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Effects of variable viscosity on unsteady natural convection hydromagnetic fluid flow over an isothermal sphere in a rotating system

Research Article

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Abstract: Unsteady laminar free convection flow over an isothermal sphere immersed in incompressible fluid rotating system taking into account the effects of temperature-dependent of viscosity has been investigated in this study. The rotating of the sphere initiates the motion of the fluid. This study is divided in two parts, namely: the mathematical modelling of the problem using equations governing fluid flows and the determination of dimensionless parameters of interest and their effects on velocity and temperature profiles, surface skin friction and heat transfer. The mathematical modelling involves non-linear partial differential equations which are solved using numerical methods such as the finite difference method and then the results are displayed in graphs once implemented in MATLAB. These graphical numerical results express the effects of viscous variation parameter, Grashof number, Magnetic parameter, rotational parameter on primary velocity, secondary velocity, surface skin friction and heat transfer. This study is useful and can be applied but not limited to study problems which occur in engineering and industrial fields, meteorology, biomechanics, and epidemiology.

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Keywords: Laminar flow • Hydromagnetic • Viscosity dependent of temperature • Isothermal sphere • Rotating system

1. Introduction

Temperature is one of the factors that causes variation of viscosity of a given fluid. Several researches have been done about the effects of fluid flow viscosity past stationary isothermal sphere, but the case where the system is rotating needs to be investigated. [1], [2], [3] studied laminar free convection flow from an isothermal sphere immersed in a fluid. [1], [2] focused their studies on viscosity dependent of the temperature while Mamun et al [3] focused the study on thermal conductivity proportional to a linear function of temperature. Mwangi et al [1] established that the viscosity to be an inverse function of temperature and Molla et al [2] considered that the viscosity as linear function of temperature. Numerical results of the flow variables have been obtained using finite difference method. [1], [2] observed that an increase in magnetic parameter has a retarding influence on fluid velocity, skin friction and rate of heat

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transfer while an increase in viscous variation parameter leads to a decrease in fluid velocity but leads to an increase in temperature. Mamun et al [3] obtained that increasing the values of thermal conductivity variation parameter leads to the increase of the velocity and the temperature distributions. Zhang and Schubert [4] studied magnetohydrodynamics in rapidly rotating spherical systems. These authors focused on the dynamic feature key of rotating spherical sphere, the interaction between the Coriolis and Lorentz forces and the resulting effect on convection and magnetohydrodynamics (MHD) processes. They observed that a rapid rotation and small viscosity in the fluid parts of planetary cores, causes severe difficulties in modeling planetary dynamos. [5], [6] studied effects of viscous dissipation on MHD natural convection flow over a sphere in the presence of magnetic field and heat generation. Alam et al [5] assumed thermal conductivity of the fluid to be constant but Haque et al [6] considered thermal conductivity dependent of temperature and both used finite difference method to solve numerically the equations governing fluid flow problems. They observed that as viscous dissipation parameter, heat generation parameter and thermal conductivity variation parameter increase, both velocity and temperature profiles increase as well. An increase in the values of magnetic parameter leads to a decrease in velocity distribution, skin friction and rate of heat transfer but the temperature increases. Davino et al [7] studied the effect of viscoelasticity of the suspending liquid on the rotation period of the sphere by means of three-dimensional finite element simulations. The analysis of transients shows a clear correlation between rotation rate and the development of first normal stress difference. They used numerical methods to predict the effect of viscoelasticity on the rotation of inertialess. Kim [8] investigated laminar flow past a sphere rotating in the transverse direction in order to understand the effect of the rotation characteristics of flow over the sphere. The rotation affects the flow unsteadiness, and its effect depends on the rotational speed and the Reynolds number. Rahma et al [9] studied the boundary layer flow of unsteady incompressible and viscous fluid passing over a porous sphere with forced convection. The boundary layer equations are converted into non-similar boundary layers and solved numerically using the Keller-Box method. The outcome of this study revealed that when the magnetic parameter, convection parameter, and porosity parameter increase and the permeability parameter decreases, the velocity profiles increase while the temperature profiles decrease. Iva et al [10] investigated numerically MHD free convection heat and mass transfer flow over a vertical porous plate in a rotating system with hall current, heat source and suction. The governing equations are then solved using explicit finite difference method. They determined the effects of various parameters such as magnetic parameter, heat source parameter, Grashof number, modified Grashof number, hall parameter, Prandtl number, Schmidt number on the velocity and temperature profiles. The effects of variable viscosity of nanofluid flow over a permeable wedge embedded in saturated non-Darcy porous medium with chemical reaction and thermal radiation have been studied by James et al [11]. These authors observed that the heat transfer rate and the velocity profiles decrease with the decrease of variable viscosity parameter, whereas temperature increases. They also noticed that both velocity and temperature profiles decrease with an increase in buoyancy ratio parameter values. Chepkonga et al [12] studied fluid flow and heat transfer through a vertical cylindrical collapsible tube in presence of magnetic field and a spherical obstacle. The non linear partial differential equations governing the fluid flow has been transformed into non linear ordinary differential equations and solved simultaneously with the boundary value problem 4th order collocation MATLAB library. The authors observed that an increase in density of the obstacle in presence of transverse magnetic field slows down the velocity of the fluid and does not produce significant change on the rate of heat transfer. Mwangi et al [13] investigated the effects of variable viscosity on unsteady natural convection hydromagnetic flow past an isothermal sphere. They assumed the viscosity to be a linear function of temperature and used the finite difference method. They obtained the effects of dimensionless parameters such as magnetic parameter, Reynolds number, Grashof number, viscous variation parameter on both primary and secondary velocity profiles, temperature profile, skin friction and heat transfer.
Despite the number of studies being done on different areas of MHD in a rotating system by the aforesaid work, no study has been done on the effects of variable viscosity on unsteady natural convection hydromagnetic fluid flow over an isothermal sphere in a rotating system.

