Int. J. Adv. Appl. Math. and Mech. 5(2) (2017) 25 – 29 (ISSN: 2347-2529)
Journal homepage: www.ijaamm.com

IJAAMM

International Journal of Advances in Applied Mathematics and Mechanics

Mean labeling on degree splitting graph of star graph

Research Article

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Received 18 August 2017; accepted (in revised version) 07 November 2017

Abstract: In this paper, we define mean labeling for degree splitting of star graphs. We consider the mean labeling for degree splitting graph of single star and two star graphs. Also, we say that a degree splitting graph for n star is a mean graph if $n < 3$. Then the mean labeling for degree splitting graph of two star with a wedge in common is also given. And the degree splitting graph of three star graph with wedge in common is a mean graph.

MSC: 05C78

Keywords: Mean graph • Star Graph • Degree splitting graph

1. Introduction

In [1], it is proved that $DS(K_n)$, $DS\left(K_n^c\right)$, $DS\left(K_{1,n}\right)$, $DS(C_n)$, $DS(nK_2)$, $DS\left(B_{n,n}\right)$, $DS\left(C_3\hat{O}K_{1,n}\right)$ are mean graphs.

Definition 1.1.
A graph with p vertices and q edges is said to be a mean graph if there exists a function f from the vertex set of G to $\{0,1,2,\cdots,q\}$ such that the induced map f^* from the edge set of G to $\{1,2,\cdots,q\}$ defined by

$$f^*(e=uv) = \begin{cases} \dfrac{f(u)+f(v)}{2} & \text{if } f(u)+f(v) \text{ is even;} \\ \dfrac{f(u)+f(v)+1}{2} & \text{if } f(u)+f(v) \text{ is odd} \end{cases}$$

then the resulting edges get distinct labels from the set $\{1,2,\cdots,q\}$.

Definition 1.2.
Let $G=(V,E)$ be a graph with $V = S_1 \cup S_2 \cup \cdots S_t \cup T$ where each S_i is a set of vertices having atleast two vertices of the same degree and $T = V \cup_{i=1}^{t} S_i$. The degree splitting graph of G denoted by $DS(G)$ is obtained from G by adding vertices $u_1, u_2, \cdots, u_t$ and joining to each vertex of S_i for $1 \le i \le t$.

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Theorem 1.1.
Degree splitting graph of single star graph $K_{1,n}$ is a mean graph for all n.

Proof. Mean labeling for $DS(K_{1,1})$ is given in Fig. 1.
Let $G = K_{1,n}, n \geq 2$. The vertex and edge set of G when $n \geq 2$ is given by,

$$V(G) = \{u, u_i : 1 \leqslant i \leqslant n\}$$
$$DS(V(G)) / V(G) = \{w\}$$
$$E(G) = \{uu_i : 1 \leqslant i \leqslant n\}$$
$$DS(E(G)) / E(G) = \{wu_i : 1 \leqslant i \leqslant n\}$$

The mean labeling for vertex and edge set of G is, $f : DS(V(G)) \to \{0, 1, 2, \cdots, q\}$, $f^* : DS(E(G)) \to \{1, 2, \cdots, q\}$

$$f(u) = q$$
$$f(w) = 0$$
$$f(u_i) = q + 1 - 2i \text{ for } 1 \leqslant i \leqslant n$$

The corresponding edge labels, uu_i is $q + 1 - i$ for $1 \leq i \leq n$; wu_i is $\frac{q}{2} + 1 - i$ for $1 \leq i \leq n$. Acknowledging the fact that the number of edges in a degree splitting graph of $G = K_{1,n}, n \geq 2$ is $2n$, the edge labels of uu_i is $\{q, q-1, \cdots, q-n+1 = n+1\}$ and the edge labels of wu_i is $\{n, n-1, \cdots, 1\}$. Hence the vertex and edge labels are distinct. Therefore, degree splitting graph of single star graph is a mean graph. □

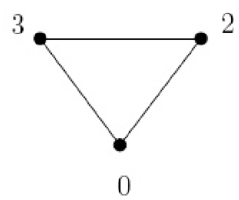

Fig. 1.

