Int. J. Adv. Appl. Math. and Mech. 6(4) (2019) 41 – 49 (ISSN: 2347-2529)
Journal homepage: www.ijaamm.com

International Journal of Advances in Applied Mathematics and Mechanics

Fluid flow and heat transfer through a vertical cylindrical collapsible tube in presence of magnetic field and an obstacle

Research Article

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Received 12 April 2019; accepted (in revised version) 20 May 2019

Abstract: Fluid flow coupled with heat transfer through a vertical cylindrical Collapsible tube in the presence of a magnetic field and an obstacle has been investigated. The governing equations for this flow are the equations of continuity, motion and energy. These are non linear partial differential equations and has been transformed into non-linear ordinary differential equations by introducing a similarity transformation. The resulting equations are solved simultenously in the bvp4c MATLAB library to obtain the profiles and also the rate of heat transfer has been calculated. The effects of varying the Reynolds number, Hartmann number, Eckert number, Unsteadiness parameter and Prandtl number on fluid temperature , fluid velocity and the rate of heat transfer are presented in the form of tables and graphs and has been discussed. Variation in the various parameters is observed to change the fluid primary velocity, temperature and the rate of heat transfer. This kind of result is important due to its widespread application in physical, biological and applied sciences.

MSC: 80Axx • 92Bxx

Keywords: Collapsible tube • Similarity transformation • Unsteadiness

1. Introduction

Studies that are related to the transient fluid flows that produced by simple wall motion have attracted lot of interest for several years due to its practical importance in understanding engineering and physiological flow problems. The entire tube in human body are flexible and also made of collapsible walls. Collapsing is caused by change in pressure referred to as transmural pressure. Mathematical modeling of this kind of flow provides important information to medical practitioners in order to understand the complexity of collapsible tube flows. Research done by Marzo and Luo[1] established that increase in external pressure up to a certain critical value makes the structure buckle. Makinde [2] investigated flow in Collapsible tubes, he used pertubation technique and Hermite-Pade approximations. It was established that temperature decreases with increase in prandtl number. Luo et al [3] investigated the unsteady behavior and linear stability of the flow in a Collapsible tube by using a fluid beam model. It was established that for high values of Reynolds number interesting cascade of instabilities is discovered because the wall stiffness is reduced. Varshney et al [4] investigated the effects of magnetic field on the blood flow in the artery having multiple stenoses. Radial transformation was used together with finite difference. It was established that the presence of stenoses and

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magnetic fields affected the flow. Marchandise et al [5] investigated an accurate modeling of unsteady flows in collapsible tubes. One-dimensional Runge-Kutta discontinuous Galerkin method coupled with lumped parameter models for the boundary conditions was used. Sankar et al[6] carried out Mathematical modeling of a complex system for MHD flow in Hemodynamics. Finite difference method was used to solve the pdes, it was established that velocity decreases with an increase in the magnetic field and pressure gradient. Prakash et al [7] investigated Radiative heat transfer to blood flow through a stenotic artery in the presence of magnetic field. It was established that magnetic properties of RBCs play vital roles in the increase of blood viscosity during exposure to magnetic field. Malekzadeh et al [8] established that pressure drop changes proportionally to the square of the product of the magnetic field and the sine of which the magnetic field was applied. Aminfar et al [9] established that the necessary drugs are bound with Ferrofluids that helps in concentrating the drug in a certain area by the magnificent application of magnetic fields. Siviglia and Toffolon [10] investigated multiple states for flow through a Collapsible tube with discontinuities. It was established that the complexity of the fluid-structure gives Collapsible tubes their specific dynamic features. The numerical solution was obtained by using finite volume method of the path conservative type. Kozlovsky et al [11] utilized a computational model for the evaluation of the geometry of the deformed cross-sectional area due to negative transmural pressure. Odejide [12] solved the nonlinear equation arising from the model by using the perturbation series. It was established that increasing the Prandtl number leads to an increase or decrease in heat transfer across the wall. Odejide [12] investigated unsteady fluid structure interactions in a collapsible wall micro-channel. The set of equations were coupled non linear partial differential equations which were solved numerically using a segregated approach with fully-implicit time stepping and second finite-difference discretization. The Reynolds number was varied independently to explore the unsteady behavior of the flow. It was established that nonlinear tension does not change the qualitative response but only affects pre-factors in the scaling. Anand and Christov [13] investigated steady low Reynolds number flow of a generalized Newtonian fluid through a slender elastic tube. Non-Newtonian effects of biofluids was captured by using the power-law rheological model. Perturbation approach yields analytical solutions for flow and deformation. Mehdari et al [14] investigated analytical model of an unsteady fluid flow through an elastic tube. The fluid was considered to be Newtonian and Incompressible, they took into consideration large Reynolds number and a small aspect ratio, the tube was assumed to be having a small shell, which they considered to be the source of asymmetric vibration. Ali et al [15]investigated flow of magnetic particles with isothermal heat in the the blood. They used Laplace and Hankel transformation techniques to solve the equations. It was noted that velocity profiles increase with an increase in Grashoff number.
Previous studies on collapsible tubes have not consider the problem of presence of obstacle and magnetic fields affecting the fluid flow. This phenomena should be emphasized for a model to adequately address the complexity of blood flow. This research has addressed the effect of the obstacle to the flow. Numerical and graphical results obtained using the collocation method are validated through comparison with numerical results generated using the inbuilt MATLAB boundary value solver.

