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Impacts of hall and heat transfer on peristaltic blood flow of a MHD Jeffrey fluid in a vertical asymmetric porous channel

Research Article

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Abstract: In this paper, we investigate the effects of hall and heat transfer on peristaltic blood flow of a MHD (Magneto hydrodynamic) Jeffrey fluid in a porous asymmetric channel. The flow is analysed by long wavelength and low Reynolds number approximations. The reduced equations are solved by using the Adomian Decomposition Method and the expressions for velocity, stream function and pressure gradient are obtained. The effect of pertinent parameters are illustrated graphically.

MSC: 76A05 • 76Z05

Keywords: Peristaltic transport • MHD Jeffrey fluid • Hall effect

1. Introduction

The Hall effect principle was discovered by an American physicist named Edwin Hall in 1879. The topic of Hall effect in magneto hydrodynamics is a recent trend because of its important influence of the electromagnetic force. The Hall effect parameter is defined as the ratio between the electron-cyclotron frequency and the electron-atom-collision frequency. The principle of Hall effect is used to determine the efficiency of some devices such as power generators and heat exchangers. Hayat et al. [1]studied the hall effects on peristaltic flow of a Maxwell fluid in a porous medium. Eladahab et al. [2] analysed the effects of hall and ion-slip currents on peristaltic transport of a couple stress fluid. The effects due to the hall current become significant when the hall parameter is high. This situation occurs as a result of high magnetic field or low collision frequency. In dealing with weak and moderate magneticfields, the Hall effect is ignored and the results give good agreement with experimental data. Further Eldahab [3] studied Hall currents and ion-slip effects on the MHD peristaltic transport. Some studies pertaining to the peristaltic transport with Hall effect are mentioned in Refs[4–6]. Therefore, Hall currents are important and they have remarkable effects on the magnitude and the direction of the current density and consequently on the magnetic force term. These facts make the impact of Hall current on the flow worth studying.

The heat transfer analysis plays a significant role because it provides us information about the blood flow rate. Heat transfer takes place in the human body by conduction, convection, evaporation and radiation. Bio-heat transfer finds applications in laser therapy, cryosurgery, cancer tumour treatment and hypothermia. The study of heat transfer in peristaltic flows is important in some biomedical processes like hemodialysis and oxygenation. The effects of Hall and heat transfer are likely to be important in many situations as well as in engineering applications in areas like

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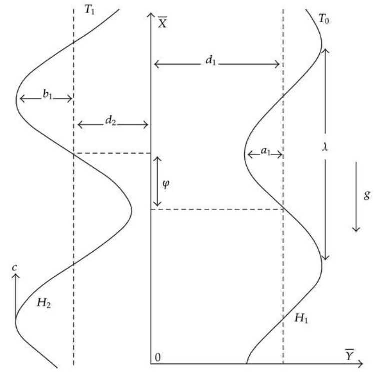

Fig. 1. A physical sketch of the problem

power generators, MHD and Hall accelerators, electric transformers, refrigeration coils and heating elements.Srinivas and Kothandapani[7] analysed the Hall currents and heat transfer effects on peristaltic transport through a porous medium. Prabhakar Reddy[8] studied the MHD flow over a vertical moving porous plate with viscous dissipation by considering double diffusive convection in the presence of chemical reaction.

The topic of Hemodynamics or blood flow problems have received a great attention due to its application in physiopathology. The analysis of blood flow in peristalsis has caught the attention of many physiologists. Basically peristaltic mechanism consists of expansion and contraction activities which propel the material forward. Some examples of peristaltic phenomenon include transport of urine from kidney to gallbladder, transport of bile in the bile duct, transport of cilia, mixing of food in digestive tract, vasomotion of small blood vessels etc. This process has many industrial applications such as rollers, hose and tube pumps whereas peristaltic pumps used in dialysis, open-heart by pass and heart-lung machines are few biomedical applications. Latham [9] was the first to examine the peristaltic motion of fluid in a pump. Later, Shapiro et.al [10] examined the peristaltic flow with long wavelength and low Reynolds number assumptions Some studies relating to the peristaltic flow of Newtonian and non-Newtonian fluids are given in Refs [11–13],Hayman and Subhash[14]

A non-Newtonian fluid model that has attracted many researchers is the Jeffrey fluid as this is found to be a better model for physiological fluids. Jeffrey fluid model is a significant generalization of Newtonian fluid model as the latter one can be deduced as a special case of the former. Several researchers have studied Jeffrey fluid flows under different conditions. Kothandapani and Srinivasa [15]have studied the MHD peristaltic transport of a Jeffrey fluid in an asymmetric channel. Vajravelu et.al [16] investigated the peristaltic transport of a conducting Jeffrey fluid. Hayat and Ali [17] have considered the peristaltic flow of a Jeffrey fluid in a tube. Vajravelu et.al [18] has analysed the peristaltic transport of Jeffrey fluid with heat transfer in a vertical porous stratum.

