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Analytical investigation of Jeffery-Hemal flow with magnetic field by differential transform method

Research Article

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Abstract: The aim of this research paper is to show a new reliable method to calculate semi-analytical solution of magneto-hydrodynamic (MHD) Jeffery-Hemal [20] flow with magnetic field by using a Differential Transform Method.The approximate solution of this problem is computed here in the form of a rapidly convergent series.By taking Different values of Hartmann and Reynolds numbers for velocity profiles in divergent and convergent channels was computed. The validity and applicability of this new technique is illustrated through convergence of DTM & comparison between Differential Transform Method, Modification of Adomian Decomposition Method [15], Reproducing Kernel Hilbert Space Method[17] and Numerical solutions[12].

MSC: 35A24 • 76S05 • 46C05

Keywords: Jeffery-Hemal flow • Differential Transform Method (DTM) • Non-linear differential equation

1. Introduction

In recent years the study on fluid flow between two inclined plates in many engineering discipline and sciences is a most important problem in the field of fluid flow.This type of flow was first discussed by Hemal[13] and Jeffery [20] and obtained the mathematical formulation for Jaffery Hamel flow from Navier-stokes equation and continuity equation to describe velocity of Jeffery Hemal flow.Later on many researchers taken their keen interest to analyse the Jeffery Hemal flow with different field effect due to its importance and its role of regulator for velocity intensity for the flow. The study of this type of flow has been used in modern science applications like, the designing to maximize cooling in distributed system. J.Nagler [19] derived a mathematical formulation of Jeffery Hemal flow with considering the flow as a non-Newtonian flow.Naveed Ahmed[28] investigated a special case in which walls taken to be shirking and stretching in which Jeffery Hemal flow was flowing. Till now several techniques being developed to derive an approximate or exact solution of classical Jeffery Hemal flow problem. Vasile and Remus[32] applied Optimal Homotopy Perturbation method (OHAM) to calculate the approximate solution of MHD-JFL flow with the effect of Heat.Later on same problem with magnetic field effect was discussed by sheikholeslami, Ganji [30],Inc.M. and Akgul[17] solved the MHD-JHF problem by using RKHS method in non-parallel sides, same type of problem was solved by Moghimi[24] by using Homotopy analysis method.Singh & shodiya[31] used the modified sumudu transform technique to obtain the solution of Jeffery Hemal flow. Dongochi and Divsalar[2]introduced Duan Rach approach to obtain the solution of governing equation arising during flow & heat transfer of nano-fluid among two parallel plates with effect of thermal radiation. Vasile Marinca[32] applied OHAM to find an approximate solution for the non linear MHD-Jeffery-Hemal

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flow and shown that OHAM did not affected by variation of parameters and introduced new technique to dominance the convergence of the solution.Dogonchi and Ganji[2] investigated an analytical solution for finding heat transfer of two phase nano fluid flow between non-parellel walls with joule heating effect.Khudir[3] used Spectral-Homotopy Perturbation Method to obtain the solution of MHD Jeffery Hemal flow and also computed Boundary value problem related with MHD Jeffery Hemal flow by using Homotopy Peturbation Method.Abuteena, omanib,Ahmad Alawnehb[8] introduced a multi-step generalized Differential Tansform Method for finding the solution of time -fractional non linear block system. The effect of thermal radiation on the classical Jeffery-Hemal flow due to appoint sink or source in parallel plates was analyzed by Barzaegar[22] for the particular case in which the plates are adjusted for stretchable or sinkable.

