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Approximation by modified Gamma type operators

Research Article

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Abstract: In this paper, we give a modified form of general Gamma type operators which preserve the constant as well as linear functions. We estimate the moments of the operators and then prove the Voronovskaja type theorem. Also, direct approximation theorem, rate of convergence and weighted approximation by these operators in terms of modulus of continuity are studied. Lastly, we obtain pointwise estimate using the Lipschitz type maximal function and two parameter Lipschitz type space.

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Keywords: Gamma type operators • Modulus of continuity • Weighted approximation • Voronovskaja type asymptotic formula • Rate of convergence

1. Introduction

The approximation of functions by linear positive operators is an important research topic in general mathematics and it also provides powerful tools to application areas such as computer-aided geometric design, numerical analysis, and solutions of differential equations.

In [33], Lupas and M$\ddot{u}$ller defined and studied some approximation properties of a sequence of linear and positive operators $\{G_n\}$ defined as

$$G_n(f;x)=\int_0^\infty g_n(x,u)f\Big(\frac{n}{u}\Big)du,$$

where

$$g_n(x,u)=\frac{x^{n+1}}{n!}e^{-xu}u^n,\quad x>0.$$

Approximation properties of $\{G_n\}$ in some function spaces were studied in many papers (see [33], [38], [41]). The above operators were modified by several researchers (see [24], [28], [34]), which showed that new operators have similar approximation properties to $\{G_n\}$ (see [23], [25], [26], [27], [31], [32], [40]).

In the year 2007, Mao [35] defined the following generalized Gamma type linear and positive operators

$$\begin{aligned} M_{n,k}(f;x) &= \int_0^\infty\int_0^\infty g_n(x,u)g_{n-k}(u,t)f(t)\,du\,dt \\ &= \frac{(2n-k+1)!}{n!(n-k)!}x^{n+1}\int_0^\infty \frac{t^{n-k}}{(x+t)^{2n-k+2}}f(t)\,dt,\ x>0, \end{aligned} \tag{1}$$

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for any f for which the above integral is convergent.
Some approximation properties of $\{M_{n,k}\}$ were studied in these papers (see [5], [8], [12], [13], [14], [29], [30]).
It is observed that the operators (1) reproduce only constant functions. So here we modify the operators (1) so that they may be capable to reproduce constant as well as linear function. King [17] gave an approach for modification of the classical Bernstein polynomials and he achieved better approximation.
For $f \in C_\gamma[0,\infty) := \{f \in C[0,\infty) : |f(t)| \le \mathcal{M}(1+t)^\gamma$ for some $\mathcal{M} > 0, \gamma > 0\}$, we introduce the following King type modification of the operators (1) as follows:

$$M^*_{n,k}(f;x) = \frac{(2n-k+1)!}{n!(n-k)!}x^{n+1}\int_0^\infty \frac{t^{n-k}}{(x+t)^{2n-k+2}} f\left(\frac{nt}{n-k+1}\right)dt. \tag{2}$$

In the present paper, we study the basic convergence theorem, Voronovskaja type theorem, local approximation, rate of convergence, weighted approximation and pointwise estimation of the operators (2).

2. Preliminaries

In this section we collect some results about the operators $M^*_{n,k}$ useful in the sequel.

Lemma 2.1.
[35] For $M_{n,k}(t^m;x)$, $m = 0,1,2$, we have

1. $M_{n,k}(1;x) = 1$;
2. $M_{n,k}(t;x) = \dfrac{(n-k+1)x}{n}$;
3. $M_{n,k}(t^2;x) = \dfrac{(n-k+2)(n-k+1)x^2}{n(n-1)}$.

Lemma 2.2.
For the operators $M^*_{n,k}(f;x)$ as defined in (2), the following equalities holds:

1. $M^*_{n,k}(1;x) = 1$;
2. $M^*_{n,k}(t;x) = x$;
3. $M^*_{n,k}(t^2;x) = \dfrac{n(n-k+2)x^2}{(n-1)(n-k+1)}$.