A porous medium is a material volume consisting of solid matrix with an interconnected void. Flows through porous medium have several practical applications present in nature: flow in sand beds, in petroleum reservoir rocks, slurries, sedimentation, and so forth. Examples of natural porous media are beach sand, sandstone, limestone, the human lung, bile duct, and gall bladder with stones in small blood vessels. Flow through porous medium has been studied by a number of workers employing Darcy's law. Furthermore, the study of Magneto hydrodynamics (MHD) flow problems has gained considerable interest because of its extensive engineering and medical applications. The MHD deals with the dynamics of electrically conducting fluids. Some investigators have considered the MHD studies of Newtonian and non-Newtonian fluids in different flow geometries. Ebaid et.al.[19] have investigated the peristaltic transport in an asymmetric channel through a porous medium. Kothandapani and Srinivas [20] studied the non-linear peristaltic motion through an inclined porous channel. Mahmouda et.al [21] have studied the effect of porous medium on MHD peristaltic flow of a Jeffrey fluid. Santosh and Radhakrishnamacharya [22] investigated the Jeffrey fluid flow through a porous medium in narrow tubes.Patel and Ramakanta Meher [23] investigated the Jeffery-Hemal flow with magnetic field.

2. Mathematical formulation

Here we assume the peristaltic flow of an incompressible, viscous and electrically conducting MHD Jeffrey fluid through a vertical asymmetric two dimensional channel with porous medium. The flow is characterized by a train of sinusoidal waves travelling with a constant speed 'c' along the channel walls. The geometry of the wall channels is represented by,

$$h_1'\left(X',t'\right) = d_1 + a_1 \cos\left[\frac{2\pi}{\lambda}\left(X' - ct'\right)\right], \text{rightwall} \tag{1}$$

,

$$h_2'\left(X',t'\right) = -d_2 - b_1 \cos\left[\frac{2\pi}{\lambda}\left(X' - ct'\right) + \phi\right], \text{leftwall} \tag{2}$$

Where a_1 and b_1 are the amplitudes of waves, $\lambda$is the wavelength, $\phi\left(0 \leqslant \phi \leqslant \pi\right)$ is the phase difference, and$d_1 + d_2$is the half-width of the channel. Furthera_1,b_1,$d_1$and $d_2$satisfy the relation

$$a_1{}^2 + b_1{}^2 + 2a_1 b_1 \cos\phi \le (d_1 + d_2)^2 \tag{3}$$

A uniform magnetic field with magnetic flux density vector $B = (0,0,B_0)$isapplied. The effect of induced magnetic field is neglected by taking very low Reynolds number. The right and left walls of the channel are maintained at temperatures T_0andT_1respectively. The fundamental equations governing the flow together with the Ohm's law and Maxwell's equations taking into the effects of Hall currents are given by,

$$\nabla \cdot \overrightarrow{V} = 0 \tag{4}$$

$$-\nabla \cdot \overrightarrow{P} + \overrightarrow{V} \cdot \overline{S} + \rho g \alpha (T - T_0) - \frac{\mu}{k_1}\overrightarrow{V} + \overrightarrow{J} \wedge \overrightarrow{B} = \rho \frac{d\overrightarrow{V}}{dt} \tag{5}$$

Where $T = -pI + S$ and the extra stress tensor for Jeffrey fluid is defined as

$$S = \frac{\mu}{1+\lambda_1}\left(\dot{\gamma} + \lambda_2 \ddot{\gamma}\right) \tag{6}$$

$$\overrightarrow{J} = \sigma\left[\overrightarrow{V} \wedge \overrightarrow{B} - \frac{1}{en_e}\overrightarrow{J} \wedge \overrightarrow{B}\right] \tag{7}$$

$$\rho c_p \frac{d\overrightarrow{T}}{dt} = \kappa \nabla^2 \overrightarrow{T} + Q_0 \tag{8}$$

where$V$is the velocity vector, $P$is the pressure, $\mu$is the dynamic viscosity, $\nabla^2$is the Laplacian operator, $\rho$is the density of the fluid, $d/dt$is the material derivative, t is the time, $I$is the Cauchy stress tensor, $S$is the extra stress tensor, $\mu$is the dynamic viscosity, $\lambda_1$is the ratio of relaxation to retardation time, $\lambda_2$is the retardation time,and$\dot{\gamma}$is the shear rate.$J$is the current density, B is the total magnetic field, $\sigma$is the electric conductivity, e is the electric charge, $n_e$is the number density of electrons, $c_p$is the specific heat at constant pressure, αisthe co-efficient of linear thermal expansion of the fluid, $\kappa$is the thermal conductivity of the fluid, $Q_0$is the constant heat addition/absorption.