Many researchers have taken their keen interest on different analytical methods in the study of fluid flow problems. Zhou[33] and pukchov applied DTM to solve the problems which arises in electric circuit analysis.similarly Hossein Jafari,Marym Alipour and Hale Tejadodi [16] used DTM in finding the solution of non-linear Gas Dynemic and Klein-Gordon equations arising in fluid flow problems.Patil and Khambayat[25] used this technique for linear differential equation later it was extended by Farshid Mirzee[11] for finding the solution of a non-linear system of ordinary differential equation. Chen & Ho[6] extended it to obtain the solution of partial differential equation and Ayaz [9] developed it to derive a solution of system of partial differential equation.Shahmorad et al.[7] solved a fractional order integro differential equation with non vocal boundary conditions by using DTM.Muhmmad Usman[23] studied the effect of magnetic field between two parallels walls for unsteady two phases of nano fluid flow and heat transfer by using differential Transform Method.Ayed R.khudir [3] extended the Differential transform method to Fractional Differential Transform Method to solve irrational order fractional differential equations. Kundu, Das and Lee[4] investigated the thermal analysis for exponential fins under sensible and latent heat transfer by applying DTM.Farshid[11] extended Differential Tansform method to three-dimensional fuzzy differential transform method to obtain the solution of fuzzy partial differential equations. EI Sayed & Nour [1] extended Differential Transform method to modified fractional differential Tansform Method Combined with the Adomain Polynomials and studied the projectile motion with quadratic drag force to investigate the local path angle ,the velocity and position at any time.N.D.Patel and Meher [26],[27] computed solution of Kolmogrove-Petrovskii-Piskunov Equation using DTM and also applied it to obtain solution of governing equation which is arising in Fingero-Imbibition Phenomena in Double Phase flow through Homogeneous Porous Media.

In this paper, the solution of classical problem of Jeffery Hemal flow with Magnetic effect was derived by DTM.In special case,the solution was compared with Modification of Adomian Decomposition Method to calculate error and convergence of DTM was discussed . In addition the variation in velocity profile of Jeffrey Hemal Flow was discussed by taking different values of Reynolds number and Hartmann number in divergent and convergent channels.

2. Mathematical formulation of Jeffery-Hemal flow

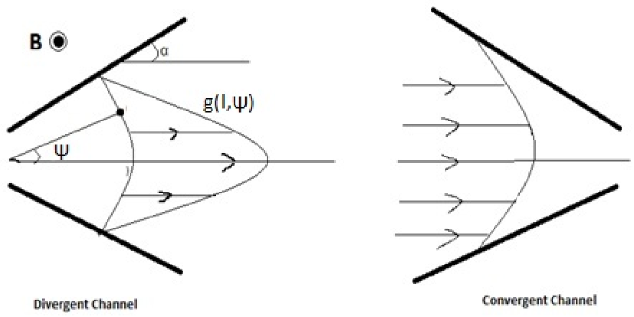

Fig. 1. The 2D flow of JHF with Magnetic Field

As shown in Fig. 1, Consider a steady two-dimensional flow of an incompressible conductive viscous fluid between two thick plates that meet at an angle & it is assumed that the flow of fluid is in horizontal direction and there is no movement in vertical direction.

For mathematical formulation, consider continuity equation and Navies-Stokes equation in polar coordinates as follows [15]:

$$\frac{\rho}{l}\frac{\partial}{\partial l}\left(l g(l,\psi)\right)=0 \tag{1}$$

$$g(l,\psi)\frac{\partial g(l,\psi)}{\partial l}=-\frac{1}{\psi}\frac{\partial p}{\partial l}+v\left[\frac{\partial^2 g(l,\psi)}{\partial l^2}+\frac{1}{l}\frac{\partial g(l,\psi)}{\partial l}+\frac{1}{l^2}\frac{\partial^2 g(l,\psi)}{\partial \psi^2}-\frac{g(l,\psi)}{l^2}\right]-\frac{\sigma B_0^2}{\rho l^2}g(l,\psi) \tag{2}$$

$$\frac{1}{\rho l}\frac{\partial p}{\partial \psi}-\frac{2v}{l^2}\frac{\partial g(l,\psi)}{\partial \psi}=0 \tag{3}$$

Where σ is the conductivity of the fluid,$g(l,\psi)$ is the velocity along radial direction, p is the fluid pressure, is the coefficient of kinematic viscosity and ρ be the fluid density, B_0 be the electromagnetic induction.

