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Anti fuzzy graph

Research Article

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Abstract: In this paper, The notions of Anti fuzzy graph are introduced, union, intersection of two Anti fuzzy graphs are introduced, in this paper we introduced some operation on Anti fuzzy graph, denoted by AFG. We study the type graph on properties Anti fuzzy graph are established here and we study Anti fuzzy graph depended of degree graph, regular graph Anti fuzzy graph and non-regular graph Anti fuzzy graph and complete Anti fuzzy graph.If G Anti fuzzy graph then $G-e$ satisfied property Anti fuzzy graph. Also we apply AFG on removable vertex from graph G Anti fuzzy graph, in other word if G Anti fuzzy graph then $G-v$ satisfied property Anti fuzzy graph. If G Anti fuzzy graph then $G*e$ satisfied property anti fuzzy graph.

MSC: 35R11 • 74H10

Keywords: Fuzzy graph • Anti fuzzy graph • Regular graph • Non regular graph • Complete graph • Operation of graphs

1. Introduction

In this section, In 1736, Euler first introduced the concept of graph theory . the theory of graphs is extremely useful tool for solving combinatorial problems in different areas such that geometry, algebra, number theory, topology, operation research, optimization and computer science, etc.. The first publications in fuzzy set theory by Zadeh [1965] and Goguen [1967, 1969] show the intention of the authors to generalize the classical notion of a set, in [1975], Rosenfeld [1] introduced the concepts of fuzzy graphs, there after many research have generalized they notions graph theory. In this paper , our aim is to introduce the notion of Anti fuzzy graph and some properties and operations, union of Anti fuzzy graph, intersection of two Anti fuzzy graph, we study of type graph on Anti fuzzy graph and properties removable edge and vertex on Anti fuzzy graph , we study properties contraction edge of Anti fuzzy graph.

2. Preliminaries

In this section, we defined Anti fuzzy graph as following.

Definition 2.1.

[1]: Let V be anon empty finite set and ,$S: V \to [0,1]$, let$M: V\times V \to [0,1]$ such that $M(x,y) \le S(x)\wedge S(y)$, $\forall (x,y) \in V\times V$. A fuzzy graph $G=(S,M)$ over the set $V$is called strong fuzzy graph if $M(x,y)=S(x)\wedge S(y)$, $\forall (x,y)\in V\times V$.

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Definition 2.2.
[5]: Let $G_1 = (S_1, M_1)$ and $G_2 = (S_2, M_2)$ be two fuzzy graph over vertex set V. Then the union of $G_1$and $G_2$is fuzzy graph $G_3 = (S_3, M_3)$ over the set$V$ where $S_3 = (S_1 \vee S_2)$, $M_3 = (M_1 \vee M_2)$, where $S_3(x) = \max\{(S_1(x))\} \forall X \in V$, and $M_3(x, y) = \max\{(M_1(x, y), M_2(x, y))\}, \forall x, y \in V$.

Definition 2.3.
[5]: Let $G_1 = (S_1, M_1)$ and $G_2 = (S_2, M_2)$ be two fuzzy graph over vertex set$V$. Then the intersection of G_1 and G_2 is fuzzy graph $G_3 = (S_3, M_3)$ over the set V where $S_3 = (S_1 \wedge S_2), M_3 = (M_1 \wedge M_2)$, where $S_3(x) = \min\{(S_1(x), S_2(x)\} \forall x \in V$ and $M_3(x, y) = \min\{(M_1(x, y), M_2(x, y)\}, \forall x, y \in V$.

Definition 2.4.
[3]: A graph $G = (V(G), E(G))$ is triple consisting finite sets:$V(G)$, the vertices set of the graph, often denoted by just V, which is a nonempty set of elements called vertices, and $E(G)$, the edges set of the graph, often denoted by just E, which is a possibly an empty set of elements called edges.

