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The binomial transforms of the generalized (s,t)-Jacobsthal matrix sequence

Research Article

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Abstract: In this paper, we study the binomial transforms of the generalized (s,t)Jacobsthal matrix sequence $\Re_{n+1}(s,t)_{n\in N}$; $(s,t)-$Jacobsthal $J_{n+1}(s,t)_{n\in N}$and, $(s,t)-$Jacobsthal Lucas $C_{n+1}(s,t)_{n\in N}$matrix sequences. After thatby using recurrence relations of them, the generating functions have beenfounded for these transforms. Finally the relations among these transforms have been demonstrated with deriving new equalities.

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1. Introduction and preliminary

There are so many studies in the literature that are concern about special integer sequences such as Fibonacci, Lucas, Pell, Jacobsthal, Padovan. You can encounter the generalizations of these sequences in all of the references. In [1] the author wrote a book about these integer sequences You can see the generalized number and matrix sequences for Fibonacci and Lucas sequences in [3, 5, 6]. Similarly the author defined number and matrix sequences which generalizes Jacobsthal and Jacobsthal Lucas sequences in [7, 8]. Some authors introduced matrix based transforms for these special sequences. Binomial transform is one of most popular transforms. You can have detailed information about binomial transform in [9, 10]. Falcon defined different binomial transforms of the$k-$Fibonacci sequence such as falling, rising binomial transforms in [4]. The authors gave binomial transform for generalized $(s,t)-$matrix sequences in [11] .The authors introduced binomial transforms for the Padovan and Perrin numbers in [12] . And in [13] binomial transforms of the $k-$Jacobsthal sequence a reintroduced. In [14] the authors gave some properties of Lucas numbers with binomial coefficients.The goal of this paper is to apply the binomial transforms to the generalized Jacobsthal and Jacobsthal Lucas matrix sequences. Also, the generating function of this transform is found by recurrence relations. Finally the relations among these transforms have been demonstrated with deriving new equalities.

In [2] , the author defined the Jacobsthal and Jacobsthal Lucas sequence as follows respectively

$$\begin{aligned} j_{n+1} &= j_n + 2j_{n-1} \quad n\geq 1,\ (j_0=0,\ \ j_1=1) \\ c_{n+1} &= c_n + 2c_{n-1} \quad n\geq 1,\ (c_0=0,\ \ c_1=1) \end{aligned}$$

Now we give some preliminaries related to our study. For a given integer sequence $X=\{x_1,x_2,...,x_n,...\}$ the binomial transform Y of the sequence X;$Y(X)=\{y_n\}$is defined by

$$y_n = \sum_{i=0}^{n} \binom{n}{i} x_i$$

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Definition 1.1.
Assume that

$$a, b \in R, \quad s > 0, \quad t \neq 0 \quad s^2 + 8t > 0$$

. The$(s,t)-$Jacobsthal sequence $\{\hat{j}_n(s,t)\}_{n\in N}$ is defined by the following recurrence relation:

$$\hat{j}_{n+1}(s,t) = s\hat{j}_n(s,t) + 2t\hat{j}_{n-1}(s,t), \qquad (\hat{j}_0(s,t) = 0, \ \hat{j}_1(s,t) = 1)$$

and, the $(s,t)-$Jacobsthal Lucas $\{\hat{c}_n(s,t)\}_{n\in N}$ is defined by the following recurrence relation

$$\hat{c}_{n+1}(s,t) = s\hat{c}_n(s,t) + 2t\hat{c}_{n-1}(s,t), \qquad (\hat{c}_0(s,t) = 2, \ \hat{c}_1(s,t) = s)$$

And the generalized (s,t)Jacobsthal sequence $\{G_n(s,t)\}_{n\in N}$ is defined by the following recurrence relation

$$G_{n+1}(s,t) = sG_n(s,t) + 2tG_{n-1}(s,t), \qquad (G_0(s,t) = a, \ \hat{c}_1(s,t) = bs)$$

in [7] .