Eq. (1) can be written of the form

$$f(\psi)\equiv lg(l,\psi) \tag{4}$$

By using the dimensionless parameters

$$\tau(\eta)=\frac{f(\psi)}{f_{\max}} \quad ,Where \quad \eta=\frac{\psi}{\alpha} \tag{5}$$

in Eq. (2) and (3) and ellimation of p, it obtains a third order ordinary differential equation for the normalized function profile $e(\eta)$ as:

$$\tau^{'''}(\eta)+2\alpha Re\tau(\eta)\tau^{'}(\eta)+(4-Ha)\alpha^2\tau^{'}(\eta)=0 \tag{6}$$

with suitable boundary conditions

$$\tau(0)=1,\ \ \tau'(0)=0,\ \ \tau(1)=0 \tag{7}$$

which comes from these facts that, we have $\frac{\partial g(l,\psi)}{\partial\psi}=0$ at centreline of the channel and$g(l,\psi)=0$ at the plates that makes the body of the channel.

3. DTM for solving ordinary differential equation

Differential Transform of function $\tau(\eta)$ can be defined as follows:

$$\zeta(k)=\frac{1}{k!}[\frac{d^k\tau(\eta)}{d\eta^k}]_{\eta=0} \tag{8}$$

Where $\tau(\eta)$ is original function and $\zeta(k)$ is the transformed function.The uppercase and lowercase letters represent the transformed and original function respectively.The inverse differential transform of $\zeta(k)$ is defined as :

$$\tau(\eta)=\sum_{0}^{\infty}\zeta(k)\eta^k \tag{9}$$

Using equation (8) in (9),it gives ,

$$\tau(\eta)=\sum_{0}^{\infty}[\frac{d^k\tau(\eta)}{d\eta^k}]_{\eta=0}\frac{\eta^k}{k!} \tag{10}$$

4. Application of Differential Transform Method to Jeffery - Hemal flow

By Applying the fundamental operations of differential transformation method to (6), it obtains,

$$(k+1)(k+2)(k+3)\zeta(k+3) \qquad +2\alpha\mathrm{Re}\sum_{m=0}^{k}(m+1)\zeta(m+1)\zeta(k-m)+(4-H)\ \alpha^2(k+1)\zeta(k+1)=0 \tag{11}$$

(11) together with the boundary conditions gives

$$\tau(\eta)=1+B\eta^2+(-\frac{1}{6}\alpha RB+\frac{1}{12}(4-H)\alpha^2B)\eta^4+(-\frac{1}{60}\alpha R(2B^2-\frac{2}{3}\alpha RB+ \\ \frac{1}{3}(H-4)\alpha^2B)+\frac{1}{30}(H-4)\alpha^2(-\frac{1}{6}\alpha RB+\frac{1}{12}(4-H)\alpha^2B)\eta^6+\dots \tag{12}$$

Where,

$$B=\frac{1}{24}\frac{1}{\alpha R}(H^2\alpha^2-4HR\alpha^3-8H\alpha^4+4R^2\alpha^2+16R\alpha^3+16\alpha^4+30H\alpha^2-60R\alpha-120\alpha^2+360 \\ -H^4\alpha^8-8H^3R\alpha^7-16H^3\alpha^8+24H^2R^2\alpha^6+96H^2R\alpha^7+96H^2\alpha^8 \\ +60H^3\alpha^6-32HR^3\alpha^5-192HR^2\alpha^6-384HR\alpha^7-256H\alpha^8-360H^2R\alpha^5 \\ -720H^2\alpha^6+16HR^3\alpha^5+128R^3\alpha^5+384R^2\alpha^6+512R\alpha^7+256\alpha^8-360H^2R\alpha^5 \\ +2880HR\alpha^5+2880HR\alpha^6+1620H^2\alpha^4-480R^3\alpha^3-2880R^2\alpha^4-5760R\alpha^5 \\ -3840\alpha^6-6480HR\alpha^3-12960H\alpha^4+6480R^2\alpha^2+25920R\alpha^3+25920\alpha^4 \\ +21600HR^2-25920R\alpha-86400\alpha^2+129600$$

Eq. (12) discusses the velocity profile of jaffery Hamel flow at different angle for different Reynolds and Hartmann number.