2. Mathematical model

The study considers a two-dimensional unsteady flow in a collapsible tube. The fluid considered is Newtonian and incompressible. The flow is taken to be along the z and r direction with velocity u and v respectively. Where z is taken to be along the axial of the main flow while r is taken to be along the radial.

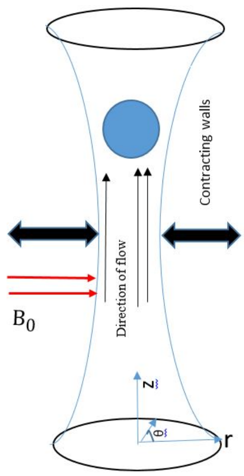

Fig. 1. Geometry of the Collapsible tube

3. Governing equations

Let's take a cylindrical coordinate system (r, θ, z) where θ is the azimuthal angle and r is the radial distance. we take a case where u_z and u_r are velocity components in z and r directions respectively. The tube's wall is at $r = a_0\sqrt{(1-\alpha t)}$, where α is a constant of dimension $[T^{-1}]$ which characterizes unsteadiness in the flow field, a_0 is the characteristic radius of the tube at time t = 0. Flow is taken to be along the z-direction, $|\alpha t|$ should not be greater than one. $R = \frac{a_0^2\alpha}{2\upsilon}$ is the local Reynolds number. The governing equations are the equations of continuity, mass and energy which are given respectively as:

$$\frac{\partial(u_z)}{\partial z} = 0. \tag{1}$$

$$\frac{\partial u_z}{\partial t} = -\frac{1}{\rho}\frac{\partial p}{\partial z} + \frac{\mu}{\rho}\left[\frac{1}{r}\frac{\partial u_z}{\partial r} + \frac{\partial^2 u_z}{\partial r^2}\right] - \frac{\sigma B_0^2 u_z}{\rho} + (1 + \frac{\rho_1}{\rho})g_z \tag{2}$$

$$\frac{\partial T}{\partial t} = \frac{K}{\rho c_p}\left(\frac{1}{r}\frac{\partial T}{\partial r} + \frac{\partial^2 T}{\partial r^2}\right) + \frac{\mu}{\rho c_p}\left[\left(\frac{\partial u_z}{\partial r}\right)^2\right] \tag{3}$$

4. Boundary conditions

The boundary conditions governing flow through a vertical Collapsible tube at the center and wall of the tube are:

$$\frac{\partial u_z}{\partial r} = 0, \;\; T = T_\infty, \;\; at \;\; r = 0; \tag{4}$$

$$u_z = 0, \;\; T = T_w, \;\; r = a(t) \tag{5}$$

Where T_w is the wall temperature and T_∞ is centerline temperature.

5. Similarity transformation

The partial differential equations are transformed into ordinary differential equations . The model of this flow consists of non linear partial differential equations as depicted by equations (2) and (3) , to use bvp4c Collocation method, its a requirement that the equations are transformed into ordinary differential equations. Continuity equation is expected to be satisfied by the introduction of the stream function $\psi(r, z, t)$.