2. Mathematical Model

Unsteady two-dimensional hydromagnetic laminar free convection fluid flow over an uniformly heated sphere of radius a and centred at the origin O in a rotating system is considered in the presence of a vertical magnetic field of strength B_0. Fig. 1 illustrates the flow configuration and the coordinates system of the study. The isothermal sphere is immersed in incompressible viscous fluid with fluid viscosity as a linear function of temperature. The z'-axis is perpendicular to the $x' - y'$ plane. The x'-axis shows the rotating fluid motion and y'-axis is normal to the sphere surface. B_0 is applied in the direction parallel to z'-axis. The rotating of the sphere at uniform angular velocity Ω about the z'-axis initiates the motion of the fluid along side with the velocity velocity U_0.

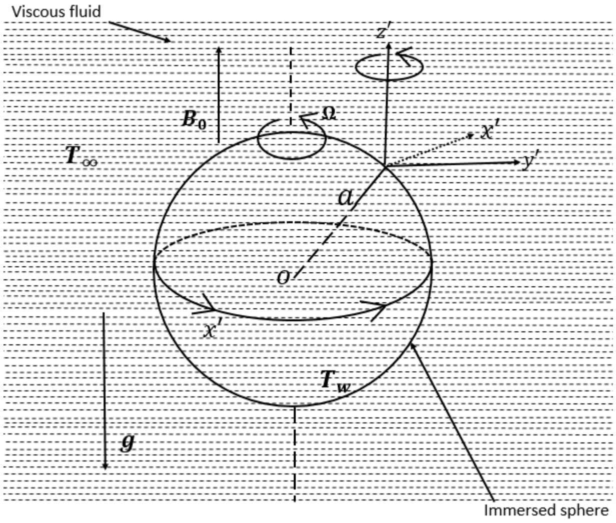

Fig. 1. Flow configuration

2.1. Governing equations in dimensional form

2.1.1. Equation of continuity

The fluid flow is considered to be incompressible, then, the expression corresponding to equation of continuity is reduced to:

$$\hat{\nabla}.\hat{q} = 0 \tag{1}$$

where $\hat{q}$ is the fluid velocity in 2-dimensions defined as $\hat{q} = u'\hat{i} + v'\hat{j}$ and $\hat{\nabla} = \hat{i}\frac{\partial}{\partial x'} + \hat{j}\frac{\partial}{\partial y'}$.
Substituting 2-dimensions form for $\hat{\nabla}$ and $\hat{q}$ in equation (1) yields:

$$\frac{\partial u'}{\partial x'} + \frac{\partial v'}{\partial y'} = 0 \tag{2}$$