5. Convergence of solution

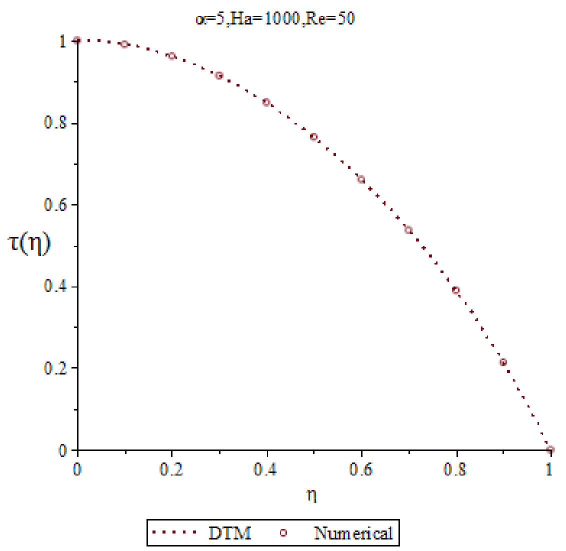

Fig. 2. Comparison between DTM and Numerical solution in divergent Channels

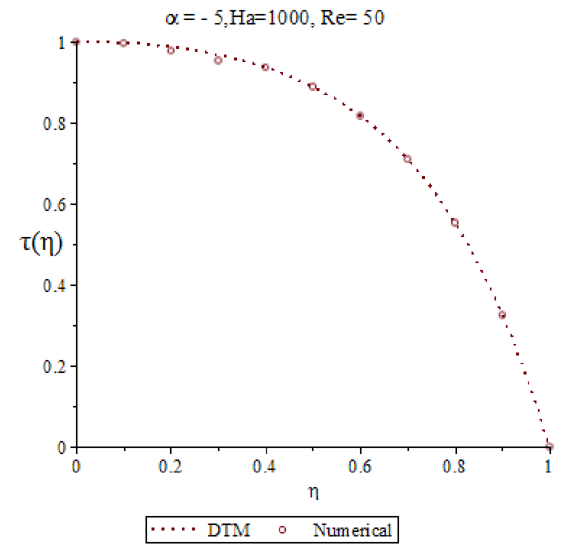

Fig. 3. Comparison between DTM and Numerical solution in convergent Channels

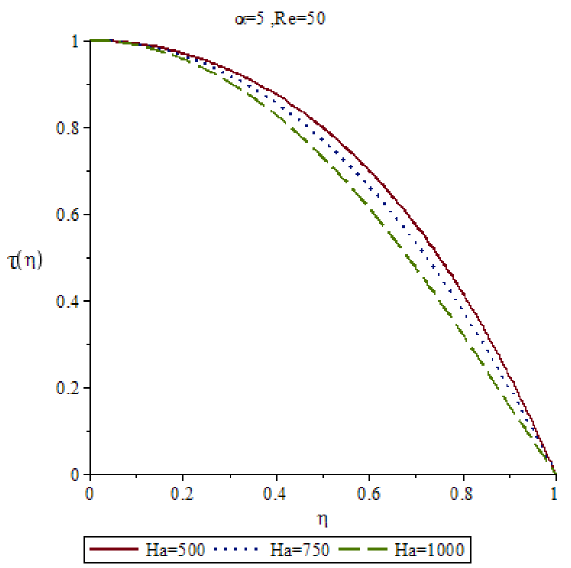

Fig. 4. Effect of Hartmann number on velosity profile in divergent channel

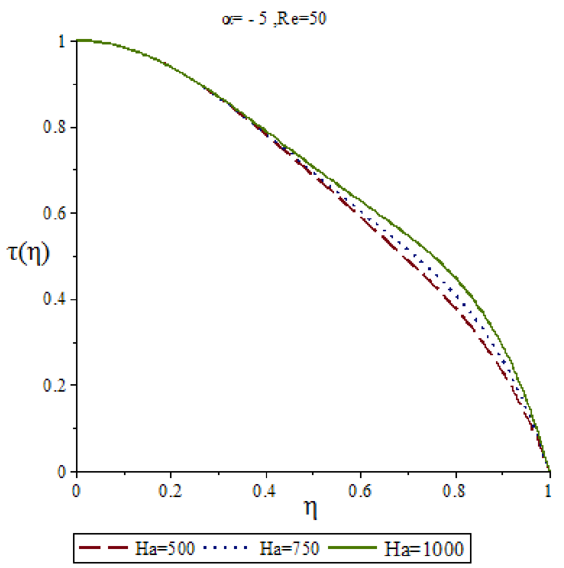

Fig. 5. Effect of Hartmann number on velosity profile in convergent channel

Theorem 5.1.
Let Ω *be an operator from a Hilbert space* H *in to* H *and let* $g(\eta)$ *be an exact solution of Eq.* (6). *Then* $\sum_{i=0}^{\infty}\zeta_i$ *which is obtained by Eq.* (9), *converges to the exact solution, if there exists a* γ, $0 \le \gamma < 1$ *such that* $\|\zeta_{k+1}\| \le \gamma \|\zeta_k\|$, $\forall k \in \mathrm{N} \cup \{0\}$.

Proof. we have, Partial sum as follows,

$S_0 = 0,$
$S_1 = S_0 + \zeta_1 = \zeta_1,$
$S_2 = S_1 + \zeta_2 = \zeta_1 + \zeta_2,$
$\vdots$
$S_n = S_{n-1} + \zeta_n = \zeta_1 + \zeta_2 + \zeta_3 + \ldots + \zeta_n$