Theorem 1.2.
Degree splitting graph of a two star graph $K_{1,m} \cup K_{1,n}$ is a mean graph for all m and n.

Proof. Let $G = K_{1,m} \cup K_{1,n}$. Without loss of generality, let us consider, $m \leq n$.
We prove the theorem by two cases, $m = n$ and $m < n$.

Case 1: $m = n$
The vertex and edge set of $G = K_{1,m} \cup K_{1,n}$ is given by,

$$V(G) = \{u, v\} \cup \{u_i : 1 \leqslant i \leqslant m\} \cup \{v_i : 1 \leqslant i \leqslant n\}$$
$$DS(V(G)) / V(G) = \{w_1, w_2\}$$
$$E(G) = \{uu_i : 1 \leqslant i \leqslant m\} \cup \{vv_i : 1 \leqslant i \leqslant n\}$$
$$DS(E(G)) / E(G) = \{w_1u, w_1v\} \cup \{u_iw_2 : 1 \leqslant i \leqslant m\} \cup \{v_iw_2 : 1 \leqslant i \leqslant n\}$$

The mean labeling for vertex and edge set of G is, $f : DS(V(G)) \to \{0, 1, 2, \cdots, q\}$, $f^* : DS(E(G)) \to \{1, 2, \cdots, q\}$

$$f(u) = q - 1$$
$$f(v) = 0$$
$$f(w_1) = q$$
$$f(u_i) = q - 2i \quad \text{for} \quad 1 \leqslant i \leqslant m$$
$$f(v_i) = 2i - 1 \quad \text{for} \quad 1 \leqslant i \leqslant n$$
$$f(w_2) = f(v_n) + 2$$

The corresponding edge labels for uu_i is $q - i$ for $1 \leq i \leq m$; vv_i is i for $1 \leq i \leq n$; w_1u is q; w_1v is $\frac{q}{2}$; u_iw_2 is $3n - i + 1$ (since, $q = 4n + 2$) for $1 \leq i \leq m$; v_iw_2 is $n + i + 1$ for $1 \leq i \leq n$. Hence, the vertex labels and the induced edge labels are distinct. Therefore, degree splitting graph of a two star graph is mean when $m = n$.

Case 2: $m < n$
The vertex and edge set of $G = K_{1,m} \cup K_{1,n}$ is given by,

$$V(G) = \{u, v\} \cup \{u_i : 1 \leqslant i \leqslant m\} \cup \{v_i : 1 \leqslant i \leqslant n\}$$
$$DS(V(G)) / V(G) = \{w\}$$
$$E(G) = \{uu_i : 1 \leqslant i \leqslant m\} \cup \{vv_i : 1 \leqslant i \leqslant n\}$$
$$DS(E(G)) / E(G) = \{u_i w : 1 \leqslant i \leqslant m\} \cup \{v_i w : 1 \leqslant i \leqslant n\}$$

The mean labeling for vertex and edge set of G is, $f : DS(V(G)) \to \{0, 1, 2, \cdots, q\}$, $f^* : DS(E(G)) \to \{1, 2, \cdots, q\}$

Subcase (a): $m = 1$
When $m = 1$ an edge uw will be added to $DS(G)$.

$$f(u) = q$$
$$f(v) = 0$$
$$f(u_1) = q - 1$$
$$f(v_i) = 2i - 1 \quad \text{for} \quad 1 \leqslant i \leqslant n$$
$$f(w) = q - 3$$

The corresponding edge labels of, uu_1 is q; vv_i is i for $1 \le i \le n$; $u_1 w$ is $q - 2$; $v_i w$ is $\frac{q}{2} + i - 2$ for $1 \le i \le n$; uw is $q - 1$ (Note, $q = 2n + 3$). Hence the vertex and edge labels are distinct, So $DS(G)$ is a mean graph.

