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Analysis of thermal boundary layer flow of viscous fluids by new similarity method

Research Article

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Abstract: A unified analysis is derived for the treatment of the laminar unsteady boundary layer equations with heat conductive mass transfer to establish conditions under which similarity solutions are possible. The method of new similarity technique is applied to derive the various conditions under which similarity variables for the unsteady thermal boundary layer flows is exist. Controlled similarity equations reduce to some well known flow equations. Numerical solution is discussed in detail.

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Keywords: Thermal boundary layer • Similarity analysis • Viscous fluids • Heat conductive mass transfer

1. Introduction

When a heated fluid is in motion it carries heat along in the form of internal energy, a mode of energy transport called convection. Generally, the convection can be subdivided into two parts: Natural (or free) Convection and Forced Convection. For free or natural convection, the flow velocity depends on both the fluid mechanical properties of the system and the heat transfer processes that occur. Free convection is driven by buoyant forces that result from density differences in the convecting fluid. In most situations, the density gradients are caused by temperature variations in the fluid. Examples of free convection include a single-phase closed loop thermos phone, a Trombe wall (a passive solar heating device), and the shimmering visible above a paved highway on a hot summer day. The term forced convection refers to heat transport that results from fluid motion caused by external means such as a pump, a fan, or atmospheric winds. The flow velocity depends on the fluid mechanical properties of the system and not (ideally) on the heat transfer processes that occur in the system. Typical examples of forced convection include forced-air heating of homes, liquid filled active solar heating systems, and nuclear reactor cooling (during normal operation). In other words, there is also another kind of heat transfer between the fluid and its surroundings by conduction; the mechanism of heat flow in which energy is transported from a body of higher temperature to a body of lower temperature by the drift of electrons. Heat transfer in fluids is, therefore, usually caused by interplay between convection and conduction. In some situations a fluid is caused to move by external forces such as a blower, a pump or gravity. The energy transport arise in such a situation is known as forced convection. For example, in a liquid-to-liquid exchanger, a pump is used to force the fluid to flow over the tube bundles. In a gas-cooled nuclear reactor, the hot gas from the

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reactor core is circulated over the tubes of the steam generator by special blowers. In the cooling of air automobile radiator, air is forced to flow over the hot radiator tubes as a result of the motion of the car and the fan.

In the case where no external influences are present, temperature differences can still create motion in the fluid. In general, free convection is caused by motion created by anybody force within the system in which the heat transfer takes place. Energy transfer by free convection occurs in many engineering and industrial applications. Heat transfer from a hot radiator to heat a room, refrigeration coils, and electric heating elements are a few examples. The partial differential equations governing the motion of fluid flow problems are usually non linear in nature and hence cannot be solved easily. Whenever possible these differential equations are reduced to ordinary differential equations with the help of some short of transformations known as similarity transformations.

Numerous studies have been made of the thermal boundary layer equations for non-isothermal surfaces. Chambre [4] has examined the structure of the thermal boundary layer with distributed heat sources. Harmett and Eckert [7] have studied the heat transfer, skin friction, and required coolant flows for transpiration cooling. Sparrow and Gregg [12] have studied the problems of constant-fluid properties and variable-fluid properties [13] in free convection. In another paper Sparrow and Gregg [11] investigated laminar film condensation on a vertical plate using the techniques of boundary layer theory. Sparrow [14] found a series solution for the thermal boundary layer on a non-isothermal surface subjected to non-uniform free stream velocity. Siegel [2] Sieg has employed the method of characteristics to obtain solutions to the time dependent free convection equations of momentum and energy placed in integral form. Eichhorn [3] has considered the effect of mass transfer on free convection. Yang [1] has obtained similarity solutions for free convection on vertical plates and cylinders using the free parameter method, Koh, Sparrow and Harmett [8] have considered the two-phase flow problem in laminar film condensation which arises when induced motions of the vapor are included. Kaviany and Mittal [5] have made an experimental and analytical study of the heat transfer rate from an isothermal plate placed next to a high permeability porous media. In [16] , a new similarity analysis method was proposed on extensive investigation for heat transfer of laminar free convection, and collected in [17].