By choosing suitable values on a; b, we will obtain $(s,t)-$ Jacobsthal sequence;the $(s,t)-$ Jacobsthal Lucas sequence by the generalized $(s,t)-$ Jacobsthal sequence:

$$a = b = 1 \Rightarrow \{G_n(s,t) = \hat{j}_{n+1}(s,t)$$
$$a = 2, \quad b = 1 \Rightarrow \{G_n(s,t) = \hat{c}_n(s,t)$$

Definition 1.2.
Assume that

$$a, b \in R, \quad s > 0, \quad t \neq 0 \quad s^2 + 8t > 0$$

. Generalized (s,t)Jacobsthal matrix sequence $\Re_{n+1}(s,t)_{n\in N}$; $(s,t)-$Jacobsthal $J_{n+1}(s,t)_{n\in N}$ and, $(s,t)-$Jacobsthal Lucas $C_{n+1}(s,t)_{n\in N}$matrix sequences are defined by the following recurrence relations in [8] :

$$\Re_{n+1}(s,t) = s\Re_n(s,t) + 2t\Re_{n-1}(s,t) \tag{1}$$
$$J_{n+1}(s,t) = sJ_n(s,t) + 2tJ_{n-1}(s,t) \tag{2}$$
$$C_{n+1}(s,t) = sC_n(s,t) + 2tC_{n-1}(s,t) \tag{3}$$

with initial conditions

$$\Re_0 = \begin{bmatrix} bs & 2a \\ at & (b-a)s \end{bmatrix} \text{ and } \Re_1 = \begin{bmatrix} bs^2 + 2at & 2bs \\ bst & 2at \end{bmatrix}$$
$$J_0(s,t) = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \text{ and } J_1(s,t) = \begin{bmatrix} s & 2 \\ t & 0 \end{bmatrix}$$
$$C_0(s,t) = \begin{bmatrix} s & 4 \\ 2t & -s \end{bmatrix} \text{ and } C_1(s,t) = \begin{bmatrix} s^2 + 4t & 2s \\ st & 4t \end{bmatrix}$$

By choosing suitable values on a; b, we will obtain $(s,t)-$Jacobsthal matrix sequence;the $(s,t)-$Jacobsthal Lucas matrix sequence by the generalized (s,t)Jacobsthal matrix sequence:

$$a = b = 1 \Rightarrow \{\Re_n(s,t) = J_{n+1}(s,t)$$
$$a = 2, \quad b = 1 \Rightarrow \{\Re_n(s,t) = C_n(s,t)$$

In the rest of this paper, for convenience we will use the symbols $\hat{j}_n$, $\hat{c}_n$,G_n, J_n, C_n, $\Re_n$instead of $\hat{j}_n(s,t)$,$\hat{c}_n(s,t)$, $G_n(s,t)$, $J_n(s,t)$, $C_n(s,t)$, $\Re_n(s,t)$ respectively.

Proposition 1.1.
The relations between the number sequences and their matrix sequences are given as

$$J_n = \begin{bmatrix} \hat{j}_{n+1} & 2\hat{j}_n \\ t\hat{j}_n & 2t\hat{j}_{n-1} \end{bmatrix}, C_n = \begin{bmatrix} \hat{c}_{n+1} & 2\hat{c}_n \\ t\hat{c}_n & 2t\hat{c}_{n-1} \end{bmatrix} \textit{ and } \Re_n = \begin{bmatrix} G_{n+1} & 2G_n \\ tG_n & 2tG_{n-1} \end{bmatrix}.$$

2. Binomial transform of the $(s,t)-$Generalized and $(s,t)-$Jacobsthal, $(s,t)-$Jacobsthal Lucas matrix sequences

In this section, the binomial transforms of the generalized $(s,t)-$matrix sequence, $(s,t)-$Jacobsthal and $(s,t)-$Jacobsthal Lucas matrix sequences will be introduced.