The equations that govern the fluid motion for unsteady flow of an incompressible magneto-Jeffrey fluid in the Cartesian co-ordinate system may be represented as,

$$\frac{\partial U'}{\partial X'} + \frac{\partial V'}{\partial Y'} = 0 \tag{9}$$

$$\rho\left(\frac{\partial U'}{\partial t'} + U'\frac{\partial U'}{\partial X'} + V'\frac{\partial U'}{\partial Y'}\right) = -\frac{\partial P'}{\partial X'} + \frac{\partial S'_{xx}}{\partial X'} + \frac{\partial S'_{xy}}{\partial Y'} - \frac{\mu}{k_1}U' + \rho g \alpha\left(T' - T_0'\right) + \frac{\sigma B_0'^2}{1+m^2}\left(mV' - U'\right) \tag{10}$$

$$\rho\left(\frac{\partial V'}{\partial t'} + U'\frac{\partial V'}{\partial X'} + V'\frac{\partial V'}{\partial Y'}\right) = -\frac{\partial P'}{\partial Y'} + \frac{\partial S'_{xy}}{\partial X'} + \frac{\partial S'_{yy}}{\partial Y'} - \frac{\mu}{k_1}V' - \frac{\sigma B_0'^2}{1+m^2}\left(mU' + V\right) \tag{11}$$

$$\rho C_p'\left(\frac{\partial T'}{\partial t'} + U'\frac{\partial T'}{\partial X'} + V'\frac{\partial T'}{\partial Y'}\right) = \kappa\left(\frac{\partial^2 T'}{\partial X'^2} + \frac{\partial^2 T'}{\partial Y'^2}\right) + Q_0' \tag{12}$$

Where $m = \frac{\sigma B_0{}'}{en_e}$ is the Hall parameter, with the following dimensional boundary conditions,

$$\left.\begin{aligned} \psi' &= \frac{q'}{2}, \quad \frac{\partial \psi'}{\partial y'} = 0, \quad T' - T_0 = 0 \text{ at } y = h_1 \\ \psi' &= -\frac{q'}{2}, \quad \frac{\partial \psi'}{\partial y'} = 0, \quad T' - T_0 = 1 \text{ at } y = h_2 \end{aligned}\right\} \tag{13}$$

Now let us consider (X, Y)as the laboratory frame and(x, y)as the wave frame which are taken as the unsteady and steady motion respectively. The following transformations are done between the wave frame $(x, y)$and the fixed frame (X, Y)respectively

$$x' = X' - ct', \quad y' = Y', \quad u'(x', y') = U' - c, \quad v'(x', y') = V'$$

Introducing the following non-dimensional variables,

$$\left.\begin{array}{l} x = \dfrac{x'}{\lambda}, \quad y = \dfrac{y'}{d_1}, \quad u = \dfrac{u'}{c}, \quad v = \dfrac{\lambda v'}{d_1 c}, \quad Re = \dfrac{\rho d_1 c}{\mu}, \quad \delta = \dfrac{d_1}{\lambda}, \quad p = \dfrac{{d_1}^2 p'}{\lambda \mu c}, \quad t = \dfrac{ct'}{\lambda}, \quad \psi = \dfrac{\psi'}{cd_1}, \quad F = \dfrac{q'}{cd_1} \\ k = \dfrac{{k_1}'}{{d_1}^2}, \quad h_1 = \dfrac{{h_1}'}{d_1}, \quad h_2 = \dfrac{{h_2}'}{d_1}, \quad a = \dfrac{{a_1}'}{d_1}, \quad b = \dfrac{{a_2}'}{d_1}, \quad \theta = \dfrac{T' - T_0}{T_1 - T_0}, \quad \Pr = \dfrac{\mu {C_p}'}{\kappa}, \quad \beta = \dfrac{Q_0 d_1^2}{\kappa (T_1 - T_0)} \\ Gr = \dfrac{\rho g \alpha d_1^2 (T_1 - T_0)}{\mu c}, \quad M = \dfrac{\sigma B_0^2 d_1^2}{\mu} \end{array}\right\} \tag{14}$$

Where $\rho$is the fluid density, $\mu$is thedynamic viscosity, $Re$is the Reynolds number, δ is the wave number, F is the dimensionless mean flow, k is the porosity parameter, Pris the Prandtl number, βisthe dimensionless heat sink/source parameter,$M$is the magnetic parameter, Gr is the Grashoff's number.
Using the above defined non-dimensional variables in Eqns. (9) – (12) we get,