In this paper, we present a unified analysis of the laminar boundary layer equations with heat convection and mass transfer to establish conditions under which similarity solutions are possible. We employ new similarity technique and discuss unsteady flow problem. A new similarity analysis method with a new set of dimensionless similarity variables are derived systematically. The present new similarity analysis method has following advantages:

- More convenient for consideration and treatment of the variable physical properties,
- More convenient for analysis and investigation of the two-dimensional velocity field,
- More convenient for satisfaction of the interfacial mass transfer matching conditions in the numerical calculation and for rigorous investigation of mass transfer for two-phase film flows with three- point boundary problem.

First, a system of dimensionless similarity variables, such as Reynolds number, dimensionless coordinate variable and dimensionless velocity components, is derived and determined through the analysis with the typical basis conservation equations. In derivation of the dimensionless similarity variables, it is never necessary to induce the stream function ÏĹ,intermediate variable $f(\eta)$, and its derivatives. In this way, we attempt to determine the similarity solution for the problem of thermal boundary layer flow of viscous fluid using the present new similarity analysis technique.

The paper is ordered as follows. In Section 2, we introduce some basic definitions. In Section 3, the fractional complex transform and the analysis of new iterative method is briefly presented. In Section 4, the approximate solutions, plots and absolute errors of the time fractional coupled KdV and mKdV systems are presented. Section 5 is the conclusions.

2. Formulation of the problem

The governing equations for the unsteady boundary layer are [Refer: Schlichting [9]:

$$\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}=0 \tag{1}$$

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+u\frac{\partial u}{\partial y}=\frac{\partial U}{\partial t}+U\frac{\partial U}{\partial x}+g\beta(T-T_\infty)+v\frac{\partial}{\partial y}\left\{\left(\frac{\partial u}{\partial y}\right)^{n-1}\frac{\partial u}{\partial y}\right\} \tag{2}$$

$$\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial x}+u\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2} \tag{3}$$

Subject to the boundary conditions:

$$y=0\Rightarrow u=0,\ v=V_w(x,t),\ T=T_\infty+W(x,t) \tag{4}$$

$$y \to \infty \Rightarrow u = U(x,t),\ T \to T_\infty \tag{5}$$

Equations (1) to (3) forms system of non-linear partial differential equations with boundary conditions (4) to (5) and hence its exact solution is indeed difficult. It is now desirable to transform the said system of partial differential equations into system of ordinary differential equations. To do this we follow the new similarity analysis based on the principal of quantitative grade analysis. (Surawala et al.[6]). According to this method we need to select physical independent variables.

3. Selection of physical independent variables dominating the physical phenomenon

Observing governing equations (1) to (2), it is seen that the whole physical independent variables dominating the physical phenomenon of laminar boundary layer can be obtained and expressed as the following form:

$$F_0(\mu,\rho,\beta,T,U,x,t) = 0 \tag{6}$$

The above seven physical variable i.e. $(n = 7)$ are whole independent physical variables.

3.1. Select basis dimension system

For investigation of the given problem the following four physical dimensions are taken as basic dimensions: time[s], length[m], mass[kg], temperature[k]. While the dimensions of the above independent physical variables μ,ρ,β,T,U,x and t can be described by the basic dimensions, i.e. $\left[\frac{kg}{ms}\right]$, $\left[\frac{kg}{m^3}\right]$,$[k^{-1}]$,$[k]$, $\left[\frac{m}{s}\right]$,$[m]$ and $[s]$ respectively. The above basic four dimensions are $[s]$, $[m]$, $[kg]$ and $[k]$ i.e. $(r = 4)$. Here the number of the related dimensionless similarity physical parameters should be $n - r = 7 - 4 = 3$. According to the Buckingham's $\pi-$theorem, (Buckingham [10]) the dimensional analysis thus yield the result

$$F_0 = f(\pi_1,\pi_2,\pi_3) \tag{7}$$

Where π_1,π_2 and π_3 and are the dimensionless similarity parameters and F_0 is the suitable dimensionless physical phenomenon (variable).