2.1.2. Equation of motion

The expression of the equation of motion for rotating system is written as:

$$\rho\left(\frac{\partial \hat{q}}{\partial t'} + (\hat{q}.\hat{\nabla})\,\hat{q} + 2\hat{\Omega}\times\hat{q}\right) = -\hat{\nabla}P + \mu\nabla^2\hat{q} + \hat{F} \tag{3}$$

where $\frac{\partial \hat{q}}{\partial t'}$ is the temporal acceleration, $((\hat{q}.\hat{\nabla})\,\hat{q})$ is the convective acceleration, $2\hat{\Omega}\times\hat{q}$ is the Coriolis force, $\hat{\nabla}P$ is the pressure gradient, $\upsilon\nabla^2\hat{q}$ is the force due to viscosity and $\hat{F}$ represents the body forces vector. In two-dimensional flow in which $\hat{\Omega} = (0,0,\Omega)$ and $\hat{q} = (u', v', 0)$, the components of momentum equation with respect to x' and y'- directions are therefore given by;

Momentum equation in x'-direction

$$\frac{\partial u'}{\partial t'} + u'\frac{\partial u'}{\partial x'} + v'\frac{\partial u'}{\partial y'} - 2\Omega v' = \frac{1}{\rho}\frac{\partial}{\partial y'}\left(\mu(T)\frac{\partial u'}{\partial y'}\right) + \frac{\mu}{\rho}\left(\frac{\partial^2 u'}{\partial x'^2} + \frac{\partial^2 u'}{\partial y'^2}\right) + g\beta(T' - T_\infty)Sin\left(\frac{x'}{a}\right) - \frac{\delta_0 B_0^2 u'}{\rho} \tag{4}$$

By introducing the physical principle conservation of mass (2) in the left hand side of the equation (4), equation (4) can be written as;

$$\frac{\partial u'}{\partial t'} + \frac{\partial (u'^2)}{\partial x'} + \frac{\partial (u'v')}{\partial y'} - 2\Omega v' = \frac{1}{\rho}\frac{\partial}{\partial y'}\left(\mu(T)\frac{\partial u'}{\partial y'}\right) + \frac{\mu}{\rho}\left(\frac{\partial^2 u'}{\partial x'^2} + \frac{\partial^2 u'}{\partial y'^2}\right) + g\beta(T' - T_\infty)Sin\left(\frac{x'}{a}\right) - \frac{\delta_0 B_0^2 u'}{\rho} \tag{5}$$

Where $\mu(T)$ is the viscosity of the fluid which depends on the fluid temperature.

Momentum equation in y'-direction

$$\frac{\partial v'}{\partial t'}+u'\frac{\partial v'}{\partial x'}+v'\frac{\partial v'}{\partial y'}+2\Omega u'=\frac{\mu}{\rho}\left(\frac{\partial^2 v'}{\partial x'^2}+\frac{\partial^2 v'}{\partial y'^2}\right)-\frac{\delta_0 B_0^2 v'}{\rho} \tag{6}$$

By introducing again the physical principle conservation of mass (2) in the left hand side of the equation (6), equation (6) can be written as;

$$\frac{\partial v'}{\partial t'}+\frac{\partial (u'v')}{\partial x'}+\frac{\partial (v'^2)}{\partial y'}+2\Omega u'=\frac{\mu}{\rho}\left(\frac{\partial^2 v'}{\partial x'^2}+\frac{\partial^2 v'}{\partial y'^2}\right)-\frac{\delta_0 B_0^2 v'}{\rho} \tag{7}$$

2.1.3. Equation of energy

The energy equation is given as:

$$\frac{\partial T'}{\partial t'}+u'\frac{\partial T'}{\partial x'}+v'\frac{\partial T'}{\partial y'}=\frac{K}{\rho C_p}\left(\frac{\partial^2 T'}{\partial y'^2}\right) \tag{8}$$

Substituting (2) in (8), we have:

$$\frac{\partial T'}{\partial t'}+\frac{\partial (u'T')}{\partial x'}+\frac{\partial (v'T')}{\partial y'}=\frac{K}{\rho C_p}\left(\frac{\partial^2 T'}{\partial y'^2}\right) \tag{9}$$

Equations (2), (5), (7) and (9) are the equations governing the hydromagnetic flow problem with initial and boundary conditions defined as;

$$\begin{cases} t=0 \; : \; u'=0, \quad v'=0, \quad T'=T_\infty \\ t>0 \; : \; u'=u_0', \quad v'=0, \quad T'=T_w \quad for \quad y'=0 \\ \quad\quad\; : \; u'\longrightarrow 0, \; v'\longrightarrow 0, \; T'\longrightarrow T_\infty \quad for \quad y'\longrightarrow\infty \end{cases} \tag{10}$$