Subcase (b): $m \neq 1$

$$f(u) = q$$
$$f(v) = 0$$
$$f(u_i) = q + 1 - 2i \quad \text{for} \quad 1 \leqslant i \leqslant m$$
$$f(v_i) = 2i - 1 \quad \text{for} \quad 1 \leqslant i \leqslant n$$
$$f(w) = f(u_m) - 1$$

The corresponding edge labels for uu_i is $q - i + 1$ for $1 \le i \le m$; vv_i is i for $1 \le i \le n$; wu_i is $q - m - i + 1$ for $1 \le i \le m$; wv_i is $\frac{q}{2} - m + i - 1$ for $1 \le i \le n$(since, $q = 2m + 2n$). Hence, the vertex labels and the induced edge labels are distinct. Therefore, degree splitting graph of a two star graph is mean when $m < n$.

Thus we have proved that degree splitting graph of $K_{1,m} \cup K_{1,n}$ is a mean graph for all m and n. □

Lemma 1.1.
The degree splitting graph of three star graph $K_{1,\ell} \cup K_{1,m} \cup K_{1,n}$ is not a mean graph for any ℓ, m and n.

2. Wedge

Definition 2.1.
A Wedge is defined as an edge connecting two components of a graph, denoted as $\wedge.\omega(G\wedge) < \omega(G)$.
$K_{1,m} \cup K_{1,n}$ is a two star and is a two component or a disconnected graph, whereas $K_{1,m} \wedge K_{1,n}$ is a two star but a connected graph. Which means adding a wedge to a disconnected graph with two components becomes a connected or a single component graph. And a disconnected graph with three components and two wedges becomes a connected or a single component graph.

Note: In this paper all wedge is considered to connect the non-pendent vertices.

Theorem 2.1.
The degree splitting graph of two star graph $K_{1,m} \wedge K_{1,n}$ is a mean graph for all m and n.

Proof. Let $G = K_{1,m} \wedge K_{1,n}$. Without loss of generality, let us consider, $m \le n$.
We prove the theorem by two cases, $m = n$ and $m < n$.

Case 1: $m = n$
The vertex and edge set of $G = K_{1,m} \wedge K_{1,n}$ is given by,

$$V(G) = \{u, v\} \cup \{u_i : 1 \leqslant i \leqslant m\} \cup \{v_i : 1 \leqslant i \leqslant n\}$$
$$DS(V(G))\,/\,V(G) = \{w_1, w_2\}$$
$$E(G) = \{uu_i : 1 \leqslant i \leqslant m\} \cup \{vv_i : 1 \leqslant i \leqslant n\} \cup \{uv\}$$
$$DS(E(G))\,/\,E(G) = \{w_1u, w_1v\} \cup \{u_iw_2 : 1 \leqslant i \leqslant m\} \cup \{v_iw_2 : 1 \leqslant i \leqslant n\}$$

The mean labeling for vertex and edge set of G is, $f : DS(V(G)) \to \{0, 1, 2, \cdots, q\}$, $f^* : DS(E(G)) \to \{1, 2, \cdots, q\}$

$$f(u) = q$$
$$f(v) = 2$$
$$f(u_i) = q + 1 - 2i \quad \text{for} \quad 1 \leqslant i \leqslant m$$
$$f(v_i) = 2i - 1 \quad \text{for} \quad 1 \leqslant i \leqslant n$$
$$f(w_1) = 0$$
$$f(w_2) = f(u_m) - 1$$

The corresponding edge labels for uu_i is $q - i + 1$ for $1 \le i \le m$; vv_i is $i + 1$ for $1 \le i \le n$; uv is $\frac{q}{2} + 1$; w_1u is $\frac{q}{2}$; w_1v is 1; u_iw_2 is $q - m - i + 1$ for $1 \le i \le m$; v_iw_2 is $m + i$ for $1 \le i \le n$ (since, $q = 4n+3 = 4m+3$). Hence, the vertex labels and the induced edge labels are distinct. Therefore, degree splitting graph of a two star graph with wedge is mean when $m = n$.