3.2. Determination of the dimensionless similarity parameters π_1,π_2 and π_3

Before determination of the dimensionless similarity physical parameters of the physical phenomenon, we should select the physical variables. Each physical parameter is constituted by $r + 1 = 4 + 1 = 5$ physical variables. Then the dimensionless similarity parameters π_1,π_2 and π_3 can be expressed as the following equations, respectively:

$$\pi_1 : \mu^{a_1} \times U^{b_1} \times x^{c_1} \times T^{d_1} \times \beta = 0 \tag{8}$$
$$\pi_2 : \mu^{a_2} \times U^{b_2} \times x^{c_2} \times T^{d_2} \times \rho = 0 \tag{9}$$
$$\pi_3 : \mu^{a_3} \times U^{b_3} \times x^{c_3} \times T^{d_3} \times t = 0 \tag{10}$$

By using dimensional analysis, the following dimensional equation is obtained for the dimensionless similarity parameter π_1:

$$\left[\frac{kg}{ms}\right]^{a_1} \cdot \left[\frac{m}{s}\right]^{b_1} \cdot [m]^{c_1} \cdot [k]^{d_1} \cdot [k^{-1}] = 0$$

Obviously, the indexes a_1 to d_1 be suitable to the following equations:

For dimension $[kg]$ balance: $a_1 = 0$
For dimension $[m]$ balance: $-a_1 + b_1 + c_1 = 0$
For dimension $[s]$ balance: $-a_1 - b_1 = 0$
For dimension $[A]$ balance: $d_1 - 1 = 0$
The solutions are $a_1 = 0,\ b_1 = 0,\ c_1 = 0,\ d_1 = 1$

Then the dimensionless similarity parameter π_1 is

$$\pi_1 = T\beta \tag{11}$$

Similarly for the dimensionless similarity parameter π_2:

$$\left[\frac{kg}{ms}\right]^{a_2}\cdot\left[\frac{m}{s}\right]^{b_2}\cdot[m]^{c_2}\cdot[k]^{d_2}\cdot\left[\frac{kg}{m^3}\right]=0$$

Then the solutions are $a_2=-1,\ b_2=1,\ c_2=1,\ d_2=0$
Hence the dimensionless similarity parameter π_2 is

$$\pi_2=\frac{\rho Ux}{\mu}\ \text{ or } \pi_2=\frac{Ux}{\upsilon} \tag{12}$$

At last in similar way we can find from the relation π_3 from the relation

$$\left[\frac{kg}{ms}\right]^{a_3}\cdot\left[\frac{m}{s}\right]^{b_3}\cdot[m]^{c_3}\cdot[k]^{d_3}\cdot[s]=0$$

which is written as:

$$\pi_3=\frac{Ut}{x} \tag{13}$$

4. Investigation of the dimensionless similarity variable on the velocity field

For further investigation of dimensionless similarity variables on the velocity field of unsteady boundary layer laminar forced convection boundary layer MHD-UCM fluids, the method applied is based on the basic momentum conversation equation.

4.1. Derivation of dimensionless coordinate variable

For investigation of the dimensionless similarity variables on the velocity field, first it is necessary to determine the related dimensionless coordinate variable. In the boundary layer for unsteady convection, the velocity component u can be regarded to have a quantitative grade equivalent to that of the mainstream velocity U and time t. that is

$$u\propto\frac{U}{t} \tag{14}$$

Meanwhile, with quantity grade analysis, (1) can be approximately rewritten as the following form:

$$\frac{u}{x}+\frac{v}{y}\propto 0 \tag{15}$$

Using quantity grade analysis, (2) can be approximately expressed as the following quantity grade:

$$\frac{1}{t}\left(\frac{U}{t}\right)+\frac{1}{x}\cdot\frac{U}{t}\left(\frac{U}{t}\right)+\frac{Uy}{tx}\cdot\frac{U}{ty}\propto\frac{U}{t}+\frac{U^2}{x}+\nu\left(\frac{U}{t}\right)^n\frac{1}{y^{n+1}}$$

$$y\propto\left[x\nu\frac{U^{n-2}}{t^{n-2}}\right]^{\frac{1}{n+1}} \tag{16}$$

According to the quantity grade relationship in (16), we can set the dimensionless variable η as follows:

$$\eta=\frac{y}{x}\left[\frac{t^{n-2}x^n}{\nu U^{n-2}}\right]^{\frac{1}{n+1}} \tag{17}$$