2.2. Governing equations in non-dimensional form

The non-dimensional variables used to non-dimensionalize the governing equations (2), (5), (7) and (9) are given as follows;

$x=\frac{x'}{a},\; y=\frac{Gr^{\frac{1}{4}}y'}{a},\; u=\frac{\rho a}{\mu}Gr^{-\frac{1}{2}}u',\; v=\frac{\rho a}{\mu}Gr^{-\frac{1}{2}}v',\; \theta=\frac{T'-T_\infty}{T_w-T_\infty},\; t'=\frac{U_0 t}{a}$

In addition, $\mu(T)$ is taken as a linear function of temperature and is defined by $\mu(T)=\mu_\infty[1+\gamma^*(T-T_\infty)]$ where γ^* is a constant defined as $\gamma^*=\frac{1}{\mu}(\frac{\partial\mu}{\partial T})_f$ and $\gamma=\frac{1}{\mu}(\frac{\partial\mu}{\partial T})_f(T_w-T_\infty)$ representing viscous variation parameter.
Substituting these, we get the following non-dimensional equations:

2.2.1. Continuity equation

$$\frac{\partial u}{\partial x}+Gr^{\frac{1}{4}}\frac{\partial v}{\partial y}=0 \tag{11}$$

2.2.2. Momentum equation in x-direction

$$\frac{Re\,a^2}{Gr^{\frac{1}{2}}U_0^2}\frac{\partial u}{\partial t}+\frac{\partial (u^2)}{\partial x}+Gr^{\frac{1}{4}}\frac{\partial (uv)}{\partial y}-2Er\,v=\gamma\frac{\partial\theta}{\partial y}\frac{\partial u}{\partial y}+(1+\gamma\theta)\frac{\partial^2 u}{\partial y^2}+\frac{1}{Gr^{\frac{1}{2}}}\frac{\partial^2 u}{\partial x^2}+\frac{1}{Gr^{\frac{7}{16}}}\frac{\partial^2 u}{\partial y^2}+\theta\,Sin(x)-M\,u \tag{12}$$

2.2.3. Momentum equation in y-direction

$$\frac{Re\,a^2}{Gr^{\frac{1}{2}}U_0^2}\frac{\partial v}{\partial t}+\frac{\partial (uv)}{\partial x}+Gr^{\frac{1}{4}}\frac{\partial (v^2)}{\partial y}+2Er\,u=\frac{1}{Gr^{\frac{1}{2}}}\frac{\partial^2 v}{\partial x^2}+\frac{1}{Gr^{\frac{7}{16}}}\frac{\partial^2 v}{\partial y^2}-M\,v \tag{13}$$

2.2.4. Energy equation

$$\frac{Re\,a^2}{Gr^{\frac{1}{2}}U_0^2}\frac{\partial\theta}{\partial t}+\frac{\partial (u\theta)}{\partial x}+Gr^{\frac{1}{4}}\frac{\partial (v\theta)}{\partial y}=\frac{1}{Pr\,Gr^{\frac{1}{4}}}\frac{\partial^2\theta}{\partial y^2} \tag{14}$$

Where Gr, Pr, Re, M and Er are called dimensionless parameters, which are defined as follows:

$$G_r=\frac{g\beta(T_w-T_\infty)a^3}{v^2},\quad P_r=\frac{\mu C_p}{K},\quad Re=\frac{\rho a U_0}{\mu},\quad M=\frac{\delta_0 B_0^2 a^2}{\mu Gr^{\frac{1}{2}}},\quad \text{and}\quad Er=\frac{\rho\Omega a^2}{\mu Gr^{\frac{1}{2}}}$$

The initial and boundary conditions defined in equation (10) become:

$$\begin{cases} t=0 \; : \; u=0, \quad v=0, \quad \theta=0 \\ t>0 \; : \; u=u_0, \quad v=0, \quad \theta=1 \quad for \quad y=0 \\ \quad\quad\; : \; u\longrightarrow 0, \; v\longrightarrow 0, \; \theta\longrightarrow 0 \quad for \quad y\longrightarrow\infty \end{cases} \tag{15}$$