Proof. For $x \in [0,\infty)$, in view of Lemma 2.1, we have

$$M^*_{n,k}(1;x) = 1.$$

Next, for $f(t) = t$, we get

$$M^*_{n,k}(t;x) = \frac{(2n-k+1)!}{n!(n-k)!}x^{n+1}\int_0^\infty \frac{t^{n-k}}{(x+t)^{2n-k+2}}\left(\frac{nt}{n-k+1}\right)dt = x.$$

Proceeding similarly, we have

$$\begin{aligned} M^*_{n,k}(t^2;x) &= \frac{(2n-k+1)!}{n!(n-k)!}x^{n+1}\int_0^\infty \frac{t^{n-k}}{(x+t)^{2n-k+2}}\left(\frac{nt}{n-k+1}\right)^2 dt \\ &= \frac{n(n-k+2)x^2}{(n-1)(n-k+1)}. \end{aligned}$$

□

Remark 2.1.
For every $x \in [0,\infty)$, we have

$$M^*_{n,k}((t-x);x) = 0$$

and

$$\begin{aligned} M^*_{n,k}\left((t-x)^2;x\right) &= \frac{(2n-k+1)x^2}{(n-1)(n-k+1)} \\ &= \zeta_{n,k}(x), \text{(say)}. \end{aligned}$$

Lemma 2.3.
For $f \in C_B[0,\infty)$ *(space of all real valued bounded and uniformly continuous functions on* $[0,\infty)$ *endowed with norm* $\| f \|_{C_B[0,\infty)} = \sup\limits_{x\in[0,\infty)} |f(x)|)$, $\| M^*_{n,k}(f) \| \le \| f \|$.

Proof. In view of (2) and Lemma 2.2, the proof of this lemma easily follows. □

For $C_B[0,\infty)$, let us define the following Peetre's K-functional:

$$K_2(f,\delta) = \inf_{g\in W^2} \{\| f-g \|_{C_B[0,\infty)} + \delta \| g'' \|_{C_B[0,\infty)}\},$$

where $\delta > 0$ and $W^2 = \{g \in C_B[0,\infty) : g', g'' \in C_B[0,\infty)\}$. By, p. 177, Theorem 2.4 in [2], there exists an absolute constant $\mathcal{M} > 0$ such that

$$K_2(f,\delta) \le \mathcal{M}\omega_2(f,\sqrt{\delta}), \tag{3}$$

where

$$\omega_2(f,\sqrt{\delta}) = \sup_{0<|h|\le\sqrt{\delta}} \sup_{x\in[0,\infty)} | f(x+2h) - 2f(x+h) + f(x) |$$

is the second order modulus of smoothness of f.

3. Main results

In this section we establish approximation properties in several settings. For the reader's convenience we split up this section in more subsections.

3.1. Voronovskaja type theorem

Theorem 3.1.
Let f be bounded and integrable on $[0,\infty)$, *second derivative of f exists at a fixed point* $x \in [0,\infty)$, *then we have*

$$\lim_{n\to\infty} n\left(M^*_{n,k}(f;x) - f(x)\right) = x^2 f''(x).$$

Proof. Using Taylor's theorem, we have

$$f(t) = f(x) + (t-x)f'(x) + \frac{1}{2}f''(x)(t-x)^2 + \xi(t,x)(t-x)^2, \tag{4}$$

where $\xi(t,x)$ is the Peano form of the remainder and $\lim\limits_{t\to x}\xi(t,x) = 0$.
Applying $M^*_{n,k}(f,x)$ to (4), we get

$$n\left(M^*_{n,k}(f;x) - f(x)\right) = nf'(x)M^*_{n,k}((t-x);x) + \frac{1}{2}nf''(x)M^*_{n,k}\left((t-x)^2;x\right) + nM^*_{n,k}\left(\xi(t,x)(t-x)^2;x\right).$$

In view of Remark 2.1, we have

$$\lim_{n\to\infty} nM^*_{n,k}((t-x);x) = 0 \tag{5}$$

and

$$\lim_{n\to\infty} nM^*_{n,k}\left((t-x)^2;x\right) = 2x^2. \tag{6}$$

Now, we shall show that

$$\lim_{n\to\infty} nM^*_{n,k}\left(\xi(t,x)(t-x)^2;x\right) = 0.$$

Applying the Cauchy-Schwarz inequality, we have

$$M^*_{n,k}\left(\xi(t,x)(t-x)^2;x\right) \le \sqrt{M^*_{n,k}(\xi^2(t,x);x)}\sqrt{M^*_{n,k}((t-x)^4;x)}. \tag{7}$$

We observe that $\xi^2(x,x) = 0$ and $\xi^2(.,x) \in C_\gamma[0,\infty)$. Then, it follows that

$$\lim_{n\to\infty} M^*_{n,k}(\xi^2(t,x);x) = \xi^2(x,x) = 0. \tag{8}$$

Now, from (7) and (8) we obtain

$$\lim_{n\to\infty} nM^*_{n,k}\left(\xi(t,x)(t-x)^2;x\right) = 0. \tag{9}$$

From (5), (6) and (9), we get the required result. □

3.2. Direct result

Theorem 3.2.
For every $x \in [0,\infty)$ and $f \in C_B[0,\infty)$, there exist an absolute constant $\mathcal{M}$ such that

$$| M^*_{n,k}\left(f;x\right) - f(x) | \le \mathcal{M}\omega_2\left(f, \sqrt{\zeta_{n,k}(x)}\right),$$

