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The approximate solutions of time-fractional Burger's and coupled time-fractional Burger's equations

Research Article

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Abstract: In this paper, we extend the fractional Sumudu decomposition method (FSDM) to solve nonlinear fractional partial differential equations. The time fractional Burger's equations and coupled time fractional Burger's equations with initial conditions are chosen to illustrate our method. As a result, we successfully obtain some available approximate solutions of them. The results reveal that the proposed method is very effective and simple for obtaining approximate solutions of nonlinear fractional partial differential equations. The fractional derivatives are considered in the Caputo sense.

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Keywords: Fractional Burger's equations • Sumudu transform • Adomian decomposition method • Caputo fractional derivative

1. Introduction

Burger's equation is a fundamental partial differential equation from fluid mechanics. It occurs in various areas of applied mathematics, such as modeling of dynamics, heat conduction, and acoustic waves. It is named for Johannes Martinis Burgers (1895-1981). It is very rare that a real life applications can be modeled by a single partial differential equation, usually it takes a system of coupled partial differential equations to yield a complete model [1].

Fractional calculus was utilized as an excellent instrument to discover the hidden aspects of various material and physical processes that deal with derivatives and integrals of arbitrary orders [2, 3]. The theory of fractional differential equations translates the reality of nature excellently in a better and systematic manner [4–6]. In recent years, many authors have investigated partial differential equations of fractional order by various techniques such as homotopy analysis technique [7, 8], variational iteration method [9–11], homotopy perturbation method [12], and Laplace homotopy perturbation method [13]. This article considers the efficiency of fractional Sumudu decomposition method (FSVIM) to solve time fractional Burger's equation and coupled time fractional Burger's equations. The FSVIM is a graceful coupling of two powerful techniques namely VIM and Sumudu transform algorithms and gives more refined convergent series solution.

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2. Preliminaries

Some fractional calculus definitions and notation needed [2, 14, 15] in the course of this work are discussed in this section

Definition 2.1.
A real function $\varphi(\mu), \mu > 0$, is said to be in the space $C_\vartheta, \vartheta \in R$ if there exists a real number $q, (q > \vartheta)$, such that $\varphi(\mu) = \mu^q \varphi_1(\mu)$, where $\varphi_1(\mu) \in C[0,\infty)$, and it is said to be in the space C_ϑ^m if $\varphi^{(m)} \in C_\vartheta, m \in N$.

Definition 2.2.
The Riemann Liouville fractional integral operator of order $\delta \geq 0$, of a function $\varphi(\mu) \in C_\vartheta, \vartheta \geq -1$ is defined as

$$I^\delta \varphi(\mu) = \begin{cases} \frac{1}{\Gamma(\delta)} \int_0^\mu (\mu-\tau)^{\delta-1} \varphi(\tau) d\tau, & \delta > 0, \mu > 0, \\ \varphi(\mu), & \delta = 0, \end{cases} \tag{1}$$

where $\Gamma(\cdot)$ is the well-known Gamma function.

Properties of the operator I^δ, which we will use here, are as follows:
For $\varphi \in C_\vartheta, \vartheta \geq -1, \delta, \gamma \geq -1$, then

1. $I^\delta I^\gamma \varphi(\mu) = I^{\delta+\gamma} \varphi(\mu)$.
2. $I^\delta I^\gamma \varphi(\mu) = I^\gamma I^\delta \varphi(\mu)$.
3. $I^\delta \mu^m = \dfrac{\Gamma(m+1)}{\Gamma(\delta+m+1} \mu^{\delta+m}$.

Definition 2.3.
The fractional derivative of $\varphi(\mu)$ in the Caputo sense is defined as

$$\begin{aligned} D^\delta \varphi(\mu) &= I^{m-\delta} D^m \varphi(\mu) \\ &= \frac{1}{\Gamma(m-\delta)} \int_0^\mu (\mu-\tau)^{m-\delta-1} \varphi^{(m)}(\tau) d\tau, \end{aligned} \tag{2}$$

for $m-1 < \delta \leq m, m \in N, \mu > 0, \varphi \in C_{-1}^m$.

The following are the basic properties of the operator D^δ:

1. $D^\delta I^\delta \varphi(\mu) = \varphi(\mu)$.
2. $D^\delta I^\delta \varphi(\mu) = \varphi(\mu) - \sum_{k=0}^{m-1} \varphi^{(k)}(0) \dfrac{\mu^k}{k!}$.