Definition 2.1.
Let $\Re_n$, J_n and C_n be the $(s,t)-$generalized,$(s,t)-$Jacobsthal and $(s,t)-$Jacobsthal Lucas matrix sequences, respectively. The binomial transforms of these matrix sequences can be expressed as follows:

i The binomial transform of the generalized $(s,t)-$matrix sequence is

$$B_n = \sum_{i=0}^{n} \binom{n}{i} \Re_i$$

ii The binomial transforms of $(s,t)-$Jacobsthal matrix sequence is

$$\hat{J}_n = \sum_{i=0}^{n} \binom{n}{i} J_i$$

iii The binomial transforms of $(s,t)-$Jacobsthal Lucas matrix sequence is

$$\hat{C}_n = \sum_{i=0}^{n} \binom{n}{i} C_i$$

Lemma 2.1.
For $n \geqslant 0$, the following equalities are hold:

i $B_{n+1} = \sum_{i=0}^{n} \binom{n}{i} (\Re_i + \Re_{i+1})$

ii $\hat{J}_{n+1} = \sum_{i=0}^{n} \binom{n}{i} (J_i + J_{i+1})$

iii $\hat{C}_{n+1} = \sum_{i=0}^{n} \binom{n}{i} (C_i + C_{i+1})$

Proof. Firstly, in here we will just prove (i), since (ii) and (iii) can be proved by using the same method.By using the definition of binomial transform and the well-known binomial equality $\binom{n+1}{i} = \binom{n}{i} + \binom{n}{i-1}$ and $\binom{n}{n+1} = 0$ it is obtained that

$$\begin{aligned} B_{n+1} &= \sum_{i=0}^{n+1} \binom{n+1}{i} \Re_i \\ &= \Re_0 + \sum_{i=1}^{n+1} \left[\binom{n}{i} + \binom{n}{i-1} \right] \Re_i \\ &= \Re_0 + \sum_{i=1}^{n+1} \binom{n}{i} \Re_i + \sum_{i=0}^{n} \binom{n}{i} \Re_{i+1} \\ &= \sum_{i=0}^{n} \binom{n}{i} (\Re_i + \Re_{i+1}) \end{aligned}$$

which is the desired result.
Note that B_{n+1} is also written as $B_{n+1} = B_n + \sum_{i=0}^{n} \binom{n}{i} \Re_{i+1}$ □

Theorem 2.1.
For $n \geqslant 0$, the sequences $\{B_n\}$, $\{\hat{J}_n\}$, $\{\hat{C}_n\}$ are verified the following recurrence relations

a) $B_{n+2} = (s+2)B_{n+1} - (s+1-2t)B_n$
with initial conditions

$$B_0 = \begin{bmatrix} bs & 2a \\ at & (b-a)s \end{bmatrix}, \; B_1 = \begin{bmatrix} bs^2+bs+2at & 2bs+2a \\ bst+at & (b-a)s+2at \end{bmatrix}$$

b) $\hat{J}_{n+2} = (s+2)\hat{J}_{n+1} - (s+1-t)\hat{J}_n$

$$\hat{J}_0 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \text{and} \hat{J}_1 = \begin{bmatrix} s+1 & 2 \\ t & 1 \end{bmatrix}$$

c) $\hat{C}_{n+2} = (s+2)\hat{C}_{n+1} - (s+1-t)\hat{C}_n$

$$\hat{C}_0 = \begin{bmatrix} s & 4 \\ 2t & -s \end{bmatrix} \text{and} \hat{C}_1 = \begin{bmatrix} s^2+4t & 2s \\ st & 4t \end{bmatrix}$$

Proof. We only prove the first case because the other cases can be proved with the same way. From Lemma 2.1, we have

$$\begin{aligned} B_{n+1} &= \sum_{i=0}^{n} \binom{n}{i} (\Re_i + \Re_{i+1}) \\ &= \Re_0 + \Re_1 + \sum_{i=1}^{n} \binom{n}{i} (\Re_i + \Re_{i+1}) \\ &= \Re_0 + \Re_1 + (s+1) \sum_{i=1}^{n} \binom{n}{i} \Re_i + 2t \sum_{i=1}^{n} \binom{n}{i} \Re_{i-1} \pm (s+1)\Re_0 \end{aligned}$$

