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A comparison study between two approaches for solution of Urysohn integral equation by using statistical method

Research Article

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Abstract: In this paper a comparison study between two approaches for solution of Urysohn integral equation by using statistical method to test the significance of the results. Numerical examples are considered to verify the effectiveness of the proposed derivations and numerical solutions are compared with the existing methods available in the literature. Several illustrative examples with numerical simulations are provided to support the theoretical claims.

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1. Introduction

Many problems of mathematical physics can be started in the form of integral equations. These equations also occur as reformulations of other mathematical problems such as partial differential equations and ordinary differential equations. Numerical simulation in engineering science and in applied mathematics has become a powerful tool to model the physical phenomena, particularly when analytical solutions are not available then very difficult to obtain. Therefore, the study of integral equations and methods for solving them are very useful in application. In recent years, there has been a growing interest in the Volterra integral equations arising in various fields of physics and engineering [1], e.g. potential theory and Dirichlet problems, electrostatics, the particle transport problems of astrophysics and reactor theory, contact problems, diffusion problems, and heat transfer problems.

In [2] J. A. Zarnan produce a novel approach for solution Urysohn integral equation by using Chebyshev polynomials and in [3] a novel approach for solution Urysohn integral equation by using Hermite polynomials is introduced by the same researcher.

However, in this paper we study:

1. The significance for each approach
2. A comparison between these two approaches, which of them is the better for the solution.
3. We use the result from illustrated examples in [2, 3] to find the significance.

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2. Urysohn integral equation

The Urysohn integral equation of the second kind, which considered in [2, 3] is given by:

$$x(t) = f(t) + \int_0^1 K(t, s, x(s))\, ds \tag{1}$$

1. The function $x(t)$ may be expanded by a finite series of Chebyshev polynomialas follows:

$$x(t) = \sum_{a=0}^{\infty} c_n T_n(t) \tag{2}$$

where $c_n = (x(t), T_n(x))$. We conceder a truncated series Eq.(2) as:

$$x_N(t) = \sum_{a=0}^{\infty} c_n T_n(t) = C^T T(t) \tag{3}$$

where C and T are two vectors given by:

$$C = (c_0, c_1, c_2, \ldots, c_N), \quad T(t) = (T_0(t), T_1(t), \ldots T_N(t))^T \tag{4}$$

Then by substituting $x_N(t)$ into Eq. (1), we get

$$C^T T(t) = f(t) + \int_0^1 K(t, s, C^T T(t)) ds \tag{5}$$

Now we use the Chebyshev collocation method which is a matrix method based on the Chebyshev collocation points depended by

$$t_j = -1 + \frac{2}{n}, \quad j = 0, 1, 2, \ldots . N \tag{6}$$

We collocate Eq. (5) with the points (6) to obtain

$$C^T T(t_j) = f(t_j) + \int_0^1 K(t_j, s, C^T T(t)) ds \tag{7}$$

The integral terms in Eq. (7) can be found using composite Trapezoidal integration technique as:

$$\int_0^1 K\left(t_j, s, C^T T(t)\right) ds \approx \frac{h}{2}\left(g(s_0) + g(s_m) + 2\sum_{k=1}^{m-1} g(s_k)\right) \tag{8}$$

where $g(s) = K\left(t_j, s, C^T T(s)\right)$ and $h = \frac{1}{m}$ for an arbitrary $s_i = ih, i = 0, 1, \ldots m$. Therefore Eq. (6) together with Eq. (7) gives an $(N+1)\, x\, (N+1)$ of linear algebraic equations, which can be solved for $c_k, k = 0, 1, \ldots N$. Hence the unknown function $x_N(t)$ can be found.

2. To determine an approximate solution of (1), $x(t)$ is approximated in the Hermite polynomial basis on $[0, 1]$ as:

$$x(t) = \sum_{i=0}^{n} a_i H_{i,n}(t) \tag{9}$$

Where $a_i, i = 0, 1,$ âĂę$, n$ are unknown constants to be determined using Newton-Raphson method. Substituting (9) in (1), we obtain:

$$\sum_{i=0}^{n} a_i H_{i,n}(t) = f(t) + \int_0^1 K\left(t, s, \sum_{i=0}^{n} a_i H_{i,n}(t)\right) ds \tag{10}$$

Now we put $t = t_j, j = 0, 1,$ âĂę$, n$ in (10), $t_j's$ being chosen as suitable distinct points in $[0, 1]$, such that $t_0 = 0, t_n = 1$, and $t_j = t_0 + jh$, where $h = (1-0)/n$. Putting $t = t_j$, we obtain the nonlinear system:

$$\sum_{i=0}^{n} a_i H_{i,n}(t_j) = f(t_j) + \int_0^1 K\left(t_j, s, \sum_{i=0}^{n} a_i H_{i,n}(t_j)\right) ds \tag{11}$$

The nonlinear system (11) can be solved by standard methods for the unknown constant $a_I's$. These $a_i, i = 0, 1, \ldots n$ are then used in (3) to obtain the unknown function *x(t)* approximately.

3. Correlation coefficient

The correlation coefficient will appear on the screen that shows the regression equation information. The quantity r, called the linear correlation coefficient, measures the strength and the direction of a linear relationship between two variables. The linear correlation coefficient is sometimes referred to as the Pearson product moment correlation coefficient in honor of its developer Karl Pearson.
The mathematical formula for computing r is:

$$r = \frac{n\sum^{x} y - \sum^{x}\sum^{y}}{\sqrt{\left(n\sum^{x^2} - (\sum^{x})^2\right)\left(n\sum^{y^2} - \left(\sum^{y^2}\right)\right)}} \tag{12}$$

where n is the number of pairs of data and $-1 \leqslant r \leqslant +1$. The $+$ and $-$ signs are used for positive linear correlations and negative linear correlations, respectively.

4. Statistical calculating

In this section, we calculate the values of correlation coefficient r for the examples in [2] and [3] and the results are given in Table 1 and Table 2.

Table 1. The values of r for examples in [2]

Example	r
1	0.932329784
2	0.998437788

Table 2. The values of r for examples in [3]

Example	r
1	0.977030488
2	0.992300944
3	0.994009984

5. Conclusion

1. From Table 1, we show that the pair values of exact and approximate solutions (x and y) have a strong positive linear correlation, r is closed to $+1$.

2. From Table 2, we show that the pair values of exact and approximate solutions (x and y) have a strong positive linear correlation, r is closed to $+1$.

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