4.2. Derivation for dimensionless velocity components

After determination of the dimensionless coordinate variable η , we focus on the appropriate forms of the dimensionless velocity components. So, first, set $F(\eta)$ as the dimensionless velocity component in x coordinate and suppose the velocity component u is directly proportional to $F(\eta)$, mainstream velocity U and the p^{th} power of $\left[\frac{t^{n-2}x^n}{\nu U^{n-2}}\right]$. In addition, set $G(\eta)$ as the dimensionless velocity component in y coordinate and suppose the velocity component v is directly proportional to $G(\eta)$, mainstream velocity U and the q^{th} power of $\left[\frac{t^{n-2}x^n}{\nu U^{n-2}}\right]$. Then we have the following supposed equations about the two-dimensional velocity components u and v, respectively:

$$u=U\left[\frac{t^{n-2}x^n}{\nu U^{n-2}}\right]^pF(\eta) \tag{18}$$

$$v = U\left[\frac{t^{n-2}x^n}{\nu U^{n-2}}\right]^q G(\eta) \tag{19}$$

Combining equations (12) and (17), we have

$$\frac{\partial\eta}{\partial x} = -\frac{1}{n+1}x^{-1}\eta \tag{20}$$

$$\frac{\partial\eta}{\partial y} = \frac{1}{x}\left[\frac{t^{n-2}x^n}{\nu U^{n-2}}\right]^{\frac{1}{n+1}} \tag{21}$$

$$\frac{\partial\eta}{\partial t} = \frac{n-2}{n+1}t^{-1}\eta \tag{22}$$

From (18) to (22) immediately we can find $\frac{\partial u}{\partial t}, \frac{\partial u}{\partial x}, \frac{\partial u}{\partial y}, \frac{\partial^2 u}{\partial y^2}\cdots$etc. and substituting all these in equation (2) and simplifying we get,

$$\frac{U}{t}(n-2)\left[\frac{t^{n-2}x^n}{\nu U^{n-2}}\right]^p\left[pF(\eta)+\frac{\eta}{n+1}\frac{dF(\eta)}{d\eta}\right]+\frac{U^2}{x}x^{-1}\left[\frac{t^{n-2}x^n}{\nu U^{n-2}}\right]^{2p}F(\eta)\left[npF(\eta)-\frac{\eta}{n+1}\frac{dF(\eta)}{d\eta}\right]$$
$$+\frac{U^2}{x}\left[\frac{t^{n-2}x^n}{\nu U^{n-2}}\right]^{q+p+\frac{1}{n+1}}G(\eta)\frac{dF(\eta)}{d\eta}=\frac{\partial U}{\partial t}+U\frac{\partial U}{\partial x}+g\beta(T-T_\infty)$$
$$+\nu n\frac{U^n}{x^{n+1}}\left[\frac{t^{n-2}x^n}{\nu U^{n-2}}\right]^{np+1}\left(\frac{dF(\eta)}{d\eta}\right)^{n-1}\frac{d^2F(\eta)}{d\eta^2}$$

The above equation is divided by $\left[\frac{t^{n-2}x^n}{\nu U^{n-2}}\right]^{2p}\frac{U^2}{xt}$ we get,

$$\frac{x}{U}(n-2)\left[\frac{t^{n-2}x^n}{\nu U^{n-2}}\right]^{-p}\left[pF(\eta)+\frac{\eta}{n+1}\frac{dF(\eta)}{d\eta}\right]+tF(\eta)\left[npF(\eta)-\frac{\eta}{n+1}\frac{dF(\eta)}{d\eta}\right]$$
$$+t\left[\frac{t^{n-2}x^n}{\nu U^{n-2}}\right]^{q-p+\frac{1}{n+1}}G(\eta)\frac{dF(\eta)}{d\eta}=\frac{xt}{U^2}\left[\frac{t^{n-2}x^n}{\nu U^{n-2}}\right]^{-2p}\frac{\partial U}{\partial t}+\frac{xt}{U}\left[\frac{t^{n-2}x^n}{\nu U^{n-2}}\right]^{-2p}\frac{\partial U}{\partial x}+\frac{xt}{U^2}\left[\frac{t^{n-2}x^n}{\nu U^{n-2}}\right]^{-2p}$$
$$\times g\beta(T-T_\infty)+\nu n\frac{U^{n-2}t}{x^n}\left[\frac{t^{n-2}x^n}{\nu U^{n-2}}\right]^{(n-2)p+1}\left(\frac{dF(\eta)}{d\eta}\right)^{n-1}\frac{d^2F(\eta)}{d\eta^2} \tag{23}$$