From Definition 2.1, it is obtained that

$$B_{n+1} = (s+1)B_n + 2t \sum_{i=1}^{n} \binom{n}{i} \Re_{i-1} + \Re_1 - s\Re_0 \tag{4}$$

putting $n-1$ instead of n in (4) we have

$$\begin{aligned} B_n &= (s+1)B_{n-1} + 2t \sum_{i=1}^{n-1} \binom{n-1}{i} \Re_{i-1} + \Re_1 - s\Re_0 \\ &= sB_{n-1} + \sum_{i=0}^{n-1} \binom{n-1}{i} \Re_i + 2t \sum_{i=1}^{n-1} \binom{n-1}{i} \Re_{i-1} + \Re_1 - s\Re_0 \\ &= sB_{n-1} + \sum_{i=1}^{n} \left[\binom{n-1}{i-1} + 2t \binom{n-1}{i} \right] \Re_{i-1} + \Re_1 - s\Re_0 \\ &= sB_{n-1} + \sum_{i=1}^{n} \left[\binom{n-1}{i-1} + 2t \binom{n-1}{i} \pm 2t \binom{n-1}{i-1} \right] \Re_{i-1} + \Re_1 - s\Re_0 \\ &= sB_{n-1} + \sum_{i=1}^{n} \left[(1-2t) \binom{n-1}{i-1} + 2t \binom{n}{i} \right] \Re_{i-1} + \Re_1 - s\Re_0 \\ &= (s+1-2t)B_{n-1} + 2t \sum_{i=1}^{n} \binom{n}{i} \Re_{i-1} + \Re_1 - s\Re_0 \end{aligned}$$

From this equality we have

$$2t \sum_{i=1}^{n} \binom{n}{i} \Re_{i-1} + \Re_1 - s\Re_0 = B_n - (s+1-2t)B_{n-1}$$

By substituting this expression in (4), we obtain

$$B_{n+1} = (s+2)B_n - (s+1-2t)B_{n-1} \tag{5}$$

which completes the proof. □

The characteristic equation of the binomial transforms of the generalized$(s,t)-$matrix sequence B_n is $\lambda^2 - (s+2)\lambda + (s-2t+1) = 0$.The roots of this equation are

$$\lambda_1 = \frac{s+2+\sqrt{s^2+8t}}{2}, \qquad \lambda_2 = \frac{s+2-\sqrt{s^2+8t}}{2}$$

Binet formula are well known in the special integer sequences theory. Binetformula allows us to express the nth term in function of the roots of $\lambda_1$and$\lambda_2$of the characteristic equation, associated the recurrence relation (5). So the Binet formula for B_n can be expressed as

$$B_n = \frac{X{\lambda_1}^n - Y{\lambda_2}^n}{\lambda_1 - \lambda_2}$$

$$X = \begin{bmatrix} bs^2 - \lambda_2 bs + 2at & 2bs - 2a\lambda_2 \\ bst - \lambda_2 at & 2at - \lambda_2(b-a)s \end{bmatrix} \tag{6}$$

$$Y = \begin{bmatrix} bs^2 - \lambda_1 bs + 2at & 2bs - 2a\lambda_1 \\ bst - \lambda_1 at & 2at - \lambda_1(b-a)s \end{bmatrix} \tag{7}$$