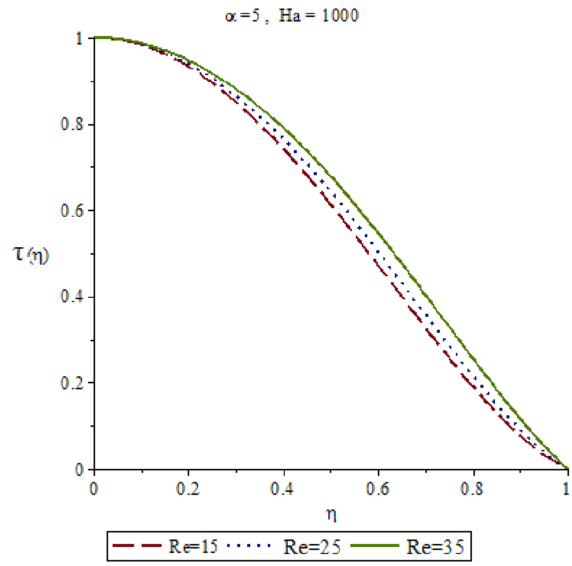

Fig. 6. Effect of Reynolds number on velosity profile in divergent channel

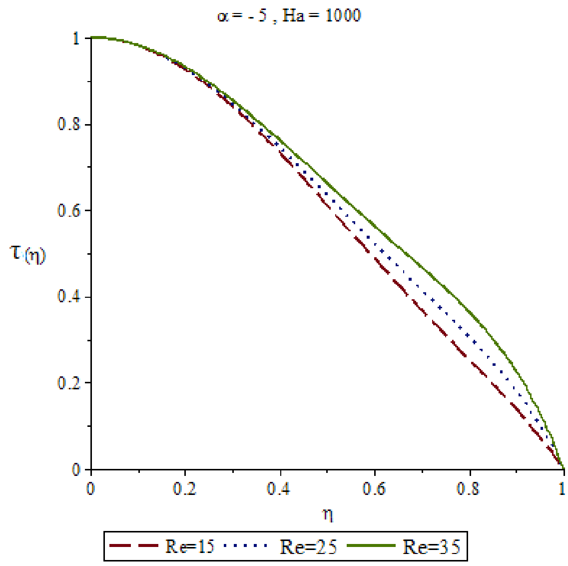

Fig. 7. Effect of Reynolds number on velosity profile in convergent channel

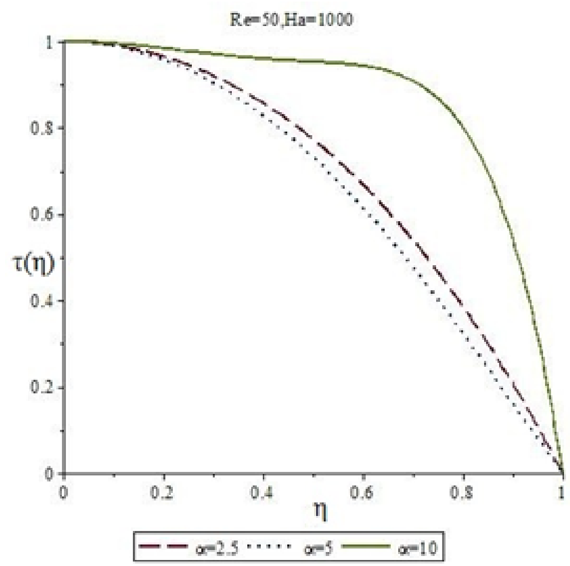

Fig. 8. Effect of different angle on velosity profile with Ha=1000 and Re=50 in divergent channel

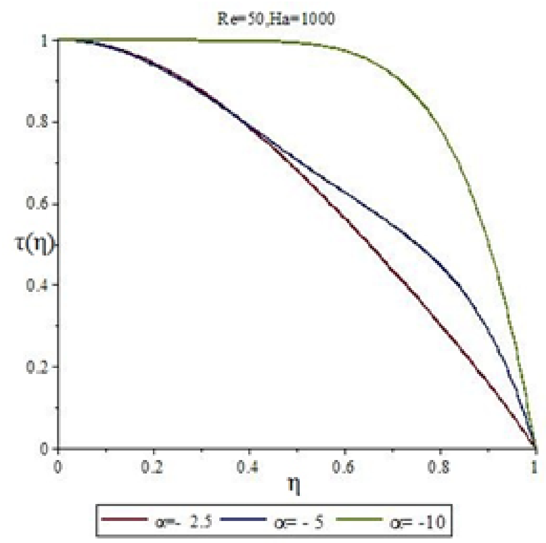

Fig. 9. Effect of different angle on velosity profile with Ha=1000 and Re=50 in covergent channel

and we will show that $\{S_n\}_{n=0}^{\infty}$ is a Cauchy sequence in a Hilbert Space H.
Now for
$\|S_{n+1}-S_n\| = \|\zeta_{n+1}\| \leq \gamma\|\zeta_n\| \leq \gamma^2\|\zeta_{n-1}\| \leq \ldots \leq \gamma^{n+1}\|\zeta_0\|$ for every $n,m \in N$, $n \geq m$ we have

$$\begin{aligned}\|S_n-S_m\| &= \|(S_n-S_{n-1})+(S_{n-1}-S_{n-2})+\cdots+(S_{m-2}-S_{m+1})+(S_{m+1}-S_m)\|\\ &\leq \|S_n-S_{n-1}\|+\|S_{n-1}-S_{n-2}\|+\cdots+\|S_{m-2}-S_{m+1}\|+\|S_{m+1}-S_m\|\\ &\leq \gamma^n\|\zeta_0\|+\gamma^{n-1}\|\zeta_0\|+\gamma^{n-2}\|\zeta_0\|+\cdots+\gamma^{m+2}\|\zeta_0\|+\gamma^{m+1}\|\zeta_0\|\\ &\leq \left(\gamma^{m+1}+\gamma^{m+2}+\cdots\right)\|\zeta_0\|\\ &= \frac{\gamma^{m+1}}{1-\gamma}\|\zeta_0\|\end{aligned}$$