Definition 2.5.
[4]: A complete graph is a simple graph in which each pair of distinct vertices is joined by an edge. If the complete graph has vertices $v_1, ..., v_n$ then the edges set can be given by $E = \{(v_i, v_j) : i \neq j, i, j = 1, ..., n\}$. It follows that the graph has $\frac{1}{2}n(n-1)$ edges .

3. Operations of Anti fuzzy graphs

In this section we establish some concepts of defined Anti fuzzy graph and union , intersection of two Anti fuzzy graph.

Definition 3.1.
: Let G be simple graph A fuzzy sub set $M of G$ is called Anti fuzzy graph such that $M(x, y) \leq S(x).S(y), \forall (x, y) \in G$. An Anti fuzzy graph $AfG = (S, M)$ over the set V is called strong Anti fuzzy graph if $M(x, y) = S(x).S(y), \forall (x, y) \in G$.

Definition 3.2.
: Let $AFG_1 = (S_1, M_1)$ and $AFG_2 = (S_2, M_2)$ be two Anti fuzzy graph over vertex set V. Then the union of $AFG_1$and AFG_2 is Anti fuzzy graph $AFG_3 = (S_3, M_3)$ over the set V where$S_3 = (S_1 \vee S_2), M_3 = (M_1 \vee M_2)$, where$S_3(x) = \max\{(S_1(x), S_2(x))\}, \forall x \in V$and $M_3(x, y) = \max\{(M_1(x, y), M_2(x, y))\}, \forall x, y \in V$.

Definition 3.3.
: Let$AfG_1 = (S_1, M_1)$ and $AFG_2 = (S_2, M_2)$ be two Anti fuzzy graph over vertex set V. Then the intersection of AfG_1 and AFG_2 is fuzzy graph $AfG_3 = (S_3, M_3)$ over the set V where$S_3 = (S_1 \wedge S_2), M_3 = (M_1 \wedge M_2)$, where $S_3(x) = \min\{(S_1(x), S_2(x))\} \forall x \in v$ and $M_3(x, y) = \min\{(M_1(x, y), M_2(x, y))\} \forall x, y \in V$.

Now we define Anti fuzzy subgraph.

Definition 3.4.
: Let G be simple graph , let $AFH = (S_i, M_i)$ and $AfG = (S, M)$ is two Anti fuzzy graph over set V then AfH is called Anti fuzzy subgraph of Anti fuzzy graph such that $S_i(x) \leq S(x), M_i(x, y) \leq M(x, y), \forall (x, y) \in AFG$.

Now we define null-identity Anti fuzzy graph.

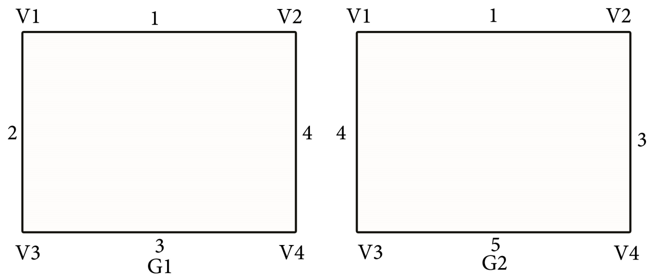
Definition 3.5.
: Let G be simple graph , the Anti fuzzy graph $AF\emptyset \in AF(V, E)$ is called null Anti fuzzy graph such that denoted by $\emptyset^{-} if AF\emptyset(e) = (0), \forall e \in E$.

Definition 3.6.
: Let G be simple graph , the Anti fuzzy graph $AFI \in AF(V, E)$ is called identity Anti fuzzy graph such that denoted by $I^{-} if AFI(e) = (1), \forall e \in E$.