Proof. Let $g \in W^2$ and $x, t \in [0,\infty)$. Using Taylor's series, we have

$$g(t) = g(x) + g'(x)(t-x) + \int_x^t (t-v)g''(v)dv.$$

Applying $M^*_{n,k}$ on both sides and using Lemma 2.2, we get

$$M^*_{n,k}(g;x) - g(x) = M^*_{n,k}\Big(\int_x^t (t-v)g''(v)dv; x\Big).$$

Obviously, we have $\left|\int_x^t (t-v)g''(v)dv\right| \le (t-x)^2\|g''\|$.
Therefore

$$| M^*_{n,k}(g;x) - g(x) | \le M^*_{n,k}((t-x)^2;x) \| g'' \| = \zeta_{n,k}(x) \| g'' \| .$$

Since $| M^*_{n,k}(f;x) | \le \|f\|$, we have

$$\begin{aligned} | M^*_{n,k}(f;x) - f(x) | &\le | M^*_{n,k}(f-g;x) | + | (f-g)(x) | + | M^*_{n,k}(g;x) - g(x) | \\ &\le 2\|f-g\| + \zeta_{n,k}(x)\|g''\|. \end{aligned}$$

Finally, taking the infimum over all $g \in W^2$ and using (3) we obtain

$$| M^*_{n,k}(f;x) - f(x) | \le \mathcal{M}\omega_2\left(f, \sqrt{\zeta_{n,k}(x)}\right),$$

which proves the theorem. □

3.3. Rate of convergence

Let $\omega_a(f,\delta)$ denote the modulus of continuity of f on the closed interval $[0,a]$, $a > 0$ and defined as

$$\omega_a(f,\delta) = \sup_{|t-x|\le\delta} \sup_{x,t\in[0,a]} |f(t) - f(x)|.$$

We observe that for a function $f \in C_B[0,\infty)$, the modulus of continuity $\omega_a(f,\delta)$ tends to zero.
Now, we give a rate of convergence theorem for the operators $M^*_{n,k}$.

Theorem 3.3.
Let $f \in C_B[0,\infty)$ and $\omega_{a+1}(f,\delta)$ be its modulus of continuity on the finite interval $[0,a+1] \subset [0,\infty)$, where $a > 0$. Then, we have

$$|M^*_{n,k}(f;x) - f(x)| \le 6\mathcal{M}_f(1+a^2)\zeta_{n,k}(a) + 2\omega_{a+1}\left(f, \sqrt{\zeta_{n,k}(a)}\right),$$

where $\zeta_{n,k}(a)$ is defined in Remark 2.1 and $\mathcal{M}_f$ is a constant depending only on f.

Proof. For $x \in [0,a]$ and $t > a+1$. Since $t - x > 1$, we have

$$|f(t) - f(x)| \le \mathcal{M}_f(2 + x^2 + t^2) \le \mathcal{M}_f(t-x)^2(2 + 3x^2 + 2(t-x)^2) \le 6\mathcal{M}_f(1+a^2)(t-x)^2.$$

For $x \in [0,a]$ and $t \le a+1$, we have

$$|f(t) - f(x)| \le \omega_{a+1}(f, |t-x|) \le \left(1 + \frac{|t-x|}{\delta}\right)\omega_{a+1}(f,\delta)$$

with $\delta > 0$.
From the above, we have

$$|f(t) - f(x)| \le 6\mathcal{M}_f(1+a^2)(t-x)^2 + \left(1 + \frac{|t-x|}{\delta}\right)\omega_{a+1}(f,\delta),$$

for $x \in [0,a]$ and $t \ge 0$.
Applying Cauchy-Schwarz inequality, we have

$$\begin{aligned} |M^*_{n,k}(f;x) - f(x)| &\le 6\mathcal{M}_f(1+a^2)(M^*_{n,k}(t-x)^2;x) + \omega_{a+1}(f,\delta)\left(1 + \frac{1}{\delta}(M^*_{n,k}(t-x)^2;x)^{\frac{1}{2}}\right) \\ &\le 6\mathcal{M}_f(1+a^2)\zeta_{n,k}(a) + 2\omega_{a+1}\left(f, \sqrt{\zeta_{n,k}(a)}\right), \end{aligned}$$

on choosing $\delta = \sqrt{\zeta_{n,k}(a)}$. This completes the proof of the theorem. □