Since the stream function satisfies the equation of continuity. Then, there is need to look for a similarity transformation. The goal of similarity transformation is that it looks for the type of potential flows for which similar solutions exists.

The similarity solutions to the unsteady two dimensional boundary layer equation is much complex due to the fact that the three variables r,z,t need to be reduced to a single variable say η .

The assumption taken into account is that the velocity is purely along z axis and depends on r , z and t.

The following transformations are used to transform the equations to obtain the non-dimensional numbers. The transformation for velocity and temperature are given as:

$$u_z = -\frac{Q}{z}\frac{1}{\delta^{(m+1)}}f(\eta) \tag{6}$$

and,

$$\frac{\omega(\eta)}{\delta^{(m+1)}} = \frac{T - T_w}{T_\infty - T_w} \tag{7}$$

This transformations were in line with the work done by [16], [17], [18] and [19].

Applying the transformations given by equations (6) and (7) on (2) and (3) the following is obtained.

$$f''(\eta) + \frac{1}{\eta}f'(\eta) + \frac{(m+1){a_0}^{1+m}}{\delta^{m+1}}\lambda f(\eta) - Ha^2 f(\eta) + \frac{{a_0}^2 z\delta^{m+1}}{\mu_0 Q}\frac{\partial p}{\partial z} - Re\frac{{a_0}^2 z\delta^{m+1}}{Q^2}\left[\left(1 + \frac{\rho_1}{\rho}\right)g_z\right] = 0 \tag{8}$$

$$\frac{1}{Pr}\left[\omega''(\eta) + \frac{1}{\eta}\omega'(\eta)\right] + \frac{(m+1)a_0^{m+1}}{\delta^{(m+1)}}\lambda\omega(\eta) + \frac{Eca_0^2}{z^2\delta^{(m+1)}}f'^2(\eta) = 0 \tag{9}$$

6. Boundary conditions transformation

The boundary conditions governing flow through a vertical Collapsible tube are as (4) and (5).
By using (6) and (7) on (4) and (5) it becomes,

$$f'(0) = 0,\ \omega(0) = \delta^{m+1}\ \ if\ \ \eta = 0 \tag{10}$$

$$f(a(t)) = 0,\ \omega(a(t)) = 0\ \ if\ \ \eta = \text{a(t)} \tag{11}$$

The non dimensional numbers obtained from the above equations are: $Re = \frac{Q\rho}{\mu_0}$ which is the Reynolds number, $Pr = \frac{c_p\mu_0}{k}$ which is the Prandtl number, $Ha = B_0 a_0\sqrt{\frac{\sigma}{\mu_0}}$ which represent the hartmann number, $Ec = \frac{Q^2/a_0^2}{c_p(T_\infty - T_w)}$ is the Eckert number, $\lambda = \frac{\rho\delta^m}{\mu_0 a_0^{m-1}}\frac{d\delta}{dt}$ is the unsteadiness parameter. Here δ is the time-dependent length scale, m is a constant related to collapsibility, f is dimensionless velocity while ω is dimensionless temperature.

7. Numerical solution

The bvp4c is a MATLAB solver that is based on the Collocation method that provides a continuous solution with a 4^{th} order accuracy. The method utilizes a mesh of points to divide the interval of integration into sub-intervals, where each sub-interval is solved based on the system of algebraic equations and the boundary equations provided. The solver estimates the error of the numerical solution on each subinterval. The solver adapts the mesh to repeat the process once the equation does not satisfy the tolerance criteria. Since BVPs can have more than one solution, BVP codes require users to supply an initial guess for the solution desired towards predicting the right solution. The codes then adapt the mesh so as to obtain an accurate numerical solution with the most number of mesh points. Coming up with a sufficiently good guess is often the hardest part of solving a BVP. The bvp4c takes an unusual approach to the control of error that helps it deal with poor guesses.
Prior to the execution of the bvp4c, the higher order non linear ODEs are reduced to first order non linear ODEs. Reduction of order is achieved by introduction of:

$$y_1 = f, \quad y_2 = f' \quad , y_3 = \omega \quad , y_4 = \omega'$$

, which reduces equations (8) and (9) and the transformed boundaries to be the following systems:

$$\begin{cases} y_1' = y_2 \\ y_2' = Ha^2 y_1 - \frac{1}{\eta} y_2 - \frac{(m+1)\ a_0^{m+1}}{\delta^{(m+1)}}\lambda y_1 - \frac{z a_0^2 \delta^{(m+1)}}{\mu_0 Q} p_z + Re\frac{z\ a_0^2\ \delta^{(m+1)}}{Q^2}[(1+\frac{\rho_1}{\rho})g_z] \\ y_3' = y_4 \\ y_4' = -\frac{1}{\eta} y_4 - Pr\frac{(m+1)\ a_0^{m+1}}{\delta^{(m+1)}}\lambda y_3 - \frac{Pr\ Ec\ a_0^2}{z^2\ \delta^{(m+1)}} y_2^2 \end{cases}$$

The combined boundary conditions for the model are,

$$\begin{cases} y_2 = 0\ ,\ y_3 = \delta^{(m+1)}\ \text{ at the center} \\ y_1 = 0\ ,\ y_3 = 0\ \ \text{ at the wall} \end{cases} \tag{12}$$

8. Results and discussions

It is seen that an increase in Reynolds number results to an increase in velocity profile of the fluid as depicted by Fig. 2. Increase in Re means reduction of viscous forces. It is also observed that increase in Re causes an increase in temperature. Viscosity and temperature are inversely proportional in liquids, a decrease in viscosity means rise in the fluid temperature.

From Fig. 3, it is noted that an increase in Hartmann number leads to a decrease in velocity. This is because applied magnetic field to the flow introduces a force known as lorentz force. Lorentz force is known to act against the direction of flow of the fluid which in turn reduces the magnitude of velocity. It is also noted that the temperature profiles increases with increase in hartmann number. Increase of Hartmann number creates an opposite flow towards the main flow. This opposite flow increases temperature due to thermal conduction.

From Fig. 4, it is observed that an increase in Eckert number has no significant effect on velocity profile. Increase in Eckert number leads to an increase in kinetic energy which only causes vibration of particles and not movement. it

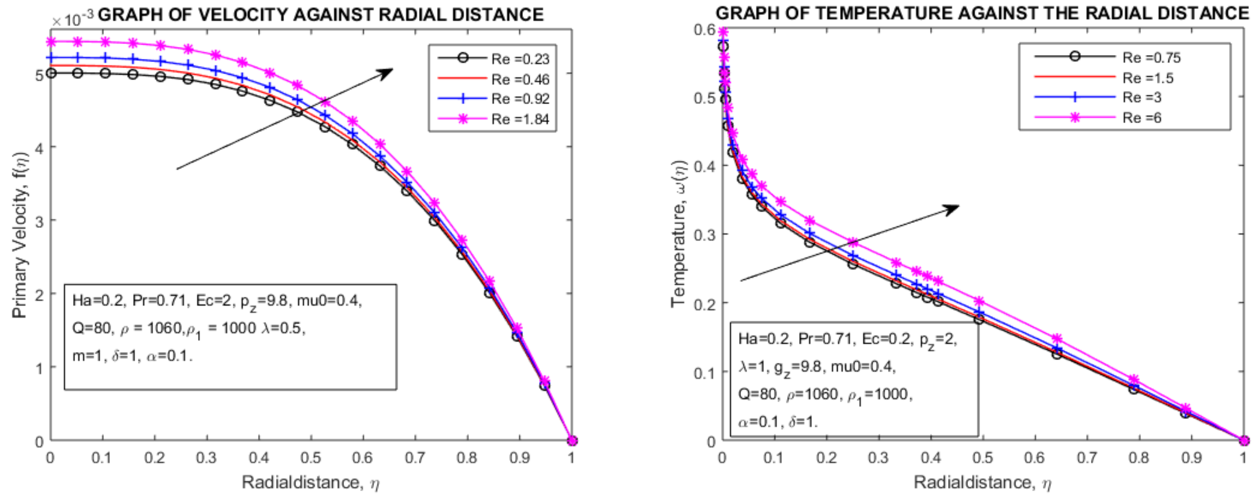

Fig. 2. Graph of dimensionless velocity and Temperature profiles for varying Reynolds number, Re