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{15}$$

$$\operatorname{Re}\delta \left[\left(\psi_y \frac{\partial}{\partial x} - \psi_x \frac{\partial}{\partial y} \right) \psi_y \right] = -\frac{\partial P}{\partial x} + \delta^2 \frac{\partial S_{xx}}{\partial x} + \frac{\partial S_{xy}}{\partial y} - \frac{(u+1)}{k} + Gr\theta + \delta \frac{mMv}{(m^2+1)} - \frac{M(u+1)}{(m^2+1)} \tag{16}$$

$$\operatorname{Re}\delta^3 \left[\left(\psi_x \frac{\partial}{\partial y} - \psi_y \frac{\partial}{\partial x} \right) \psi_x \right] = -\frac{\partial P}{\partial y} + \delta^4 \frac{\partial S_{xx}}{\partial x} + \delta^2 \frac{\partial S_{xy}}{\partial y} - \delta^2 \frac{v}{k} - \delta \frac{mM(u+1)}{(m^2+1)} - \delta^2 \frac{Mv}{(m^2+1)} \tag{17}$$

$$\operatorname{Re}\delta P_r \left[\left(\psi_y \frac{\partial}{\partial x} - \psi_x \frac{\partial}{\partial y} \right) \theta \right] = \delta^2 \frac{\partial^2 \theta}{\partial x^2} + \frac{\partial^2 \theta}{\partial y^2} + \beta \tag{18}$$

where

$$S_{xx} = \frac{2\delta}{1+\lambda_1} \left(1 + \frac{\lambda_2 c\delta}{d_1} \left(\psi_y \frac{\partial}{\partial x} - \psi_x \frac{\partial}{\partial y} \right) \right) \psi_{xy}$$
$$S_{xy} = \frac{1}{1+\lambda_1} \left(1 + \frac{\lambda_2 c\delta}{d_1} \left(\psi_y \frac{\partial}{\partial x} - \psi_x \frac{\partial}{\partial y} \right) \right) (\psi_{yy} - \delta^2 \psi_{xx})$$
$$S_{yy} = -\frac{2\delta}{1+\lambda_1} \left(1 + \frac{\lambda_2 c\delta}{d_1} \left(\psi_y \frac{\partial}{\partial x} - \psi_x \frac{\partial}{\partial y} \right) \right) \psi_{xy}$$

Approximating with the long wavelength and low Reynolds number assumptions the equations.(16) - (18) reduce to the following form after dropping terms of order δ and higher we get,

$$0 = -\frac{\partial p}{\partial x} + \frac{\partial S_{xy}}{\partial y} - \frac{(u+1)}{k} + Gr\theta - \frac{M(u+1)}{(m^2+1)} \tag{19}$$

$$0 = -\frac{\partial p}{\partial y} \tag{20}$$

$$0 = \frac{\partial^2 \theta}{\partial y^2} + \beta \tag{21}$$

With the corresponding non-dimensional boundary conditions given by,

$$\left.\begin{array}{l} \psi = \dfrac{F}{2}, \quad \dfrac{\partial \psi}{\partial y} = 0, \quad \theta = 0, \quad at \quad y = h_1 = 1 + a\cos(2\pi x) \\ \psi = -\dfrac{F}{2}, \quad \dfrac{\partial \psi}{\partial y} = 0, \quad \theta = 1, \quad at \quad y = h_2 = -d - b\cos(2\pi x + \phi) \end{array}\right\} \tag{22}$$

3. Solution of the problem

Double integration of Eq. (21) we get,

$$\theta = -\frac{\beta y^2}{2} - c_1 y - c_2 \tag{23}$$

where $c_1 = \frac{-1}{(h_2-h_1)}\left(1+\frac{\beta(h_2^2-h_1^2)}{2}\right)$ and $c_2 = \frac{\beta h_1^2}{2} + \frac{1}{(h_2-h_1)}\left(1+\frac{\beta(h_2^2-h_1^2)}{2}\right)$.
Substituting the value of c_1 and c_2 in Eq. (23) we get,

$$\theta = \frac{1}{2(h_2-h_1)}\left\{-\beta(h_2-h_1)y^2 + \left[2+\beta\left(h_2{}^2-h_1{}^2\right)\right]y + \beta h_1 h_2 (h_1-h_2) - 2h_1\right\} \tag{24}$$

We shall define the stream function ψsuch that$u = \frac{\partial\psi}{\partial y}andv = -\delta\frac{\partial\psi}{\partial x}$, equations (19) and (20) after eliminating the pressure and using the stream function can be written as,

$$\frac{\partial^4\psi}{\partial y^4} - (1+\lambda_1)\left(\frac{1}{k}+\frac{M}{1+m^2}\right)\frac{\partial^2\psi}{\partial y^2} + Gr(1+\lambda_1)\frac{\partial\theta}{\partial y} = 0 \tag{25}$$

Integrating with respect to y,

$$\frac{\partial^3\psi}{\partial y^3} - n^2\frac{\partial\psi}{\partial y} + Gr(1+\lambda_1)\theta = A_3 \tag{26}$$

Substituting the value of θ we get,

$$\frac{\partial^3\psi}{\partial y^3} - n^2\frac{\partial\psi}{\partial y} - Gr\beta(1+\lambda_1)\frac{y^2}{2} - Gr(1+\lambda_1)c_1 y = A_3 + Gr(1+\lambda_1)c_2 \tag{27}$$

where $n^2 = (1+\lambda_1)\left(\frac{1}{k}+\frac{M}{1+m^2}\right)$.