Definition 2.4.
The Mittag-Leffler function E_δ with $\delta > 0$ is defined as

$$E_\delta(z) = \sum_{m=0}^{\infty} \frac{z^\delta}{\Gamma(m\delta+1)}. \tag{3}$$

Definition 2.5.
The Sumudu transform is defined over the set of function
$A = \left\{ \varphi(\tau) / \exists M, \omega_1, \omega_2 > 0, |\varphi(\tau)| < M e^{\frac{|\tau|}{\omega_j}}, if \tau \in (-1)^j \times [0,\infty) \right\}$,
by the following formula

$$S[\varphi(\tau)] = \int_0^\infty e^{-\tau} \varphi(\omega\tau) d\tau, \omega \in (-\omega_1, \omega_2). \tag{4}$$

Definition 2.6.
The Sumudu transform of the Caputo fractional derivative is defined as

$$S[D_\tau^{m\delta} \varphi(\mu,\tau)] = \omega^{-m\delta} S[\varphi(\mu,\tau)] - \sum_{k=0}^{m-1} \omega^{-m\delta+k} \varphi^{(k)}(\mu,0), m-1 < m\delta < m. \tag{5}$$

3. Fractional Sumudu Decomposition Method (FSDM)

Let us consider a general fractional nonlinear partial differential equation of the form:

$$D_\tau^\delta \varphi(\mu,\tau) + R\left[\varphi(\mu,\tau)\right] + N\left[\varphi(\mu,\tau)\right] = g(\mu,\tau) \tag{6}$$

with the initial condition

$$\varphi(\mu,0) = f(\mu), \tag{7}$$

where $D_\tau^\delta \varphi(\mu,\tau)$ is the Caputo fractional derivative of the function $\varphi(\mu,\tau)$ defined as:

$$D_\tau^\delta \varphi(\mu,\tau) = \frac{\partial^\delta \varphi(\mu,\tau)}{\partial \tau^\delta} = \begin{cases} \frac{1}{\Gamma(m-\delta)} \int_0^\tau (\tau-\omega)^{m-\delta-1} \frac{\partial^m \varphi(\mu,\omega)}{\partial \tau^m} d\omega, & m-1<\delta<m, \\ \frac{\partial^m \varphi(\mu,\tau)}{\partial \tau^m}, & \delta = m \in N, \end{cases}$$

and R is the linear differential operator, N represents the general nonlinear differential operator, and $g(\mu,\tau)$ is the source term.

Taking the ST on both sides of (6), we have

$$S\left[D_\tau^\delta \varphi(\mu,\tau)\right] + S\left[R\left[\varphi(\mu,\tau)\right]\right] + S\left[N\left[\varphi(\mu,\tau)\right]\right] = S\left[g(\mu,\tau)\right]. \tag{8}$$

Using the property of the ST, we obtain

$$S\left[\varphi(\mu,\tau)\right] = \varphi(\mu,0) + \omega^\delta S\left[g(\mu,\tau)\right] - \omega^\delta S\left[R\left[\varphi(\mu,\tau) + N\left[\varphi(\mu,\tau)\right]\right]\right]. \tag{9}$$

Operating with the ST on both sides of (9) gives

$$\varphi(\mu,\tau) = f(\mu) + S^{-1}\left(\omega^\delta S\left[g(\mu,\tau)\right]\right) - S^{-1}\omega^\delta S\left[R\left[\varphi(\mu,\tau) + N\left[\varphi(\mu,\tau)\right]\right]\right). \tag{10}$$

Now, we represent solution as an infinite series given below

$$\varphi(\mu,\tau) = \sum_{n=0}^{\infty} \varphi_n(\mu,\tau), \tag{11}$$

and the nonlinear term can be decomposed as

$$N\left[\varphi(\mu,\tau)\right] = \sum_{n=0}^{\infty} A_n(\varphi_1,\varphi_2,\ldots,\varphi_n), \tag{12}$$

where

$$A_n(\varphi_1,\varphi_2,\ldots,\varphi_n) = \frac{1}{n!}\frac{\partial^n}{\partial \lambda^n}\left[N\left(\sum_{i=0}^{\infty} \lambda^i \varphi_i\right)\right]_{\lambda=0}.$$

Substituting (11) and (12) in (10), we get

$$\sum_{n=0}^{\infty} \varphi_n(\mu,\tau) = f(\mu) + S^{-1}\left(\omega^\delta - S\left[g(\mu,\tau)\right]\right) S^{-1}\left(\omega^\delta S\left[R\left[\sum_{n=0}^{\infty} \varphi_n(\mu,\tau)\right] + \sum_{n=0}^{\infty} A_n\right]\right). \tag{13}$$