Which implies $\lim_{n,m\to\infty}\|S_n-S_m\| = 0$ i.e., $\{S_n\}_{n=0}^{\infty}$ is a Cauchy sequence in a Hilbert space H and it convergence to S for $S \in H$ □

Definition 5.1.
For every $i \in N \cup \{0\}$, γ_i can be defined as

$$\gamma_i = \begin{cases} \frac{\|\zeta_{i+1}\|}{\|\zeta_i\|}, & \|\zeta_i\| \neq 0 \\ 0 \ , & \|\zeta_i\| = 0 \end{cases}$$

Corollary 5.1.
If $0 \leq \beta_i < 1$, $i = 1, 2, 3, \ldots$ *, then* $\sum_{i=0}^{\infty} \zeta_i$ *is converges to the exact solution* τ.

Now by corollary 5.1, since $\gamma_0 = \frac{\|\zeta_1\|}{\|\zeta_0\|} = 0 < 1$, $\gamma_1 = \frac{\|\zeta_2\|}{\|\zeta_1\|} = 0 < 1$, $\gamma_2 = \frac{\|\zeta_3\|}{\|\zeta_2\|} = 0 < 1$ Simillarly , $\gamma_n = 0$ for all n. Therefore $\sum_{i=0}^{\infty} \zeta_i$ is convergent.

6. Results and discussion

Here a few cases of MHD Jeffery-Hemal flow problem has been discussed and compared the obtained approximate solution with available Numerical results. The comparision of DTM solution with Numerical results[12] of Jeffery-Hemal flow for divergent and convergent chennals with Re=50 and Ha=1000 , is shown in Table 1 and Table 2. Flowing with the influence of, It is clarly one can see that there is good agreement between available Numerical results with Differential Transform Method for Re=50 and Ha=1000 from Fig. 2 and Fig. 3 for different values of α . The error was obtain in Table 1 and Table 2.

Fig. 4 and Fig. 5 discusses the variation of $\tau(\eta)$ for $\alpha > 0$ and $\alpha < 0$ with different Ha keeping Re=50 fixed. It appears from the graph that for $\alpha > 0$ and by decreasing the values of Ha, the velocity of flow is increasing on the other hand for $\alpha < 0$ and by decreasing the values of Ha the velocity of flow is decreasing.

A special case by taking Re=80, $\alpha = -5°$ and Ha=0 of DTM solution was compared with the available MADM, Reproducing Kernel Hilbert Space method (RKHSM) and Numerical results in Table 3 which conclude that there is great accuracy of DTM Compare to other methods.

Table 1. Comparison between DTM and Numerical solution [12] in divergent channel when Re = 50 and Ha = 1000 when $\alpha = 5^0$.

X	DTM	Numerical	Error
0	1	1	0
0.05	0.997601711	0.997605127	3.4157E-06
0.1	0.990413995	0.990427215	1.32208E-05
0.15	0.978457533	0.978485626	2.8093E-05
0.2	0.961764218	0.961810075	4.58573E-05
0.25	0.940373294	0.940436863	6.35695E-05
0.3	0.914325944	0.914403655	7.77109E-05
0.35	0.883658343	0.883742868	8.45248E-05
0.4	0.848393154	0.848473718	8.05634E-05
0.45	0.808529489	0.808592963	6.34748E-05
0.5	0.764031317	0.764064244	3.29264E-05
0.55	0.714814337	0.714805922	8.4151E-06
0.6	0.660731291	0.660677255	5.4036E-05
0.65	0.601555748	0.601462451	9.32964E-05
0.7	0.53696433	0.536852085	0.000112245
0.75	0.466517403	0.466421053	9.63502E-05
0.8	0.389638213	0.389601888	3.63246E-05
0.85	0.305590486	0.305651808	6.13218E-05
0.9	0.213454478	0.213611205	0.000156727
0.95	0.112101479	0.112250383	0.000148904
1	0	0	0

Table 2. Comparison between DTM and Numerical solution[12] in divergent channel when Re = 50 and Ha = 1000 when $\alpha = -5^0$.