According to Adomian decomposition method (ADM), we write Eq. (27) in the operator form as,

$$L_{yyy}[\psi] = n^2\psi_y + Gr\beta(1+\lambda_1)\frac{y^2}{2} + Gr(1+\lambda_1)c_1 y + A_3 + Gr(1+\lambda_1)c_2 \tag{28}$$

Applying the inverse operator $L_{yyy}^{-1} = \iiint [.]dydydy$, Eq. (28) can be written as,

$$\psi = A_0 + A_1 y + A_2\frac{y^2}{2!} + (A_3 + Gr(1+\lambda_1)c_2)\frac{y^3}{3!} + n^2 L_{yyy}^{-1}[\psi_y] + Gr\beta(1+\lambda_1)\frac{y^5}{5!} + Gr(1+\lambda_1)c_1\frac{y^4}{4!} \tag{29}$$

Now we decompose

$$\psi = \sum_{n=0}^{\infty}\psi_n \tag{30}$$

Substituting ψ into Eq. (29) we obtain,

$$\psi_0 = A_0 + A_1 y + A_2\frac{y^2}{2!} + (A_3 + Gr(1+\lambda_1)c_2)\frac{y^3}{3!} + Gr\beta(1+\lambda_1)\frac{y^5}{5!} + Gr(1+\lambda_1)c_1\frac{y^4}{4!}$$

$$\psi_{n+1} = n^2\iiint [\psi_n]_y dydydy, \text{ where } n \geqslant 0 \tag{31}$$

Therefore,

$$\psi_1 = n^2\left(A_1\frac{y^3}{3!} + A_2\frac{y^4}{4!} + (A_3 + Gr(1+\lambda_1)c_2)\frac{y^5}{5!} + Gr\beta(1+\lambda_1)\frac{y^7}{7!} + Gr(1+\lambda_1)c_1\frac{y^6}{6!}\right)$$
$$\psi_2 = n^4\left(A_1\frac{y^5}{5!} + A_2\frac{y^6}{6!} + (A_3 + Gr(1+\lambda_1)c_2)\frac{y^7}{7!} + Gr\beta(1+\lambda_1)\frac{y^9}{9!} + Gr(1+\lambda_1)c_1\frac{y^8}{8!}\right)$$
$$\vdots$$
$$\psi_n = n^{2n}\left(A_1\frac{y^{2n+1}}{(2n+1)!} + A_2\frac{y^6}{6!} + (A_3 + Gr(1+\lambda_1)c_2)\frac{y^7}{7!} + Gr\beta(1+\lambda_1)\frac{y^9}{9!} + Gr(1+\lambda_1)c_1\frac{y^8}{8!}\right)$$

According to (30) the closed form solution of ψ can be written as,

$$\psi = A_0 + \sinh(ny)\left(\frac{A_1}{n} + \frac{1}{n^3}(A_3 + Gr(1+\lambda_1)c_2)\right) + \frac{A_2}{n^2}\left(\cosh(ny)-1\right) - \frac{1}{n^2}(A_3 + Gr(1+\lambda_1)c_2)y +$$
$$Gr\beta(1+\lambda_1)\left(\sinh(ny) - y - \frac{y^3}{3!} - \frac{y^5}{5!}\right) + Gr(1+\lambda_1)c_1\left(\cosh(ny) - 1 - \frac{y^2}{2!} - \frac{y^4}{4!}\right)$$

which can be simplified as,

$$\psi = C_0 + C_1\sinh(ny) + C_2\cosh(ny) + C_3\left\{\left(\sinh(ny) - y - \frac{y^3}{3!} - \frac{y^5}{5!}\right) + \left(\cosh(ny) - 1 - \frac{y^2}{2!} - \frac{y^4}{4!}\right)\right\}$$

