + d_h(\tau)u^*(\tau) - y^*(\tau+h). \tag{28}$$

To determine $\overline{g}(\tau+h)$, we integrate (2) over $[\tau, \tau+h]$ with $\varrho(\tau)$ as an initial condition and calculate and store $\varrho(\tau+h)$ and $y^*(\tau+h)$. The resulting (27) and (28) can be used to construct $T_B^0(\tau+h)$, and calculate $u^*(\tau+h)$ at $\tau+h$.

The following two situations may arise at $\tau+h$ (after the current state $x^*(\tau+h)$) is measured):

1. $\Delta g(\tau) = 0$;
2. $\Delta g(\tau) \neq 0$.

Let us analyze case (1). It includes the following subcases: (1a) $\tau \notin T_B^0(\tau)$; (1b) $\tau \in T_B^0(\tau)$.

In case (1a), let us put $T_B^0(\tau+h) = T_B^0(\tau)$ and $u^*(\tau+h)$ equals :

1. If $\tau \notin T_B^0(\tau), \tau+h \notin T_H(\tau)$,

$$u^*(\tau+h) = \begin{cases} sign\Delta(t_*), & \text{if } t_* \notin T_B^0(\tau); \\ sign\Delta(t_*+h), & \text{if } t_* \in T_B^0(\tau). \end{cases}$$

2. If $\tau+h \notin T_B^0(\tau)$.

$$u^*(\tau+h) = \tilde{u}(\tau+h),$$

The pseudocontrol $\tilde{u}(\tau+h)$ is obtained by solving the equation

$$\sum_{j\in J_B^0(\tau)} D(I,j)\tilde{z}_j + \sum_{t\in T_B^0(\tau)} d(t)\tilde{u}(t) = g - \sum_{j\in J_H^0(\tau)} D(I,j)\tilde{z}_j + \sum_{t\in T_H^0(\tau)} d(t)\tilde{u}(t) \tag{29}$$

$$\sum_{j\in J_B^0(\tau)} G(L,j)\tilde{z}_j = \gamma - \sum_{j\in J_H^0(\tau)} G(L,j)\tilde{z}_j. \tag{30}$$

3. If $\tau+h \in T_{n0}(\tau)$.

$$u^*(\tau+h) = \begin{cases} -sign\Delta(t_*), & \text{if } t_* \notin T_B^0(\tau); \\ -sign\Delta(t_*+h), & \text{if } t_* \in T_B^0(\tau). \end{cases}$$

The stored data corresponding to the time τ are transformed into data for $\tau+h$ by setting:
$T_{n0}(\tau+h) = T_{n0}(\tau)$, $P_B(\tau+h) = P_B(\tau)$, $\nu(\tau+h) = \nu(\tau)$, $T_H(\tau+h) = T_H(\tau)\setminus\tau\cup\tau+h$, if $\tau+h \notin T_H(\tau)$, and $T_H(\tau+h) = T_H(\tau)\setminus\tau$, if $\tau+h \in T_H(\tau)$. The values $\varrho(\tau+h)$, $\psi(\tau+h)$, and $d(\tau+h)$ are calculated and stored instead of $\varrho(\tau)$, $\psi(\tau)$, and $d(\tau)$.

In case (1b), $T_B^0(\tau+h)$ is constructed by analyzing problem (15)−(18). Choosing $T_B^0(\tau)$ as an initial support $T_B^0(\tau+h)$, we used a procedure change of support [8, 9] to remove τ from the support. Using the results of iteration, we rewrite the data corresponding to $\tau+h$ and let us put

1. If $(\tau+h) \notin T_B^0(\tau+h), (\tau+h) \notin T_H(\tau)$,

$$u^*(\tau+h) = \begin{cases} sign\Delta(t_*), & \text{if } t_* \notin T_B^0(\tau+h); \\ sign\Delta(t_*+h), & \text{if } t_* \in T_B^0(\tau+h). \end{cases}$$

2. If $\tau+h \notin T_B^0(\tau)$, $(\tau+h) \in T_H(\tau)$

$$u^*(\tau+h) = \begin{cases} -sign\Delta(t_*), & \text{if } t_* \notin T_B^0(\tau+h); \\ -sign\Delta(t_*+h), & \text{if } t_* \in T_B^0(\tau+h). \end{cases}$$

3. If $\tau + h \in T_B^0(\tau + h)$,

$$u^*(\tau + h) = \widetilde{u}(\tau + h),$$

with $\widetilde{u}(\tau + h)$ is the pseudo-control.

Consider case (2). Solving equation (27), we find the vector $\widetilde{v} = (\widetilde{z}_j, j \in J_B^0(\tau), \widetilde{u}(t), t \in T_B^0(\tau))$. The following situations may arise: (2a) it holds that

$$d_{*j} < z_j < d_j^*, j \in J_B^0(\tau), |u(t)| \leq 1, t \in T_B^0(\tau); \quad (31)$$

(2b) inequalities (28) are not satisfied.

In subcase (2a), we proceed as in case (1).

In subcase (2b), the optimal support $T_B^0(\tau + h)$ of problem (15) – (18) is constructed by using procedure change of support (section 7.2) desribed above.

If $\tau \in T_B^0(\tau)$, then τ is removed from the initial support $T_B(\tau + h)$. In solving problem (23) – (26) with $\tau \notin T_B(\tau + h)$, the support is transformed on the set $T_h(\tau + h)$, which prevents τ from entering $T_B^0(\tau + h)$.