On comparing both sides of the Eq. (13), we get

$$\begin{aligned} &\varphi_0(\mu,\tau) = f(\mu) + S^{-1}\left(\omega^\delta S\left[g(\mu,\tau)\right]\right), \\ &\varphi_1(\mu,\tau) = -S^{-1}\left(\omega^\delta S\left[R\left[\varphi_0(\mu,\tau)\right] + A_0\right]\right), \\ &\varphi_2(\mu,\tau) = -S^{-1}\left(\omega^\delta S\left[R\left[\varphi_1(\mu,\tau)\right] + A_1\right]\right), \\ &\vdots \\ &\varphi_n(\mu,\tau) = -S^{-1}\left(\omega^\delta S\left[R\left[\varphi_{n-1}(\mu,\tau)\right] + A_{n-1}\right]\right), n \geq 1. \end{aligned} \tag{14}$$

Finally, we approximate the analytical solution $\varphi(\mu,\tau)$ by truncated series:

$$\varphi(\mu,\tau) = \sum_{n=0}^{\infty} \varphi_n(\mu,\tau). \tag{15}$$

4. Applications

In this section, we will implement the proposed method fractional Sumudu decomposition method (FSDM) for solving Burger's and coupled Burger's equations.

Example 4.1.
First, we consider the fractional Burger's equation

$$D_\tau^\delta \varphi(\mu,\tau) = \frac{\partial^2 \varphi(\mu,\tau)}{\partial \mu^2} - \varphi(\mu,\tau)\frac{\partial \varphi(\mu,\tau)}{\partial \mu}, \tag{16}$$

subject to initial condition

$$\varphi(\mu,0) = \mu. \tag{17}$$

From (14) and (16), the successive approximations are

$$\begin{aligned}
&\varphi_0(\mu,\tau) = \varphi(\mu,0),\\
&\varphi_n(\mu,\tau) = S^{-1}\left(\omega^\delta S\left[\frac{\partial^2 \varphi_{n-1}(\mu,\tau)}{\partial \mu^2} - A_{n-1}\right]\right), n \geq 1.
\end{aligned} \tag{18}$$

where

$$\begin{aligned}
&A_0 = \varphi_0 \varphi_{0\mu},\\
&A_1 = \varphi_0 \varphi_{1\mu} + \varphi_1 \varphi_{0\mu},\\
&A_2 = \varphi_0 \varphi_{2\mu} + \varphi_1 \varphi_{1\mu} + \varphi_2 \varphi_{0\mu},\\
&\vdots
\end{aligned}$$

In view of (17) and (18), we have

$$\begin{aligned}
&\varphi_0(\mu,\tau) = \mu,\\
&\varphi_1(\mu,\tau) = S^{-1}\left(\omega^\delta S\left[\frac{\partial^2}{\partial \mu^2}\varphi_0(\mu,\tau) - A_0\right]\right),\\
&\varphi_2(\mu,\tau) = S^{-1}\left(\omega^\delta S\left[\frac{\partial^2}{\partial \mu^2}\varphi_1(\mu,\tau) - A_1\right]\right),\\
&\varphi_3(\mu,\tau) = S^{-1}\left(\omega^\delta S\left[\frac{\partial^2}{\partial \mu^2}\varphi_2(\mu,\tau) - A_2\right]\right),\\
&\qquad\vdots
\end{aligned} \tag{19}$$

From the formulas (19), the first terms of fractional Sumudu decomposition method are given by

$$\begin{aligned}
&\varphi_0(\mu,\tau) = \mu,\\
&\varphi_1(\mu,\tau) = -\mu\frac{\tau^\delta}{\Gamma(\delta+1)},\\
&\varphi_2(\mu,\tau) = 2\mu\frac{\tau^{2\delta}}{\Gamma(2\delta+1)},\\
&\varphi_3(\mu,\tau) = -\mu\frac{\Gamma(2\delta+1)}{\Gamma^2(\delta+1)}\frac{\tau^{3\delta}}{\Gamma(3\delta+1)} - 4\mu\frac{\tau^{3\delta}}{\Gamma(3\delta+1)},\\
&\qquad\vdots
\end{aligned} \tag{20}$$