where

$$C_0 = \frac{\tanh\left[n\left(\frac{h_1-h_2}{2}\right)\right]\left(2(h_1+h_2) + \frac{Fn^2Gr(h_1+h_2)}{1+\lambda_1}\right) + Fn(h_1+h_2)}{\tanh\left[n\left(\frac{h_1-h_2}{2}\right)\right]\left(2 + \frac{n^2Gr(h_2-h_1)}{1+\lambda_1}\right) + 2n(h_2-h_1)} + Gr\beta(1+\lambda_1)$$
$$\times\begin{pmatrix}\sinh(n(h_1+h_2)) - (h_1+h_2) \\ -\dfrac{(h_1+h_2)^3}{3!} - \dfrac{(h_1+h_2)^5}{5!}\end{pmatrix} + Gr(1+\lambda_1)c_1\begin{pmatrix}\cosh(n(h_1+h_2)) - 1 \\ -\dfrac{(h_1+h_2)^2}{2!} - \dfrac{(h_1+h_2)^4}{4!}\end{pmatrix}$$

$$C_1 = \frac{\sinh\left[n\left(\frac{h_1+h_2}{2}\right)\right]\operatorname{sech}\left[n\left(\frac{h_1-h_2}{2}\right)\right](F+h_1-h_2)}{\tanh\left[n\left(\frac{h_1-h_2}{2}\right)\right]\left(2 + \frac{n^2Gr(h_2-h_1)}{1+\lambda_1}\right) + n(h_1-h_2)} + Gr\beta(1+\lambda_1)\begin{pmatrix}\sinh(n(h_1+h_2)) - (h_1+h_2) \\ -\dfrac{(h_1+h_2)^3}{3!} - \dfrac{(h_1+h_2)^5}{5!}\end{pmatrix} +$$
$$Gr(1+\lambda_1)c_1\begin{pmatrix}\cosh(n(h_1+h_2)) - 1 \\ -\dfrac{(h_1+h_2)^2}{2!} - \dfrac{(h_1+h_2)^4}{4!}\end{pmatrix}$$

$$C_2 = \frac{\cosh\left[n\left(\frac{h_1+h_2}{2}\right)\right]\operatorname{sech}\left[n\left(\frac{h_1-h_2}{2}\right)\right](F+h_1-h_2)}{\tanh\left[n\left(\frac{h_1-h_2}{2}\right)\right]\left(2 + \frac{n^2Gr(h_2-h_1)}{1+\lambda_1}\right) + n(h_2-h_1)}$$
$$+Gr\beta(1+\lambda_1)\begin{pmatrix}\sinh(n(h_1+h_2)) - (h_1+h_2) \\ -\dfrac{(h_1+h_2)^3}{3!} - \dfrac{(h_1+h_2)^5}{5!}\end{pmatrix} + Gr(1+\lambda_1)c_1\begin{pmatrix}\cosh(n(h_1+h_2)) - 1 \\ -\dfrac{(h_1+h_2)^2}{2!} - \dfrac{(h_1+h_2)^4}{4!}\end{pmatrix}$$

$$C_3 = \frac{\left(2n^2 + n^4(F-h_1+h_2) + \frac{Gr}{1+\lambda_1}\right)\tanh\left[n\left(\frac{h_1-h_2}{2}\right)\right]}{\tanh\left[n\left(\frac{h_1-h_2}{2}\right)\right]\left(2 + \frac{n^2Gr(h_2-h_1)}{1+\lambda_1}\right) + n(h_2-h_1)} + Gr\beta(1+\lambda_1)\begin{pmatrix}\sinh(n(h_1+h_2)) - (h_1+h_2) \\ -\dfrac{(h_1+h_2)^3}{3!} - \dfrac{(h_1+h_2)^5}{5!}\end{pmatrix}$$
$$+Gr(1+\lambda_1)c_1\begin{pmatrix}\cosh(n(h_1+h_2)) - 1 \\ -\dfrac{(h_1+h_2)^2}{2!} - \dfrac{(h_1+h_2)^4}{4!}\end{pmatrix}$$

The expression for velocity u is given by,

$$u = n\cosh(ny)\left(\frac{A_1}{n} + \frac{1}{n^3}(A_3 + Gr(1+\lambda_1)c_2)\right) - \frac{A_2}{n^2}(n\sinh(ny)) - \frac{1}{n^2}(A_3 + Gr(1+\lambda_1)c_2) +$$
$$Gr\beta(1+\lambda_1)\left(n\cosh(ny) - 1 - \frac{y^2}{2!} - \frac{y^4}{4!}\right) + Gr(1+\lambda_1)c_1\left(-n\sinh(ny) - y - \frac{y^3}{3!}\right) \quad (32)$$

4. Rate of Volume flow

The rate of volume flow in the wave frame of reference is given by,

$$\overline{Q}(x,t) = q + c_1d_1 + c_2d_2 \quad (33)$$

where h_1 and h_2 are functions of x.