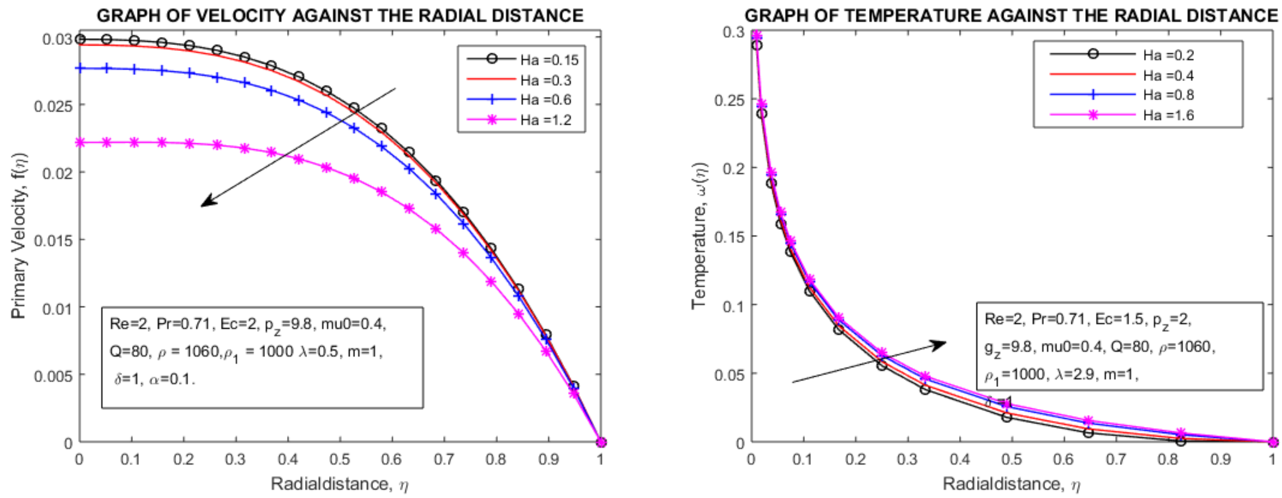

Fig. 3. Graph of dimensionless velocity and Temperature profiles for varying hartman number, Ha

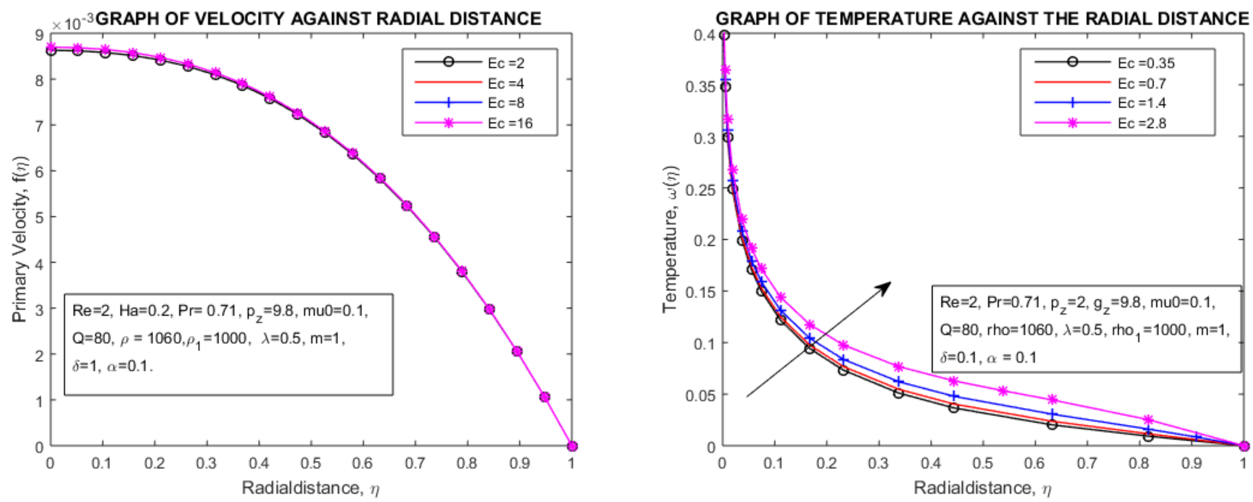

Fig. 4. Graph of dimensionless velocity and Temperature profiles for varying Eckert number, Ec

is also observed that increase in Ec leads to an increase in temperature. Increase in Ec means increase in kinetic enegy