Thererfore, the solution of (16) is given by

$$\varphi(\mu,\tau) = \mu\left[1 - \frac{\tau^\delta}{\Gamma(\delta+1)} + 2\frac{\tau^{2\delta}}{\Gamma(2\delta+1)} - \frac{\Gamma(2\delta+1)}{\Gamma^2(\delta+1)}\frac{\tau^{3\delta}}{\Gamma(3\delta+1)} - 4\frac{\tau^{3\delta}}{\Gamma(3\delta+1)} + \cdots\right] \tag{21}$$

The Eq. (21) is approximate to the form $\varphi(\mu,\tau) = \dfrac{\mu}{1-\tau}$ for $\delta = 1$, which is the exact solution of Eq. (16) for $\delta = 1$. The result is same as VIM [16].

Example 4.2.
Consider the following coupled fractional Burger's equations

$$\begin{aligned} &D_\tau^\delta \varphi(\mu,\tau) - \varphi_{\mu\mu}(\mu,\tau) - 2\varphi(\mu,\tau)\varphi_\mu(\mu,\tau) + (\varphi\psi)_\mu = 0, \\ &D_\tau^\gamma \psi(\mu,\tau) - \psi_{\mu\mu}(\mu,\tau) - 2\psi(\mu,\tau)\psi_\mu(\mu,\tau) + (\varphi\psi)_\mu = 0, \end{aligned} \tag{22}$$

subject to initial conditions

$$\begin{aligned} &\varphi(\mu,0) = \sin(\mu), \\ &\psi(\mu,0) = \sin(\mu), \end{aligned} \tag{23}$$

Taking the Sumudu transform (ST) on both sides of (22), we have

$$\begin{aligned} &S\left[D_\tau^\delta \varphi(\mu,\tau)\right] = S\left[\varphi_{\mu\mu}(\mu,\tau) + 2\varphi\varphi_\mu - (\varphi\psi)_\mu\right], \\ &S\left[D_\tau^\gamma \psi(\mu,\tau)\right] = S\left[\psi_{\mu\mu}(\mu,\tau) + 2\psi\psi_\mu - (\varphi\psi)_\mu\right]. \end{aligned} \tag{24}$$

Using the property of the Sumudu transform and the initial condition in (23), we obtain

$$\begin{aligned} &S\left[\varphi(\mu,\tau)\right] = \sin(\mu) + \omega^\delta S\left[\varphi_{\mu\mu}(\mu,\tau) + 2\varphi\varphi_\mu - (\varphi\psi)_\mu\right], \\ &S\left[\psi(\mu,\tau)\right] = \sin(\mu) + \omega^\gamma S\left[\psi_{\mu\mu}(\mu,\tau) + 2\psi\psi_\mu - (\varphi\psi)_\mu\right]. \end{aligned} \tag{25}$$

Operating with the Sumudu inverse on both sides of (25), we have

$$\begin{aligned} &\varphi(\mu,\tau) = \sin(\mu) + S^{-1}\left(\omega^\delta S\left[\varphi_{\mu\mu}(\mu,\tau) + 2\varphi\varphi_\mu - (\varphi\psi)_\mu\right]\right), \\ &\psi(\mu,\tau) = \sin(\mu) + S^{-1}\left(\omega^\gamma S\left[\psi_{\mu\mu}(\mu,\tau) + 2\psi\psi_\mu - (\varphi\psi)_\mu\right]\right). \end{aligned} \tag{26}$$

Suppose that

$$\varphi(\mu,\tau) = \sum_{n=0}^{\infty} \varphi_n(\mu,\tau), \tag{27}$$

$$\psi(\mu,\tau) = \sum_{n=0}^{\infty} \psi_n(\mu,\tau), \tag{28}$$

$$\varphi\varphi_\mu = \sum_{n=0}^{\infty} H_n, \tag{29}$$

$$\psi\psi_\mu = \sum_{n=0}^{\infty} K_n, \tag{30}$$

$$(\varphi\psi)_\mu = \sum_{n=0}^{\infty} G_n. \tag{31}$$

Substituting (27)-(31) in (26), we get

$$\begin{aligned} &\sum_{n=0}^{\infty} \varphi_n = \sin(\mu) + S^{-1}\left(\omega^\delta S\left[\frac{\partial^2}{\partial\mu^2}\left(\sum_{n=0}^{\infty}\varphi_n\right) + 2\sum_{n=0}^{\infty} H_n - \sum_{n=0}^{\infty} G_n\right]\right), \\ &\sum_{n=0}^{\infty} \psi_n = \sin(\mu) + S^{-1}\left(\omega^\delta S\left[\frac{\partial^2}{\partial\mu^2}\left(\sum_{n=0}^{\infty}\psi_n\right) + 2\sum_{n=0}^{\infty} K_n - \sum_{n=0}^{\infty} G_n\right]\right). \end{aligned} \tag{32}$$