Example 3.1.
Let $V=[v_1,v_2,v_3,v_4]$ and $E=[e_1,e_2,e_3,e_4]$. Here Anti fuzzy graph AfG such that

$$M(x,y)=\begin{cases} 0, & \text{if } v_i \text{ is not end of } e_i; \\ 1, & \text{if } v_i \text{ ist end of } e_i \end{cases}$$

$V(x_i,x_j=1,\forall(x_i,x_j)\in V$ while $V(x_i,x_j)=i,i=2,3,4,5,\forall(x_i,x_j)\notin V$ In the following graph G_1 and G_2,



Of the graph G_1 ,we apply Anti fuzzy graph

$$M(v_1v_2)=1$$

and

$$S(v_1)=1,S(v_2)=1$$

such that

$$S(v_1).S(v_2)=1$$

then

$$M(v_1v_2)\le S(v_1).S(v_2)$$

$$M(v_1v_3)=2$$

$$S(v_1)=1,S(v_3)=3$$

Then

$$M(v_1v_3)\le S(v_1).S(v_3)$$

if

$$M(v_3v_4)=3$$

$$S(v_3)=3,S(v_4)=4$$

then

$$M(v_3v_4)\le S(v_3).S(v_4)$$

if

$$M(v_2v_4)=4$$

$$S(v_2)=1,S(v_4)=4$$

then

$$M(v_2v_4)\le S(v_2).S(v_4)$$

Of the graph G_2 ,we apply Anti fuzzy graph

$$M(v_1v_2)=2$$

and

$$S(v_1) = 2, S(v_2) = 3$$

such that

$$S(v_1).S(v_2) = 6$$

then

$$M(v_1 v_2) \leq S(v_1).S(v_2)$$

if

$$M(v_1 v_3) = 4$$

$$S(v_1) = 2, S(v_3) = 4$$

then

$$M(v_1 v_3) \leq S(v_1).S(v_3)$$

if

$$M(v_3 v_4) = 5$$

$$S(v_3) = 4, S(v_4) = 5$$

then

$$M(v_3 v_4) \leq S(v_3).S(v_4)$$

if

$$M(v_2 v_4) = 3$$

$$S(v_2) = 3, S(v_4) = 5$$

then

$$M(v_2 v_4) \leq S(v_2).S(v_4)$$

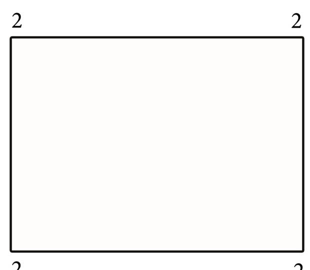
4. Anti fuzzy graph depended of degree graph

In this section we defined Anti fuzzy graph of degree and study the type graph of Anti fuzzy graph.

Definition 4.1.
: Let G be simple graph A fuzzy sub set $M of G$ is called Anti fuzzy graph such that $M(x, y) \leq nS(x).mS(y), \forall (x, y) \in G$. An Anti fuzzy graph $AfG = (S, M)$ over the set V such that $S(x), S(y)$ depended of degree graph ,$\forall (x, y) \in G, n, m$ number degree of S.

Theorem 4.1.
: Let G be 2-regular fuzzy graph then G has Anti fuzzy regular graph.

Proof. Suppose that G be 2-regular fuzzy graph , then $\sum d(v) = 2$ of each vertex of G , we must prove G has Anti fuzzy regular graph.
The condition Anti fuzzy regular graph $M(x, y) \leq nS(x).mS(y)$, such that $n, m = 2$ of the following graph G,



G

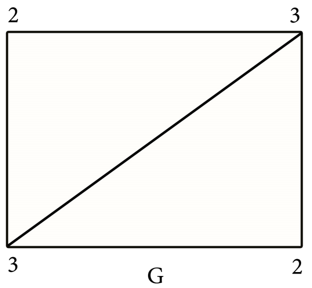
Since the graph is 2- regular graph if $M(x,y)=1$
Then $nS(x).mS(y)=4$ of each vertex of G,

$$ThusM(x,y) \leq nS(x).mS(y)$$

Then G has Anti fuzzy regular graph. □

Theorem 4.2.
Let G be not regular fuzzy graph then G has Anti fuzzy graph.