If (23) become complete similarity dimensionless equations, obviously we should have

$$-p+q+\frac{1}{n+1}=0 \tag{24}$$
$$-p=0 \tag{25}$$

Then, $p = 0$ and $q = \frac{1}{n+1}$ also the supposed equations (18) and (19) for the two-dimensional velocity components of boundary layer are expressed as, respectively,

$$u = UF(\eta) \tag{26}$$

$$v = U\left[\frac{t^{n-2}x^n}{\nu U^{n-2}}\right]^{\frac{-1}{n+1}}G(\eta) \tag{27}$$

5. Application example of new similarity analysis method

Here we apply the new similarity analysis method that is based on quantitative grade analysis and we transform continuity equation (1), momentum equation (2) and energy equation (3) into ordinary differential equations with constant boundary conditions (4) and (5) respectively.

5.1. Similarity transformation of (1)

From (26) and (27) we can find $\frac{\partial u}{\partial x}$ and $\frac{\partial u}{\partial y}$ substituting it in equation (1), we get,

$$-\eta \frac{dF(\eta)}{d\eta} + (n+1)\frac{dG(\eta)}{d\eta} = 0 \tag{28}$$

5.2. Similarity transformation of (2)

Again from (26) and (27) we can find $\frac{\partial u}{\partial t}, \frac{\partial u}{\partial x}, \frac{\partial u}{\partial y}, \frac{\partial^2 u}{\partial y^2} \cdots$etc. and substituting all these in equation (2) after simplifying we get,

$$\frac{n-2}{n+1}\frac{U\eta}{t}\frac{dF(\eta)}{d\eta} - \frac{1}{n+1}\frac{\eta U^2}{x}F(\eta)\frac{dF(\eta)}{d\eta} + \frac{U^2}{x}G(\eta)\frac{dF(\eta)}{d\eta} = \frac{\partial U}{\partial t} + U\frac{\partial U}{\partial x} + g\beta(T - T_\infty)$$
$$+\nu n\left(\frac{U}{x}\right)^n\left[\left(\frac{1}{x}\frac{t^{n-2}x^n}{\nu U^{n-2}}\right)^{n+1}\right]^{\frac{1}{n+1}}\left(\frac{dF(\eta)}{d\eta}\right)^{n-1}\frac{d^2F(\eta)}{d\eta^2}$$

The above equation is divided by $\frac{1}{n+1}\frac{U^2}{xt}$, using equation (17) and further simplifying it,

$$\left[(n-2)\frac{x}{U}\eta - t\eta F(\eta) + (n+1)tG(\eta)\right]\frac{dF(\eta)}{d\eta} = (n+1)\frac{xt}{U^2}\frac{\partial U}{\partial t} + (n+1)\frac{xt}{U}\frac{\partial U}{\partial x}$$
$$+(n+1)\frac{xt}{U^2}g\beta(T - T_\infty) + \nu n(n+1)\left(\frac{U}{x}\right)^n\frac{\eta^{n+1}}{y}\frac{xt}{U^2}\left(\frac{dF(\eta)}{d\eta}\right)^{n-1}\frac{d^2F(\eta)}{d\eta^2}$$

But consider, $U = \frac{x}{t} \Rightarrow \frac{\partial U}{\partial t} = \frac{-x}{t^2}, \frac{\partial U}{\partial x} = \frac{1}{t}$

$$\left[(n-2)\eta - \eta F(\eta) + (n+1)G(\eta)\right]\frac{dF(\eta)}{d\eta} = (n+1)g\beta + n(n+1)\nu\eta^{n+1}\left(\frac{dF(\eta)}{d\eta}\right)^{n-1}\frac{d^2F(\eta)}{d\eta^2} \tag{29}$$