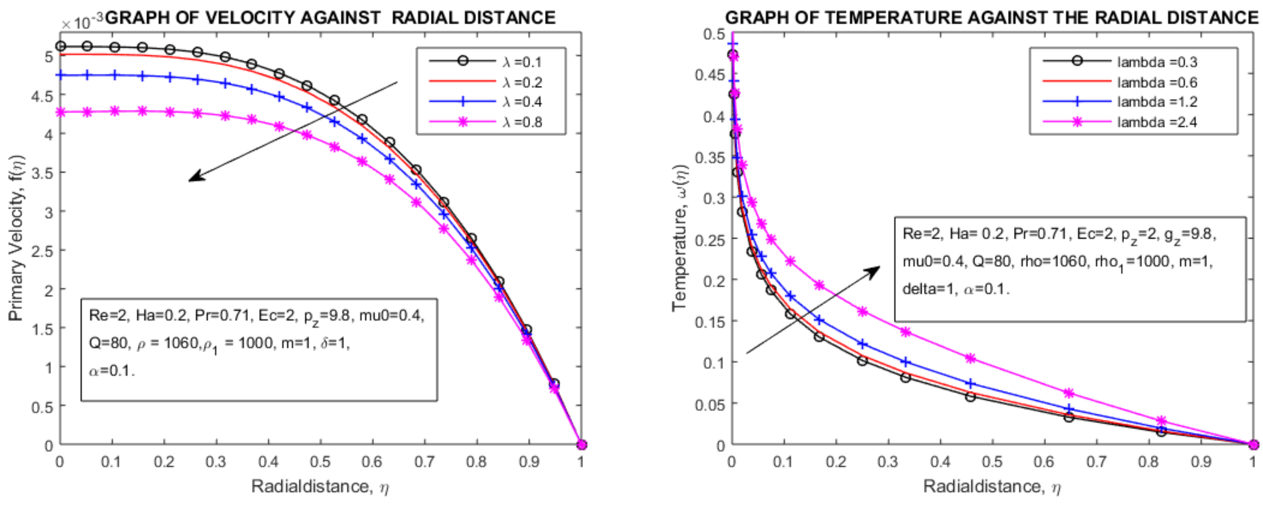

Fig. 5. Graph of dimensionless velocity and Temperature profiles for varying Unsteadiness Parameter, λ

which leads to rapid collision of particles which results in dissipation of heat and hence increase in temperature.

From Fig. 5, As λ increases the velocity of the fluid flow tends to reduce. This is because Unsteadiness parameter λ and time dependent length scale δ are directly proportional. Consequently leading leading to a decrease in the velocity which has an inverse relationship with the time dependent length scale δ, it also observed that a slight increase in the unsteadiness parameter λ causes an increase in the temperature of the fluid. λ is directly proportional to the time dependent length scale δ which on the other hand is directly proportion to the temperature of the fluid. Hence an increase of this parameter results in an increase in temperature profile.