3.4. Weighted approximation.

In this section, we obtain the Korovkin type weighted approximation by the operators defined in (2) . The weighted Korovkin-type theorems were proved by Gadzhiev [3]. A real function $\rho(x) = 1 + x^2$ is called a weight function if it is continuous on R and $\lim_{|x|\to\infty} \rho(x) = \infty$, $\rho(x) \geq 1$ for all $x \in R$.
Let $B_\rho(R)$ denote the weighted space of real-valued functions f defined on R with the property $|f(x)| \leq M_f \rho(x)$ for all $x \in R$, where M_f is a constant depending on the function f. We also consider the weighted subspace $C_\rho(R)$ of $B_\rho(R)$ given by $C_\rho(R) = \{f \in B_\rho(R) : f$ is continuous on $R\}$ and $C^*_\rho[0,\infty)$ denotes the subspace of all functions $f \in C_\rho[0,\infty)$ for which $\lim_{|x|\to\infty} \frac{f(x)}{\rho(x)}$ exists finitely.

Theorem 3.4.
For each $f \in C^*_\rho[0,\infty)$, we have

$$\lim_{n\to\infty} \| M^*_{n,k}(f) - f \|_\rho = 0.$$

Proof. From [3], we know that it is sufficient to verify the following three conditions

$$\lim_{n\to\infty} \| M^*_{n,k}(t^k) - x^k \|_\rho = 0, \ k = 0,1,2. \tag{10}$$

Since $M^*_{n,k}(1;x) = 1$, the condition in (10) holds for $k = 0$.
By Lemma 2.2, we have

$$\| M^*_{n,k}(t) - x \|_\rho = \sup_{x\in[0,\infty)} \frac{|M^*_{n,k}(t;x) - x|}{1 + x^2} = 0$$

which implies that the condition in (10) holds for $k = 1$.
Similarly, we have

$$\begin{aligned}\| M^*_{n,k}(t^2) - x^2 \|_\rho &= \sup_{x\in[0,\infty)} \frac{|M^*_{n,k}(t^2;x) - x^2|}{1 + x^2} \\ &\leq \left| \frac{(2n-k+1)}{(n-1)(n-k+1)} \right|,\end{aligned}$$

which implies that $\lim_{n\to\infty} \| M^*_{n,k}(t^2) - x^2 \|_\rho = 0$, the equation (10) holds for $k = 2$.
This completes the proof of theorem. □

3.5. Pointwise Estimates

In this section, we obtain some pointwise estimates of the rate of convergence of the operators $M^*_{n,k}$. First, we give the relationship between the local smoothness of f and the local approximation.
We know that a function $f \in C_B[0,\infty)$ is in $\mathrm{Lip}_{\mathcal{M}}(r)$ on E, $r \in (0,1]$, E$\subset [0,\infty)$ if it satisfies the condition

$$|f(t) - f(x)| \leq \mathcal{M}|t - x|^r, \ t \in [0,\infty) \ and \ x \in E,$$

where $\mathcal{M}$ is a constant depending only on r and f.

Theorem 3.5.
Let $f \in C_B[0,\infty) \cap Lip_{\mathcal{M}}(r)$, $E \subset [0,\infty)$ and $0 < r \leq 1$. Then, we have

$$|M^*_{n,k}(f;x) - f(x)| \leq \mathcal{M}\Big(\big(\zeta_{n,k}(x)\big)^{r/2} + 2(d(x,E))^r \Big), \ x \in [0,\infty),$$

where $\mathcal{M}$ is a constant depending on r and f and $d(x,E)$ is the distance between x and E defined as

$$d(x,E) = \inf\{|t - x|; t \in E\}.$$

Proof. Let $\overline{E}$ be the closure of E in $[0,\infty)$. Then, there exists at least one point $t_0 \in \overline{E}$ such that

$$d(x,E) = |x - t_0|.$$

By our hypothesis and the monotonicity of $M^*_{n,k}$, we get

$$\begin{aligned}|M^*_{n,k}(f;x) - f(x)| &\leq M^*_{n,k}(|f(t) - f(t_0)|;x) + M^*_{n,k}(|f(x) - f(t_0)|;x)\\ &\leq \mathcal{M}\left(M^*_{n,k}(|t - t_0|^r;x) + |x - t_0|^r\right)\\ &\leq \mathcal{M}\left(M^*_{n,k}(|t - x|^r;x) + 2|x - t_0|^r\right).\end{aligned}$$