Where, $T - T_\infty = W_x(x,t) = \frac{x}{t^2}$, $\nu = V_w(x,t) = \frac{1}{xt^n}$

5.3. Similarity transformation of (3)

we take the well-known form of the dimensionless temperature variable $\theta(\eta)$ as

$$\theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty} \tag{30}$$

From (20) to (22) and (30) immediately we can find $\frac{\partial u}{\partial t}, \frac{\partial u}{\partial x}, \frac{\partial u}{\partial y}, \frac{\partial^2 u}{\partial y^2} \cdots$etc. and substituting all these in equation (3) and simplifying we get,

$$\frac{n-2}{n+1}\frac{\eta}{t}(T_w - T_\infty)\frac{d\theta(\eta)}{d\eta} - \frac{1}{n+1}\frac{\eta U}{x}F(\eta)(T_w - T_\infty)\frac{d\theta(\eta)}{d\eta} + \frac{U}{x}G(\eta)(T_w - T_\infty)\frac{d\theta(\eta)}{d\eta} =$$
$$= \frac{\alpha}{x^2}(T_w - T_\infty)\left[\frac{t^{n-2}x^n}{\nu U^{n-2}}\right]^{\frac{2}{n+1}}\frac{d^2\theta(\eta)}{d\eta^2}$$

The above equation is divided by $\frac{1}{n+1}\frac{U}{xt}(T_w - T_\infty)$ using equation (17) and further simplifying it,

$$\left[(n-2)\frac{x}{U}\eta - t\eta F(\eta) + (n+1)tG(\eta)\right]\frac{d\theta(\eta)}{d\eta} = (n+1)\alpha\left(\frac{\eta}{y}\right)^2\frac{xt}{U}\frac{d^2\theta(\eta)}{d\eta^2}$$

Now, again considering $U = \frac{x}{t}$ and substituting the value of y from equation (15) we get,

$$\left[(n-2)\eta - \eta F(\eta) + (n+1)G(\eta)\right]\frac{d\theta(\eta)}{d\eta} = (n+1)\alpha\eta^2\frac{d^2\theta(\eta)}{d\eta^2} \tag{31}$$

Where, $\nu = V_w(x,t) = \left(\frac{1}{xt^3}\right)^{\frac{1}{2}}$

Up to now, by using the new similarity analysis method, the governing partial differential equations (1) to (3) have been transformed to the equations (28), (29) and (31) which are governing dimensionless ordinary differential

equations:
The boundary conditions of equations (4) and (5) are transformed to

$$\eta = 0 : F(\eta) = 0,\ G(\eta) = 1,\ \theta(\eta) = 1 \tag{32}$$

$$\eta \to \infty : F(\eta) = 1,\ \theta(\eta) = 0 \tag{33}$$

It is observe that (12), (17), (26) , (27) and (30) for the set of dimensionless similarity variables such as dimensionless coordinate variable, dimensionless velocity components are the complete similarity analysis and transformation of governing partial differential equations on thermal boundary layer flow of viscous fluids which become whole system of the similarity analysis variables of the present new similarity analysis method.

For Newtonian case i.e. for equations (28), (29) and (31) are well reduced to those derived by [15].

$$-\eta\frac{dF(\eta)}{d\eta} + 2\frac{dG(\eta)}{d\eta} = 0 \tag{34}$$

$$[-\eta - \eta F(\eta) + 2G(\eta)]\frac{dF(\eta)}{d\eta} = 2g\beta + 2\nu\eta^2\frac{d^2F(\eta)}{d\eta^2} \tag{35}$$

$$[-\eta - \eta F(\eta) + 2G(\eta)]\frac{d\theta(\eta)}{d\eta} = 2\alpha\eta^2\frac{d^2\theta(\eta)}{d\eta^2} \tag{36}$$