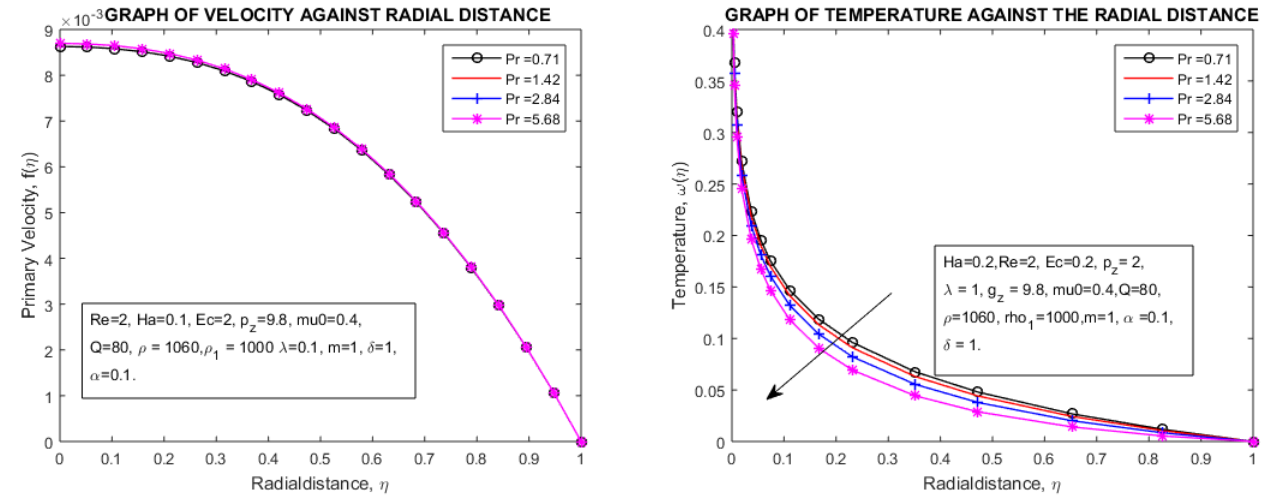

Fig. 6. Graph of dimensionless velocity and Temperature profiles for varying Prandtl number, Pr

From Fig. 6 it is seen that there is no significant change in velocity profiles when there is an increase in prandtl number. it is also observed that an increase in the Prandtl number causes a decrease in temperature. Increase in Pr implies that thermal diffusivity rate is less dominant and viscous diffusion rate is dominant. It is known that reduction in thermal diffusion leads to reduction in the temperature of the fluid that is flowing.

From Fig. 7 it is observed that an increase in density of the obstacle results to a decrease in velocity. This is because increase in density implies an increase in weight of the obstacle which in turn exerts a greater opposing force to the fluid and hence decrease in primary velocity. it is also observed that there is no significant change on temperature with increase in the density of the obstacle. This is because increase in density does not affect temperature.

The Local Nusselt number is proportional to change in temperature $\omega'(0)$ is computed and its numerical values tabulated.

From Table 1 An increase in the Reynolds number results to an increase in the rate of heat transfer. Increase in Prandtl number results to a decrease in Nusselt number Nu. Increase in Prandtl number leads to an increase in the thickness of the thermal boundary which results in reduction in heat transfer rate. Increase in hartmann number

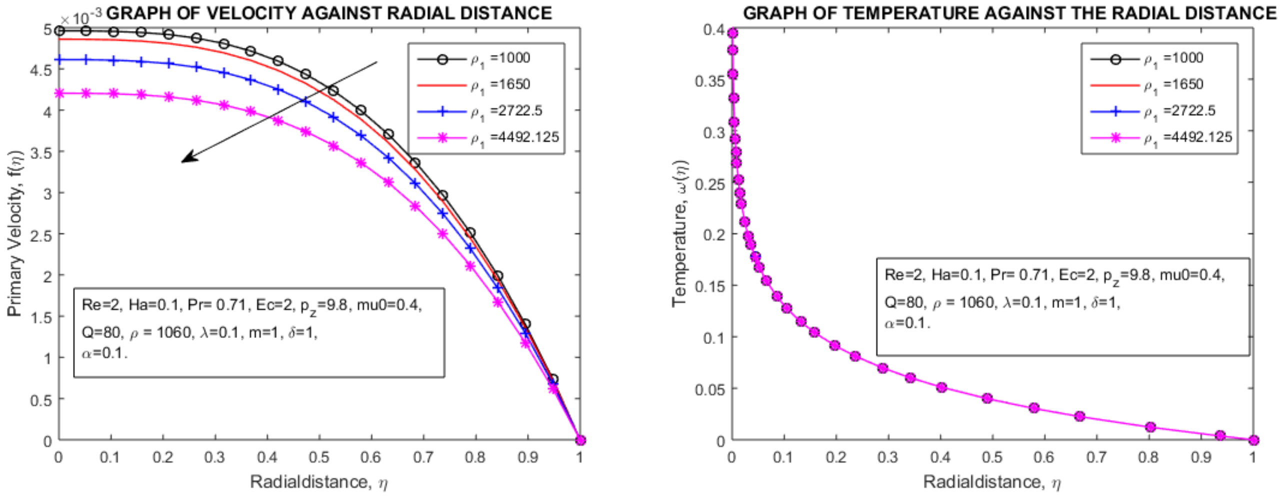

Fig. 7. Graph of dimensionless velocity and Temperature profiles for varying density of the obstacle, ρ_1