Proof. Suppose that G be not regular graph, in other word the degree of each vertex is not equal. in the following graph G look state,



G

$$M(v_1 v_2)=1$$

and

$$S(v_1)=2, S(v_2)=3$$

such that

$$nS(v_1).mS(v_2)=6$$

then

$$M(v_1 v_2) \leq nS(v_1).mS(v_2)$$

$$M(v_1 v_3)=1$$

$$S(v_1)=2, S(v_3)=3$$

then

$$M(v_1 v_3) \leq S(v_1).S(v_3)$$

if

$$M(v_3 v_4)=1$$

$$S(v_3)=3, S(v_4)=2$$

then

$$M(v_3 v_4) \leq S(v_3).S(v_4)$$

if

$$M(v_2 v_4)=1$$

$$S(v_2)=3, S(v_4)=2$$

then

$$M(v_2 v_4) \leq S(v_2).S(v_4)$$

Then G has Anti fuzzy graph. □

Theorem 4.3.
: Let G be K_n complete fuzzy graph then G has Anti fuzzy regular graph, $n = 2,3,4,5$.

Proof. Suppose that G be K_n complete fuzzy graph, if k_2 complete graph.

$$M(v_1v_2) = 1$$

and

$$S(v_1) = 1, S(v_2) = 1$$

such that

$$S(v_1).S(v_2) = 1$$

then

$$M(v_1v_2) \leq S(v_1).S(v_2)$$

if K_3 complete graph $M(v_1v_2) = 1$

$$S(v_1) = 2, S(v_2) = 2$$

Then

$$M(v_1v_2) \leq S(v_1).S(v_2)$$

if

$$M(v_1v_3) = 1$$

$$S(v_1) = 2, S(v_2) = 2$$

Then $M(v_1v_3) \leq S(v_1).S(v_3)$

$$M(v_1v_3) = 1$$

$$S(v_2) = 2, S(v_3) = 2$$

Then $M(v_2v_3) \leq S(v_2).S(v_3)$ Since complete graph is regular then degree of every vertex is equal ,then Anti fuzzy graph is verified. if k_4 complete graph,

$$M(v_1v_2) = 1$$

$$S(v_1) = 3, S(v_2) = 3$$

then

$$M(v_1v_2) \leq S(v_1).S(v_2)$$

if

$$M(v_1v_3) = 1$$

$$S(v_1) = 3, S(v_2) = 3$$

Then $M(v_1v_3) \leq S(v_1).S(v_3)$

$$M(v_2v_4) = 1$$

$$S(v_2) = 3, S(v_4) = 3$$

Then

$$M(v_2v_4) \leq S(v_2).S(v_4)$$

if

$$M(v_3v_4) = 1$$

$$S(v_3) = 3, S(v_4) = 3$$

Then

$$M(v_3v_4) \leq S(v_3).S(v_4)$$

In same method w. r. t the complete graph k_5 we apply Anti fuzzy graph.

□

Definition 4.2.
If $G = (V, E)$ and V has at least two elements, then for any vertex v of G, $G - \{v\}$ denotes the sub graph of G with vertex set $V - \{v\}$ whose edges are all those of G which are not incident with v,i.e., $G - \{v\}$ is obtained from G by removing v and all the edges of G which have v as an end. $G - \{v\}$ is referred to as a vertex deleted subgraph. If $G = (V, E)$ and e is an edge of G then $G - \{e\}$ denotes the subgraph of G having V as its vertices set.

In this section we study the graph is Anti fuzzy graph then removable vertex and removable edge and contraction edge of graph after remove vertex is remained properties Anti fuzzy graph.

Definition 4.3.
Let G be Anti fuzzy graph then $G - p$ is Anti fuzzy graph. p is vertex -edge

Theorem 4.4.
Let G be Anti fuzzy graph-regular then $G - v$ is Anti fuzzy graph.