Using equation (34) in equations (35) and (36) and simplifying we get,

$$F_1''' + \left(F_1 + \frac{\eta}{2}\right)F_1'' + \nu F_1' = 0 \tag{37}$$

$$F_2'' + \left(F_1 + \frac{\eta}{2}\right)F_2' - (F_1' - 2)F_2 = 0 \tag{38}$$

With boundary conditions are transformed as

$$\eta = 0 : F_1(0) = 0,\ F_1'(0) = 0,\ F_2(0) = 1 \tag{39}$$

$$\eta \to \infty :\ F_1'(\infty) = 1,\ F_2(\infty) = 0 \tag{40}$$

6. Numerical solution

we have utilized the 4th order Runge-Kutta method for finding the numerical solution of the coupled similarity equations (37) and (38) along with the boundary conditions (39) and (40). We have taken help of MATLAB to find numerical solution.
The numerical results of velocity profile F_1' and temperature distribution F_2 are shown graphically in Figures 1 and 2. It is observed that as η increase, velocity increase. It is to be noted that velocity is maximum for $P_r = 0.2$. Similarly, as η increase there is a fast falls in temperature. Here, the temperature is quite steadily decreasing for $P_r = 0.2$ and for other case it increased for fast with decrease the value in η.

7. Conclusions

The new similarity analysis technique is successfully employed to transform boundary layer equation governing the flow of viscous fluids. Similarity transformations and similarity equations are derived systematically. The derived dimensionless velocity components $f(\eta)$ and $G(\eta)$ based on the present new similarity analysis method are directly proportional to the related velocity components and .The obtained momentum similarity equation, which is second order non linear ordinary differential equation and energy equation which is also second order non linear ordinary differential equation with related boundary conditions. The 4th order Runge-Kutta method is employed to carry numerical solution. Graphical representation is shown using help of MATLAB. All possible conditions under which the similarity solution for present flow situation exists are automatically derived from the similarity requirement and thus the similarity solution is found in most general form.

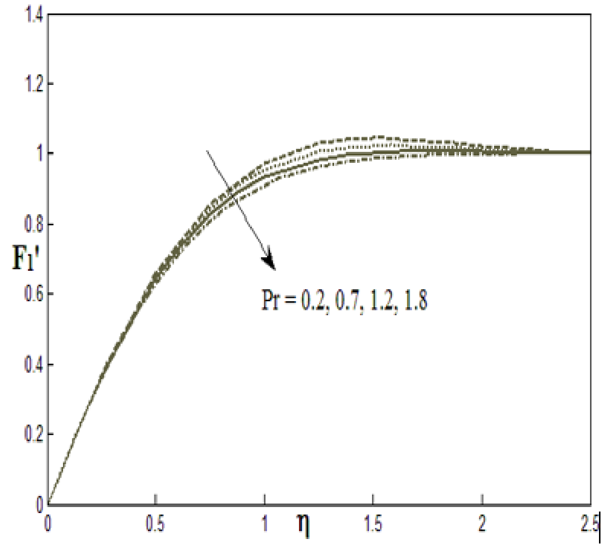

Fig. 1. Velocity profile for different values of Prandtl number

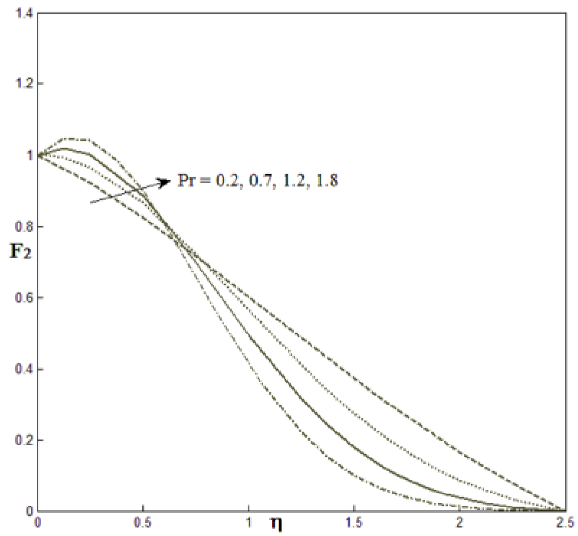

Fig. 2. Temperature profile for various Prandtl number

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