Table 1. Heat transfer values for various values of the parameter of Re, Ha, Pr, Ec and λ

Re	Ha	Pr	Ec	λ	Nu
25	0.2	1	0.2	0.35	0.3130
25	0.2	1.42	0.2	0.35	0.2821
25	0.2	2.84	0.2	0.35	0.2313
25	0.2	5.68	0.2	0.35	0.1588
4	0.2	0.71	0.2	0.35	0.1236
8	0.2	0.71	0.2	0.35	0.1753
16	0.2	0.71	0.2	0.35	0.2492
32	0.2	0.71	0.2	0.35	0.3535
25	0.2	0.71	0.2	0.35	0.3130
25	0.2	0.71	0.4	0.35	0.3111
25	0.2	0.71	0.8	0.35	0.3074
25	0.2	0.71	1.6	0.35	0.3000
25	0.2	0.71	0.2	0.35	0.3130
25	0.4	0.71	0.2	0.35	0.3130
25	0.8	0.71	0.2	0.35	0.3133
25	1.6	0.71	0.2	0.35	0.3139
25	0.2	0.71	0.2	0.35	0.3130
25	0.2	0.71	0.2	0.70	0.2835
25	0.2	0.71	0.2	1.40	0.2343
25	0.2	0.71	0.2	2.80	0.1158

Ha results in an increase in the rate of heat transfer. Thermal boundary layer thickness decreases with increase in Ha resulting in the observed increase in heat transfer. Increase in Unsteadiness parameter λ causes a decrease in Nusselt number. This implies that Nusselt number decreases with time dependent length scale. This is expected since the velocity and temperature decreases gradually with time, gradually equalizing with their respective free stream values.

9. Validation

Comparing the present study with the study done by Odejide[12] it is seen that there exist a lot of similarities. It is observed that the velocity is parabolic and increases with increase in Reynolds number, temperature of the fluid decreases as the prandtl number increases and also increase in Re results to rise in the fluid temperature.

10. Conclusions

The analysis of various parameters on unsteady fluid flow and heat transfer through a vertical cylindrical collapsible tube in the presence of magnetic field and an obstacle has been carried out. The fluid that was considered was Newtonian in nature. Magnetic field was applied in a transverse direction to the flow while the obstacle was placed in the flow such that the fluid flow was not obstructed completely.The effect of the obstacle on the flow has been investigated by varying the values of the density of the obstacle while holding the density of the fluid at a constant value. The partial differential equations governing the flow are non-linear and coupled and have solved by employing bvp4c Collocation method. In the Collocation method the Collocation points are reduced and it is observed that it does not bring significant changes to the results. Thus the scheme used in the computations is stable.

The results obtained in Chapter four shows that the fluid velocity increases with increase in Reynolds number. However an increase in Hartmann number, density of the obstacle and Unsteadiness parameter causes the velocity to decrease. Imposing a transverse magnetic field to a flow slows down the velocity of the fluid. Increasing density of the obstacle causes an increased forced retarding the motion of the fluid.

The fluid temperature increases with an increase in Reynolds number, Hartmann number, Unsteadiness parameter and Eckert number. However an increase in Prandtl number causes the temperature to decrease. An increase in Prandtl number does not impact on fluid temperature since temperature and density has no relation.

Heat transfer rate increases with rise in Reynolds number and also an increase in the Hartmann number. The heat transfer rate reduces with an increase in Prandtl number, Eckert number and Unsteadiness parameter. An increase in density of the obstacle does not produce significant change on the rate of heat transfer.
The results obtained in this study regarding velocity, temperature and heat transfer can be applied in the medical field for instance in the treatment of cancer. The study of flow through Collapsible tubes has vital applications in Bioengineering and medical fields. The study is important in gaining an understanding of the complexity of the flow.

Acknowledgements

The author(s) would like to appreciate the Pan African University for Basic Science and Technology (PAUSTI) for funding of this project. Appreciation goes to Jomo Kenyatta University of Science and Technology for technical assistance throughout the research.

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