Proof. Suppose that G be Anti fuzzy regular graph, from definition 4.3, we removable vertex of graph G when remove vertex of graph so remove two edge of each once remove vertex of graph, now apply remove vertex of 2-rgular graph , from theorem 4.1 , G be Anti fuzzy graph 2-regular , so application removable $G - v_4$

$$M(v_1 v_2) = 1$$

$$S(v_1) = 2, S(v_2) = 1$$

then

$$M(v_1 v_2) \leq S(v_1).S(v_2)$$

if

$$M(v_1 v_3) = 1$$

$$S(v_1) = 2, S(v_3) = 1$$

Then

$$M(v_1 v_3) \leq S(v_1).S(v_3)$$

Of each then remove vertex of graph has Anti fuzzy graph is remained has properties Anti fuzzy graph. □

Theorem 4.5.
Let G be Anti fuzzy graph-regular then $G - e$ is Anti fuzzy graph.

Proof. Suppose that G be Anti fuzzy regular graph, from definition 4.3, we removable edge of graph G when remove edge of graph so remove only edge of from graph, now apply remove edge of 2-rgular graph , from theorem 4.1 ,G be Anti fuzzy graph 2-regular , so application removable $G - e$

$$M(v_1 v_2) = 1$$

$$S(v_1) = 2, S(v_2) = 1$$

then

$$M(v_1 v_2) \leq S(v_1).S(v_2)$$

if

$$M(v_1 v_3) = 1$$

$$S(v_1) = 2, S(v_3) = 2$$

Then

$$M(v_1 v_3) \leq S(v_1).S(v_3)$$

$$M(v_2 v_4) = 0$$

$$S(v_2) = 1, S(v_4) = 1$$

then

$$M(v_2 v_4) \leq S(v_2).S(v_4)$$

, Of each then remove edge of graph has Anti fuzzy graph is remained has properties Anti fuzzy graph. □

Theorem 4.6.
Let G be Anti fuzzy graph-regular then $G*e$ is Anti fuzzy graph.

Proof. Suppose that G be Anti fuzzy regular graph, from definition 4.2, we contraction edge of graph G when contraction edge of graph so edge applies two edges of from graph, now apply contraction edge of 2-rgular graph , from theorem 4.1 , then G be Anti fuzzy graph 2- regular. so application contraction $G*e$

$$M(v_1v_2) = 1$$

$$S(v_1) = 2, S(v_2) = 2$$

then

$$M(v_1v_2) \leq S(v_1).S(v_2)$$

if

$$M(v_1v_3) = 1$$

$$S(v_1) = 2, S(v_3) = 2$$

Then

$$M(v_1v_3) \leq S(v_1).S(v_3)$$

$$M(v_2v_4) = 1$$

$$S(v_2) = 2, S(v_4) = 1$$

then

$$M(v_2v_4) \leq S(v_2).S(v_4)$$

Of each then contraction edge of graph has Anti fuzzy graph is remained has properties Anti fuzzy graph. □

5. Conclusions

1. we define Anti fuzzy graph as following let G be simple graph A fuzzy sub set $M of G$ is called Anti fuzzy graph such that $M(x, y) \leq S(x).S(y), \forall(x, y) \in G.$ An Anti fuzzy graph $AfG = (S, M)$ over the set V is called strong Anti fuzzy graph if $M(x, y) = S(x).S(y), \forall(x, y) \in G.$ and we study operation union and intersection null, identity on Anti fuzzy graph, we study Anti fuzzy subgraph.

2. we define Anti fuzzy graph depended degree graph as following: Let G be simple graph A fuzzy sub set $M of G$ is called Anti fuzzy graph such that $M(x, y) \leq nS(x).mS(y), \forall(x, y) \in G.$ An Anti fuzzy graph $AfG = (S, M)$ over the set V such that $S(x), S(y)$ depended of degree graph , $\forall(x, y) \in G, n, m$ number degree of $S.$ take some type graph regular - non regular - complete graph,

3. we study removable vertex and removable edge from Anti fuzzy graph is remained the graph is Anti fuzzy graph.

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