Case 2: $m < n$
The vertex and edge set of $G = K_{1,m} \wedge K_{1,n}$ is given by,

$$V(G) = \{u, v\} \cup \{u_i : 1 \leqslant i \leqslant m\} \cup \{v_i : 1 \leqslant i \leqslant n\}$$
$$DS(V(G))\,/\,V(G) = \{w\}$$
$$E(G) = \{uu_i : 1 \leqslant i \leqslant m\} \cup \{vv_i : 1 \leqslant i \leqslant n\} \cup \{uv\}$$
$$DS(E(G))\,/\,E(G) = \{u_iw : 1 \leqslant i \leqslant m\} \cup \{v_iw : 1 \leqslant i \leqslant n\}$$

The mean labeling for vertex and edge set of G is, $f : DS(V(G)) \to \{0, 1, 2, \cdots, q\}$, $f^* : DS(E(G)) \to \{1, 2, \cdots, q\}$

$$f(u) = 0$$
$$f(v) = 1$$
$$f(u_i) = 2i + 1 \quad \text{for} \quad 1 \leqslant i \leqslant m$$
$$f(v_i) = q + 2 - 2i \quad \text{for} \quad 1 \leqslant i \leqslant n$$
$$f(w) = q - 1$$

The corresponding edge labels for uu_i is $i + 1$ for $1 \le i \le m$; vv_i is $\frac{q}{2} - i + 2$ for $1 \le i \le n$; uv is 1; u_iw is $\frac{q}{2} + i$ for $1 \le i \le m$; v_iw is $q - i + 1$ for $1 \le i \le n$ (since, $q = 2m + 2n + 1$). Hence, the vertex labels and the induced edge labels are distinct. Therefore, degree splitting graph of a two star graph with wedge is mean when $m < n$.

Thus, the degree splitting graph of a two star graph with wedge $K_{1,m} \wedge K_{1,n}$ is a mean graph for all m and n. □

Theorem 2.2.
If $\ell = m \le n$ and $\ell = m = 1$ then the degree splitting graph of a three star graph with wedge $K_{1,\ell} \wedge K_{1,m} \wedge K_{1,n}$ is mean graph.

Proof. The mean labeling for the degree splitting graph of three star graph $K_{1,\ell} \wedge K_{1,m} \wedge K_{1,n}$ when $\ell = m = n = 1$ is given in Fig. 2 Next we shall consider the further cases when $n = 2, 3, \ldots$

Let $G = K_{1,1} \wedge K_{1,1} \wedge K_{1,n}$ where $n = 2, 3, \ldots$ The vertex and edge set of G are depicted below,

$$V(G) = \{u, v, w, u_1, v_1\} \cup \{w_i : 1 \leqslant i \leqslant n\}$$
$$DS(V(G))\,/\,V(G) = \{x\}$$
$$E(G) = \{uu_1, vv_1, uv, vw\} \cup \{ww_i : 1 \leqslant i \leqslant n\}$$
$$DS(E(G))\,/\,E(G) = \{u_1x, v_1x\} \cup \{w_ix : 1 \leqslant i \leqslant n\}$$

The mean labeling for vertex and edge set of G is, $f : DS(V(G)) \to \{0,1,2,\cdots,q\}$, $f^* : DS(E(G)) \to \{1,2,\cdots,q\}$

$$f(u) = 4$$
$$f(v) = 0$$
$$f(w) = 2$$
$$f(u_1) = 3$$
$$f(v_1) = 5$$
$$f(w_i) = 2i + 5 \quad \text{for} \quad 1 \leqslant i \leqslant n$$
$$f(x) = q$$

The corresponding edge labels, uu_1 is 4; vv_1 is 3; uv is 2; vw is 1; ww_i is $i+4$ for $1 \le i \le n$; u_1x is $\frac{q}{2}+2$; v_1x is $\frac{q}{2}+3$; w_ix is $\frac{q}{2}+i+3$ for $1 \le i \le n.(q = 2n+6)$. Obviously, the vertex labels and te induced edge labels are distinct. Hence, the degree splitting graph of three star graph, $DS(K_{1,\ell} \wedge K_{1,m} \wedge K_{1,n})$ is a mean graph if $\ell = m \le n$ and $\ell = m = 1$. □

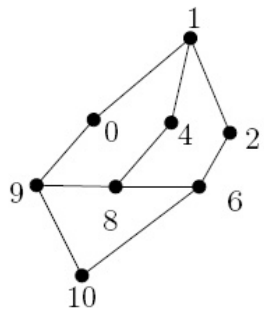

Fig. 2. $DS(K_{1,1} \wedge K_{1,1} \wedge K_{1,1})$

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