Now, applying Hölder's inequality with $p = \dfrac{2}{r}$ and $q = \dfrac{2}{2-r}$, we obtain

$$|M^*_{n,k}((f;x) - f(x)| \leq \mathcal{M}\left((M^*_{n,k}(|t - x|^2;x))^{r/2} + 2(d(x,E))^r\right),$$

from which the desired result immediate. □

Next, we obtain a local direct estimate of the operators defined in (2), using the Lipschitz-type maximal function of order r introduced by B. Lenze [18] as

$$\widetilde{\omega}_r(f,x) = \sup_{t\neq x,\ t\in[0,\infty)} \frac{|f(t) - f(x)|}{|t - x|^r}, \quad x \in [0,\infty) \text{ and } r \in (0,1]. \tag{11}$$

Theorem 3.6.
Let $f \in C_B[0,\infty)$ and $0 < r \leq 1$, then for all $x \in [0,\infty)$ we have

$$|M^*_{n,k}(f;x) - f(x)| \leq \widetilde{\omega}_r(f,x)\left(\zeta_{n,k}(x)\right)^{r/2}.$$

Proof. From the equation (11), we have

$$|M^*_{n,k}(f;x) - f(x)| \leq M^*_{n,k}(|f(t) - f(x)|;x) \leq \widetilde{\omega}_r(f,x) M^*_{n,k}(|t - x|^r;x).$$

Now, using the Hölder's inequality with $p = \dfrac{2}{r}$ and $\dfrac{1}{q} = 1 - \dfrac{1}{p}$, we obtain

$$|M^*_{n,k}(f;x) - f(x)| \leq \widetilde{\omega}_r(f,x) M^*_{n,k}((t - x)^2;x)^{r/2} \leq \widetilde{\omega}_r(f,x)\left(\zeta_{n,k}(x)\right)^{r/2}.$$

Thus, the proof is completed. □

For $a,b > 0$, Özarslan and Aktuğlu [22] consider the Lipschitz-type space with two parameters:

$$Lip^{(a,b)}_{\mathcal{M}}(r) = \left(f \in C[0,\infty) : |f(t) - f(x)| \leq \mathcal{M}\frac{|t - x|^r}{(t + ax^2 + bx)^{r/2}};\ x,t \in [0,\infty)\right),$$

where $\mathcal{M}$ is any positive constant and $0 < r \leq 1$.

Theorem 3.7.
For $f \in Lip^{(a,b)}_{\mathcal{M}}(r)$. Then, for all $x \in [0,\infty)$, we have

$$|M^*_{n,k}(f;x) - f(x)| \leq \mathcal{M}\left(\frac{\zeta_{n,k}(x)}{ax^2 + bx}\right)^{r/2}.$$

Proof. First we prove the theorem for $r = 1$. Then, for $f \in Lip^{(a,b)}_{\mathcal{M}}(1)$ and $x \in [0,\infty)$, we have

$$\begin{aligned}|M^*_{n,k}(f;x) - f(x)| &\leq M^*_{n,k}(|f(t) - f(x)|;x)\\ &\leq \mathcal{M} M^*_{n,k}\left(\frac{|t - x|}{(t + ax^2 + bx)^{1/2}};x\right)\\ &\leq \frac{\mathcal{M}}{(ax^2 + bx)^{1/2}} M^*_{n,k}(|t - x|;x).\end{aligned}$$

Applying Cauchy-Schwarz inequality, we get

$$|M_{n,k}^*(f;x) - f(x)| \le \frac{\mathcal{M}}{(ax^2+bx)^{1/2}} \left(M_{n,k}^*((t-x)^2;x)\right)^{1/2}$$
$$\le \mathcal{M}\left(\frac{\zeta_{n,k}(x)}{ax^2+bx}\right)^{1/2}.$$

Thus, the result holds for $r = 1$.
Now, we prove that the result is true for $0 < r < 1$. Then, for $f \in Lip_{\mathcal{M}}^{(a,b)}(r)$ and $x \in [0,\infty)$, we get

$$|M_{n,k}^*(f;x) - f(x)| \le \frac{\mathcal{M}}{(ax^2+bx)^{r/2}} M_{n,k}^*(|t-x|^r;x).$$

Taking $p = \frac{1}{r}$ and $q = \frac{p}{p-1}$, applying the Hölders inequality, we have

$$|M_{n,k}^*(f;x) - f(x)| \le \frac{\mathcal{M}}{(ax^2+bx)^{r/2}} \left(M_{n,k}^*(|t-x|;x)\right)^r.$$

Finally by Cauchy-Schwarz inequality, we get

$$|M_{n,k}^*(f;x) - f(x)| \le \mathcal{M}\left(\frac{\zeta_{n,k}(x)}{ax^2+bx}\right)^{r/2}.$$

Thus, the proof is completed. □

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