By choosing corresponding values on a and b in (6) and (7), we can obtain the Binet formula of $\hat{J}_n$and $\hat{C}_n$. Namely,

a) For a = b = 1; we get the Binet formula for the binomial transforms of $(s,t)-$Jacobsthal matrix sequence as $\hat{J}_{n+1} = \dfrac{A{\lambda_1}^n - B{\lambda_2}^n}{\lambda_1 - \lambda_2}$ where

$$A = \begin{bmatrix} s^2 - \lambda_2 s + 2t & 2(s-\lambda_2) \\ t(s-\lambda_2) & 2t \end{bmatrix},\ B = \begin{bmatrix} bs^2 - \lambda_1 s + 2t & 2(s-\lambda_1) \\ t(s-\lambda_1) & 2t \end{bmatrix}$$

b) For a =2, b = 1; we get the Binet formula for the binomial transforms of $(s,t)-$Jacobsthal Lucas matrix sequence as $\hat{C}_n = \dfrac{C{\lambda_1}^n - D{\lambda_2}^n}{\lambda_1 - \lambda_2}$ where

$$C = \begin{bmatrix} s^2 - \lambda_2 s + 4t & 2(s-2\lambda_2) \\ t(s-2\lambda_2) & 4t - s\lambda_2 \end{bmatrix},\ D = \begin{bmatrix} bs^2 - \lambda_1 s + 4t & 2(s-2\lambda_1) \\ t(s-2\lambda_1) & 4t - s\lambda_1 \end{bmatrix}.$$

Theorem 2.2.
The generating functions of the binomial transforms of generalized $(s,t)-$matrix sequence, $(s,t)-$Jacobsthal matrix sequence, $(s,t)-$Jacobsthal Lucas matrix sequence are
i)

$$B_n(s,t,x) = \sum_{n=0}^{\infty} B_n x^n = \tfrac{\Re_0 + x[\Re_1 - (s+1)\Re_0]}{1-(s+2)x+(s+1-t)x^2}$$
$$= \tfrac{1}{1-(s+2)x+(s+1-t)x^2} \begin{bmatrix} bs + x(2at - bs) & 2(a + x(bs - as - a)) \\ t(a + x(bs - as - a)) & (b-a)s + x(2at - s^2(b-a) - s(b-a)) \end{bmatrix}$$

ii)

$$J_n(s,t,x) = \sum_{n=0}^{\infty} \hat{j}_{n+1} x^n = \tfrac{J_0 + x[J_1 - (s+1)J_0]}{1-(s+2)x+(s+1-t)x^2}$$
$$= \tfrac{1}{1-(s+2)x+(s+1-t)x^2} \begin{bmatrix} s + x(2t-s) & 2(1-x) \\ t(1-x) & 2tx \end{bmatrix}$$

iii)

$$C_n(s,t,x) = \sum_{n=0}^{\infty} \hat{c}_n x^n = \tfrac{C_0 + x[C_1 - (s+1)C_0]}{1-(s+2)x+(s+1-t)x^2}$$
$$= \tfrac{1}{1-(s+2)x+(s+1-t)x^2} \begin{bmatrix} s + x(4t-s) & 2(2 - x(s+2)) \\ t(2 - x(s+2)) & -s + x(4t + s^2 - s) \end{bmatrix}$$

Proof. We just prove the case (i) and the others will be omitted. Let $B_n(s,t,x)$be generating function for the binomial transform of generalized $(s,t)-$Jacobsthal matrix sequence. Then,

$$B_n(s,t,x) = B_0 + xB_1 + ... + x^n B_n + ... \tag{8}$$

If we multiply $-(s+2)x$and $(s+t-1)x^2$; with the both sides of the equality (8) respectively, we obtain

$$-(s+2)\,xB_n(s,t,x) = -(s+2)\,xB_0 + -(s+2)\,x^2 B_1 + ... + -(s+2)\,x^{n+1}B_n + ... \tag{9}$$

$$(s+1-2t)x^2 B_n(s,t,x) = (s+1-2t)x^2 B_0 + (s+1-2t)x^3 B_1 + ... + (s+1-2t)x^{n+2}B_n + ... \tag{10}$$

Considering (8), (9), (10) we get the following equality

$$B_n(s,t,x)\left(1-(s+2)\,x + (s+1-t)\,x^2\right) = B_0 + x(B_1 - (s+2)B_0) \tag{11}$$