The instantaneous volume flow rate in the fixed frame is given by,

$$Q = \int_{H_2}^{H_1}[U(x,y,t)]\,dy = \int_{h_2}^{h_1}[u(x,y) + c]\,dy = q + ch_1 - ch_2 \quad (34)$$

The time mean flow over time period T at a fixed position x is defined as,

$$\overline{Q}(x,t) = \frac{1}{T}\int_0^T Q(x,y)\,dt \quad (35)$$

Using (34) and (35) we get,

$$\overline{Q}(x,t) = q + c_1 d_1 + c_2 d_2 \tag{36}$$

On defining the dimensionless mean flow in laboratory frame and F in the wave frame,

$$\Theta = \frac{\bar{Q}}{cd_1}, \quad F = \frac{q}{cd_1} \tag{37}$$

Using (36) and (37) we obtain,

$$\Theta = F + 1 + d \tag{38}$$

in which

$$F = \int_{h_2}^{h_1} u(x,y)\,dy \tag{39}$$

The pressure gradient is obtained from the dimensionless momentum equation for the axial velocity,

$$\frac{dp}{dx} = \frac{1}{1+\lambda_1}\frac{\partial^4\psi}{\partial y^4} - n^2\frac{\partial^2\psi}{\partial y^2} + Gr\theta \tag{40}$$

Integrating over one wavelength we get,

$$\Delta p_\lambda = \int_0^1 \frac{dp}{dx}\,dx \tag{41}$$

5. Numerical Results and discussion

In this section, numerical results of the problem under discussion are discussed through graphs. The graphs are plotted using the MATHEMATICA software. Fig. 2 is plotted to study the effect of heat source or sink parameter

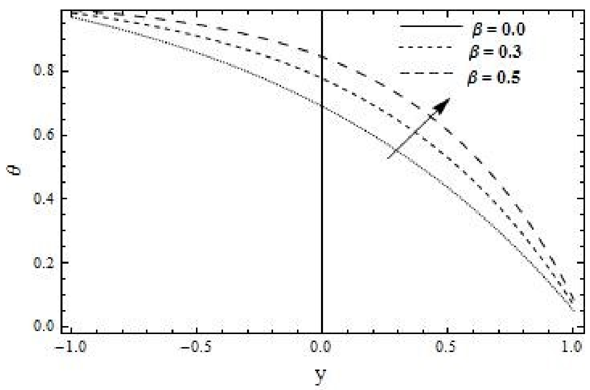

Fig. 2. Variation of temperature θ for $a = 0.1, b = 0.2, d = 1, \phi = 0$

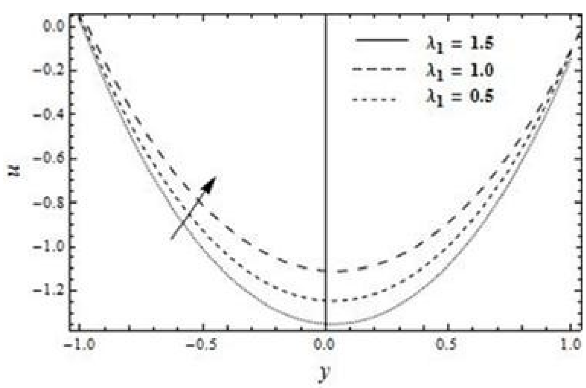

Fig. 3. Variation of velocity u for $a = 0.1, b = 0.2, d = 1, \phi = 0$

(β) on temperature (θ)it is observed that (β) enhances the temperature because both heat and temperature are directly proportional to each other. Figs. 3-5 are sketched to analyse the velocity profile for different values of Magnetic field(M), Grashoff's number (Gr) and Jeffrey parameter(λ_1). From Fig. 3 and Fig. 4 it is observed that both velocity and Grashoff's number (Gr)increase with an increase in the value of M. The effect of Jeffrey parameter(λ_1)is almost opposite as compared with the case of (Gr) and (M). The velocity profile decreases with an increase in (λ_1)as depicted in Fig. 5. The graphical results for pressure rise per wavelength (Δp_λ)verses Volume flow rate (Θ)is analysed in Figs. 6-8. It is observed that both are inversely proportional to each other. It means that pressure rise enhances for smaller values of Volume flow rate(Θ) where as pressure rise decreases for larger values of(Q). Fig. 6 shows the variation of (Δp_λ) with Volume flow rate (Θ)for different values of Jeffrey parameter(λ_1). It is found that the Volume flow rate (Θ)decreases with increasing (λ_1)in the pumping region ($\Delta p_\lambda > 0$)and free pumping region($\Delta p_\lambda = 0$), while in the co-pumping region ($\Delta p_\lambda < 0$)the flow rate increases with increase in(λ_1). Fig. 7 shows the variation of pressure rise $\Delta p_\lambda$with volume flow rate(Θ)for different values of(β). It is found that the Volume flow rate (Θ)is enhanced with an increase in the heat source or sink parameter (β)in all the three (pumping, free pumping and co-pumping) regions. Fig. 7 shows the variation of pressure rise Δp_λ with volume flow rate for different values of Grashoff's number(Gr).It is observed that an increase in (Gr)increases the Volume flow rate (Θ) in all the three (pumping, free pumping and co-pumping) regions.