5. Example

We illustrate the results obtained in this paper using the following example:

$$\begin{aligned}
&\int_0^{25} u(t)dt \to min, \dot{x}_1 = x_3, \dot{x}_2 = x_4,\\
&\dot{x}_3 = -x_1 + x_2 + u, \dot{x}_4 = 0.1x_1 - 1.01x_2,\\
&x_1(25) = x_2(25) = x_3(25) = x_4(25) = 0,\\
&0 \leq u(t) \leq 1, t \in [0, 25]. \quad (32)
\end{aligned}$$

Let be the matrix

$$H = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}, g = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}, G = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix},$$

$$\gamma = \begin{pmatrix} 0.1 \\ 0.25 \\ 2 \\ 1 \end{pmatrix}, d_* = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}, d^* = \begin{pmatrix} 2 \\ 2 \\ 2 \\ 2 \end{pmatrix}.$$

Let us consider the initial condition as:

$$x_1(0) = 0.1, x_2(0) = 0.25, x_3(0) = 2, x_4(0) = 1.$$

Problem (32) is reduced to canonical form (1) – (4) by introducing the new variable $\dot{x}_5 = u, x_5(0) = 0$. Then, the control criterion takes the form $-x_5(t^*) \to max$. A problem (32) is equivalent to LP problem of dimension 4×1000.
To construct the optimal open-loop control of problem (32). As an initial support, a set $T_B = \{5, 10, 15, 20\}$ was selected. This support corresponds to the set nonsupport zeroes of the cocontrol $T_{n0} = \{3.7259.725, 15.3, 21.3\}$. The problem was solved in 18 iterations, i.e to construct the optimal open-loop control, a support $4 \times 4-$ matrix was changed 18 times. The optimal value of the control criterion was found to be equal 6.602499.

The given data illustrate the effectiveness of the method used. In our opinion, the time it takes today to construct optimal open-loop controls is not of significant importance. It is only important that the method be able to construct a reliable solution in a reasonable time. Let us give some calculations.

At first, a characteristic of the number of iterations in various methods often differ a great deal from one another. It is more naturally to define the effectiveness of method [7] by using the number on integration of a primal or an adjoint system with insignificant volume or required operative memory. In this connection, as a unit of the complexity the time of integration of a primal or an adjoint system on the whole control interval T is taken. If a method admits to make operation in parallel then the complexity is defined by the time needed for a set of microprocessors to solve the problem.

The proposed characteristic is not absolute (exact) as it does not take into account to evaluate methods at "first approximation". Table 1 contains some information on the solution to problem (32) for other quantization periods. Of course, one can solve problem (32) by LP methods, transforming the problem (1)-(4). In doing so, one integration of the system is sufficient to form the matrix of the LP problem . However, such "static" approach is concerned with

Table 1.

h	Number of iterations	Value of the control criterion
0.25	11	6.624343
0.025	18	6.602499
0.0025	26	6.602054
0.001	32	6.602050

a large volume of required operative memory, and it is fundamentally different from the traditional "dynamical" approaches based on dynamical models (1)-(4). Then, the problem (1)-(4) was solved.
Let the perturbed system

$$\begin{aligned} &\dot{x}_1 = x_3, \dot{x}_2 = x_4, \\ &\dot{x}_3 = -x_1 + x_2 + u, \\ &\dot{x}_4 = 0.1x_1 - 1.01x_2 + w. \end{aligned} \tag{33}$$

be closed by the optimal feedback $u^* = u^0(x_1, x_2, ..., x_n)$ and be acting under a disturbance

$$w^*(t) = 0.3sint4t, t \in [0, 9.75); w^*(t) = 0, t \geq 9.75.$$

The optimal controller does not know this disturbance, but at each moment $\tau \in T_h$ it can measure a current state $x^*(\tau)$.
The realization $u^*(\tau), \tau \in T_h$ is given in Fig. 1. In Fig. 2 and 3, Trajectories obtained under the action of $u^*(\tau)$, $\tau \in T_h$ and $w^*(\tau)$ $(\tau \in T)$ are presented.

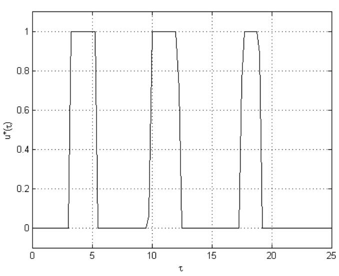

Fig. 1.

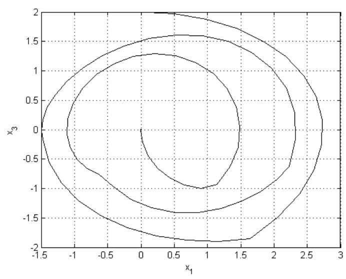

Fig. 2.

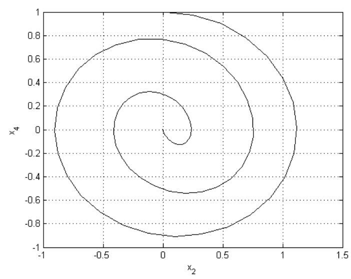

Fig. 3.

The optimal initial state is $x_1(0) = 0.1109729, x_2(0) = 0.3002507, x_3(0) = 0.9833905, x_4(0) = 1.0110008$.

6. Conclusion

An optimal control problem with free initial condition under disturbance has been considered.
The model problem, becomes a problem where we search the best of initial condition and a control which permits to bring the system of initial condition $x_0 \in X_0$ towards the final state which verify the constraint $Hx(t^*) = g$ under disturbance.
To conclude, it appears that the study and applications of adaptive methods have at least important advantage. Control law computations can be executed very quickly in real time, in particular by using parallel computers.

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