On comparing both sides of (32), we obtain

$$\begin{aligned} &\varphi_0(\mu,\tau) = \sin(\mu), \\ &\psi_0(\mu,\tau) = \sin(\mu). \end{aligned} \tag{33}$$

$$\begin{aligned} &\varphi_1(\mu,\tau) = -\sin(\mu)\frac{\tau^\delta}{\Gamma(\delta+1)}, \\ &\psi_1(\mu,\tau) = -\sin(\mu)\frac{\tau^\gamma}{\Gamma(\gamma+1)}, \end{aligned} \tag{34}$$

$$\begin{aligned} &\varphi_2(\mu,\tau) = \frac{\tau^{2\delta}}{\Gamma(2\delta+1)}\sin(\mu) - \frac{\tau^{2\delta}}{\Gamma(2\delta+1)}\sin(2\mu) + \frac{\tau^{\delta+\gamma}}{\Gamma(\delta+\gamma+1)}\sin(2\mu), \\ &\psi_2(\mu,\tau) = \frac{\tau^{2\gamma}}{\Gamma(2\gamma+1)}\sin(\mu) - \frac{\tau^{2\gamma}}{\Gamma(2\gamma+1)}\sin(2\mu) + \frac{\tau^{\delta+\gamma}}{\Gamma(\delta+\gamma+1)}\sin(2\mu), \end{aligned} \tag{35}$$

$$\vdots$$

and so on.

Therefore, the solution of (22) is given by

$$\begin{aligned}
\varphi(\mu,\tau) &= \sin(\mu)\left[1-\frac{\tau^{\delta}}{\Gamma(\delta+1)}+\frac{\tau^{2\delta}}{\Gamma(2\delta+1)}\cdots\right] \\
&\quad +\sin(2\mu)\left[-\frac{\tau^{2\delta}}{\Gamma(2\delta+1)}+\frac{\tau^{\delta+\gamma}}{\Gamma(\delta+\gamma+1)}\cdots\right], \\
\psi(\mu,\tau) &= \sin(\mu)\left[1-\frac{\tau^{\gamma}}{\Gamma(\gamma+1)}+\frac{\tau^{2\gamma}}{\Gamma(2\gamma+1)}\cdots\right] \\
&\quad +\sin(2\mu)\left[-\frac{\tau^{2\gamma}}{\Gamma(2\gamma+1)}+\frac{\tau^{\delta+\gamma}}{\Gamma(\delta+\gamma+1)}\cdots\right].
\end{aligned} \tag{36}$$

Setting $\delta=\gamma$ in (36), we obtain

$$\begin{aligned}
\varphi(\mu,\tau) &= \sin(\mu)\left[1-\frac{\tau^{\delta}}{\Gamma(\delta+1)}+\frac{\tau^{2\delta}}{\Gamma(2\delta+1)}\cdots\right] \\
\psi(\mu,\tau) &= \sin(\mu)\left[1-\frac{\tau^{\gamma}}{\Gamma(\gamma+1)}+\frac{\tau^{2\gamma}}{\Gamma(2\gamma+1)}\cdots\right] \\
&= E_{\delta}(\tau^{\delta})\sin(\mu), \\
&= E_{\gamma}(\tau^{\gamma})\sin(\mu).
\end{aligned} \tag{37}$$

The Eq. (37) is approximate to the form $\varphi(\mu,\tau)=\psi(\mu,\tau)=e^{-\tau}\sin(\mu)$ for $\delta=\gamma=1$, which is the exact solution of Eq. (22) for $\delta=\gamma=1$. The result is same as q-HATM [8] and HPM [12].

5. Conclusion

The coupling of Adomian decomposition method (ADM) and the Sumudu transform (ST) in the sense of Caputo fractional derivative, proved very effective to solve fractional partial differential equations. The proposed algorithm provides the solution in a series form that converges rapidly to the exact solution if it exists. From the obtained results, it is clear that the FSDM yields very accurate solutions using only a few iterates. As a result, the conclusion that comes through this work is that FSDM can be applied to other fractional partial differential equations of higher order, due to the efficiency and flexibility in the application as can be seen in the proposed examples.

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