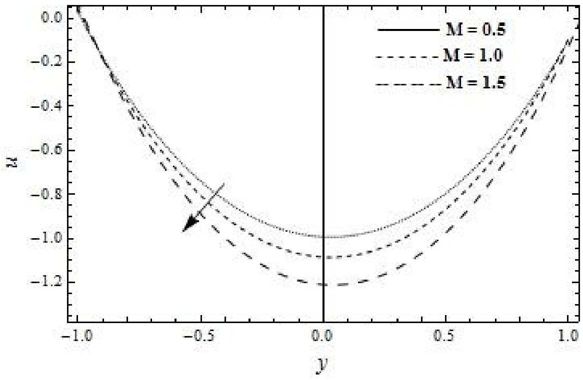

Fig. 4. Variation of velocity u for $a = 0.1, b = 0.2, d = 1, \phi = 0$

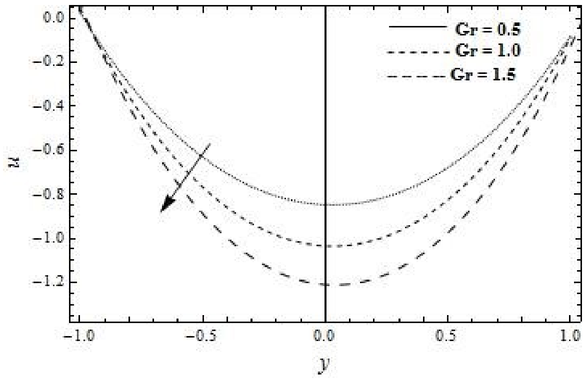

Fig. 5. Variation of velocity u for $a = 0.1, b = 0.2, d = 1, \phi = 0$

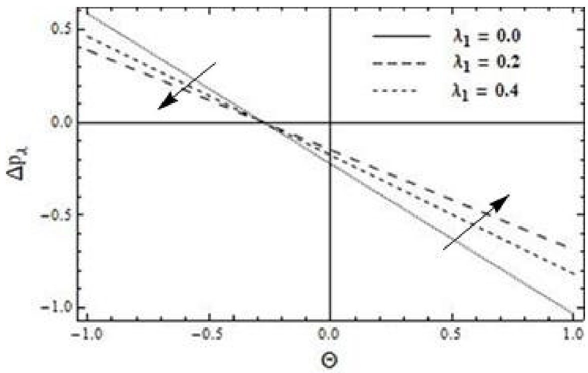

Fig. 6. Variation of pressure rise per wavelength Δp_λ for $a = 0.1, b = 0.2, d = 1, \phi = 0$

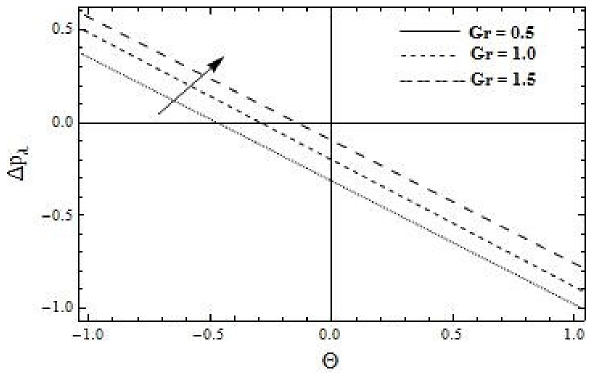

Fig. 7. Variation of pressure rise per wavelength Δp_λ for $a = 0.1, b = 0.2, d = 1, \phi = 0$

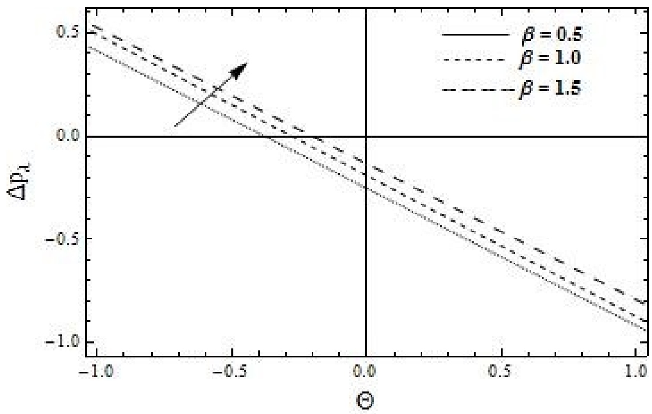

Fig. 8. Variation of pressure rise per wavelength Δp_λ for $a = 0.1, b = 0.2, d = 1, \phi = 0$

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