Finally, from Theorem 2.1, Definition 2.1, and (11) we have the desired result. □

We can get the following relations between the generalized (s; t)- matrix sequence, $(s,t)-$Jacobsthal and $(s,t)-$Jacobsthal Lucas matrix sequences and the generating functions of the binomial transforms of these sequences, respectively.

i) Let $r(x) = \frac{\Re_0 + x[\Re_1 - s\Re_0]}{1-sx-tx^2}$be the ordinary generating function of the sequence $\{\Re_n\}$: By using the transformation of $\frac{x}{1-x} f\left(\frac{x}{1-x}\right)$ we have the generating function of the binomial transform sequence $\{B_n\}$ in Theorem 2.2-(i).

ii) Let $j(x) = \frac{J_0 + x[J_1 - sJ_0]}{1-sx-tx^2}$be the ordinary generating function of the sequence $\{J_n\}$: By using the transformation of $\frac{x}{1-x} j\left(\frac{x}{1-x}\right)$ we have the generating function of the binomial transform sequence $\{J_n\}$in Theorem 2.2-(ii).

iii) Let $c(x) = \frac{C_0 + x[C_1 - sC_0]}{1 - sx - tx^2}$ be the ordinary generating function of the sequence $\{C_n\}$: By using the transformation of $\frac{x}{1-x} c\left(\frac{x}{1-x}\right)$ we have the generating function of the binomial transform sequence$\{C_n\}$ in Theorem 2.2-(iii).

Theorem 2.3.
Let $m, n \in \mathbb{N}$; then $\hat{J}_{m+n} = \hat{J}_m \hat{J}_n$.

Proof. We use the induction method. Let $n = 0$, then we get $\hat{J}_{m+0} = \hat{J}_m \hat{J}_0 = \hat{J}_m I$.Assume that $\hat{J}_{m+n} = \hat{J}_m \hat{J}_n$ for $n \leqslant N$. Then we obtain

$$\begin{aligned}
\hat{J}_{m+N+1} &= (s+2)\hat{J}_{m+N} - (s+1-t)\hat{J}_{m+N-1} \\
&= (s+2)\hat{J}_m \hat{J}_N - (s+1-t)\hat{J}_m \hat{J}_{N-1} \\
&= \hat{J}_m \left[(s+2)\hat{J}_N - (s+1-t)\hat{J}_{N-1}\right] \\
&= \hat{J}_m \hat{J}_{N+1}
\end{aligned}$$

□

Theorem 2.4.
Let $n \in \mathbb{N}$, then

$$\begin{aligned}
&\Re_{n+1} = \Re_1 J_n \\
&J_{n+1} = J_1 J_n \\
&C_{n+1} = C_1 J_n
\end{aligned}$$

Proof. The proof is easily obtained by using mathematical induction method. □

Theorem 2.5.
The relations among the transforms B_n, $\hat{J}_n$ and $\hat{C}_n$ can be demonstrated by the following equalities:

$$\begin{aligned}
&B_{n+1} - B_n = \Re_1 \hat{J}_n \\
&\hat{J}_{n+1} - \hat{J}_n = J_1 \hat{J}_n \\
&\hat{C}_{n+1} - \hat{C}_n = C_1 \hat{J}_n
\end{aligned}$$

Proof. By considering Definition 2.1, Lemma 2.1, we get

$$B_{n+1} = \sum_{i=0}^{n} \binom{n}{i} (\Re_i + \Re_{i+1}) = B_n + \sum_{i=0}^{n} \binom{n}{i} \Re_{i+1}.$$

By Theorem 2.4,

$$B_{n+1} - B_n = \sum_{i=0}^{n} \binom{n}{i} \Re_{i+1} = \sum_{i=0}^{n} \binom{n}{i} (\Re_1 J_n) = \Re_1 \hat{J}_n$$

This completes the proof of i): The others are made by using the same method. □

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