η	DTM	Numerical	Error
0	1	1	0
0.05	0.999200258	0.999197382	2.8763E-06
0.1	0.99676823	0.9967567	1.15294E-05
0.15	0.992604115	0.99257822	2.58952E-05
0.2	0.986536946	0.986491495	4.54509E-05
0.25	0.97831763	0.978248943	6.86875E-05
0.3	0.96760922	0.967516581	9.26396E-05
0.35	0.953974393	0.953861798	0.000112595
0.4	0.936860153	0.936737927	0.000122227
0.45	0.915579754	0.915465238	0.000114515
0.5	0.889291835	0.889208343	8.34925E-05
0.55	0.856976784	0.856949974	2.68098E-05
0.6	0.817410306	0.817461268	5.09623E-05
0.65	0.769134223	0.769269319	0.000135096
0.7	0.710424487	0.710623399	0.000198912
0.75	0.639256411	0.63946237	0.000205959
0.8	0.553267116	0.553387455	0.000120339
0.85	0.449715208	0.449646802	6.84057E-05
0.9	0.325437655	0.325141349	0.000296305
0.95	0.176803902	0.176465418	0.000338484
1	0	0	0

Table 3. Comparison of DTM with MADM[15] , RKHSM[17] for e(n) when Re = 80 , $\alpha = -5°$ and Ha=0. Error 1 : | **DTM - Numerical** |, Error2 : | **MADM - Numerical** | and Error3 : | **RKSM - Numerical** |.

η	DTM	Numerical	MADM	RKHSM	Error 1	Error 2	Error 3
0	1	1	1	1	0.00E+00	0.00E+00	0.00E+00
0.1	0.996121388	0.9967567	0.99532538	0.99595999	6.35E-04	1.43E-03	7.97E-04
0.2	0.983941748	0.986491495	0.98065602	0.983275	2.55E-03	5.84E-03	3.22E-03
0.3	0.96176774	0.967516581	0.95405542	0.96017	5.75E-03	1.35E-02	7.35E-03
0.4	0.926570689	0.936737927	0.91229608	0.923519	1.02E-02	2.44E-02	1.32E-02
0.5	0.873676921	0.889208343	0.85085949	0.86845826	1.55E-02	3.83E-02	2.08E-02
0.6	0.79633424	0.817461268	0.76393614	0.78809	2.11E-02	5.35E-02	2.94E-02
0.7	0.685154535	0.710623399	0.64442552	0.67314	2.55E-02	6.62E-02	3.75E-02
0.8	0.527432523	0.553387455	0.48393614	0.51198735	2.60E-02	6.95E-02	4.14E-02
0.9	0.306340629	0.325141349	0.27278547	0.291558267	1.88E-02	5.24E-02	3.36E-02
1	0	0	0	0	0	0	0

Fig. 6 and Fig. 7 discusses the variation of $\tau(\eta)$ for $\alpha > 0$ and $\alpha < 0$ and with different Re keeping Ha = 50 fixed. It appears from the graph that for both convergent & divergent channel , if value of Reynolds number is decreasing ,the velocity of flow decreasing.

From Fig. 8 and Fig. 9, one can observed that for divergent chanel when α is increasing the velocity $\tau(\eta)$ increases,on other hand for convergent chanel when α is increasing the velocity $\tau(\eta)$ decreases.

7. Conclusion

Here, Differential Transform Method has been applied successfully to find an semi-analytical solution of MHD Jeffery -Hemal flow and studied its convergence to show the efficiency of the method. It shows from Table 1, Table 2 and Table 3, that the obtained result using DTM is in excellent agreement with RKHSM and MHPM.The comparison and convergence of the method confirms the validity of the approach.

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