2.3. Method of solution

We used the Direct Numerical Scheme (DNS) and applied it to solve the equations (12), (13), (14) subjected to the initial and boundary conditions (15). Considering the following transformations $x = X$, $y = Y$, $U = \frac{u}{x}$, $V = \frac{v}{y}$ then $u = UX$ and $v = VY$.
Substituting these transformations in equations (12), (13), (14) yields:

$$\frac{Re\,a^2}{Gr^{\frac{1}{2}}U_0^2}\frac{\partial U}{\partial t}+X\frac{\partial (U^2)}{\partial X}+2U^2+Gr^{\frac{1}{4}}\left(UV+YV\frac{\partial U}{\partial Y}+YU\frac{\partial V}{\partial Y}\right)-2Er\,V\frac{Y}{X}=\gamma\frac{\partial \theta}{\partial Y}\frac{\partial U}{\partial Y}+(1+\gamma\theta)\frac{\partial^2 U}{\partial Y^2}+\frac{1}{Gr^{\frac{1}{2}}}\frac{\partial^2 U}{\partial X^2}$$

$$+\frac{1}{Gr^{\frac{7}{16}}}\frac{\partial^2 U}{\partial Y^2}+\theta\frac{SinX}{X}-MU \tag{16}$$

Equation (16) is the momentum equation in x-direction when applying the DNS.

$$\frac{Re\,a^2}{Gr^{\frac{1}{2}}U_0^2}\frac{\partial V}{\partial t}+UV+XV\frac{\partial U}{\partial X}+XU\frac{\partial V}{\partial X}+Gr^{\frac{1}{4}}\left(2V^2+Y\frac{\partial (V^2)}{\partial Y}\right)+2Er\,U\frac{X}{Y}=\frac{1}{Gr^{\frac{1}{2}}}\frac{\partial^2 V}{\partial X^2}+\frac{1}{Gr^{\frac{9}{16}}}\frac{\partial^2 V}{\partial Y^2}-MV \tag{17}$$

Equation (17) is the momentum equation in y-direction when applying the DNS.

$$\frac{Re\,a^2}{Gr^{\frac{1}{2}}U_0^2}\frac{\partial \theta}{\partial t}+UX\frac{\partial \theta}{\partial X}+\theta U+X\theta\frac{\partial U}{\partial X}+Gr^{\frac{1}{4}}\left(YV\frac{\partial \theta}{\partial Y}+\theta V+Y\theta\frac{\partial V}{\partial Y}\right)=\frac{1}{Pr\,Gr^{\frac{1}{4}}}\frac{\partial^2 \theta}{\partial Y^2} \tag{18}$$

Equation (18) is the energy equation when applying the DNS.
The physical quantities of interest are the surface skin-friction coefficient C_f and the local Nusselt number Nu that are respectively defined by:

$$C_f=\frac{2\tau_w}{\rho U_\infty^2} \quad \text{and} \quad Nu=\frac{a\,q_w}{K(T_w-T_\infty)} \tag{19}$$

$$where \quad \tau_w=\left(\mu(T)\frac{\partial u'}{\partial y'}\right)_{y'=0} \quad \text{and} \quad q_w=-K\left(\frac{\partial T'}{\partial y'}\right)_{y'=0} \tag{20}$$

Equations defined in (19) written in non-dimensional form are given by;

$$C_f=2(1+\gamma)Gr^{\frac{1}{4}}\left(\frac{\partial u}{\partial y}\right)_{y=0} \tag{21}$$

$$Nu=-Gr^{\frac{1}{4}}\left(\frac{\partial \theta}{\partial y}\right)_{y=0} \tag{22}$$

Applying the DNS, we obtain:

$$C_f=2(1+\gamma)Gr^{\frac{1}{4}}X\left(\frac{\partial U}{\partial Y}\right)_{Y=0} \tag{23}$$

$$Nu=-Gr^{\frac{1}{4}}\left(\frac{\partial \theta}{\partial Y}\right)_{Y=0} \tag{24}$$

And equation (15) can be written as;
Initial condition:

$$t=0\ :\ U=0,\ \ V=0,\ \ \theta=0 \tag{25}$$

Boundary conditions:

$$\begin{cases} t>0\ :\ U=u_0, & V=0, & \theta=1 & for \quad Y=0 \\ \quad\quad\ :\ U\longrightarrow 0, & V\longrightarrow 0, & \theta\longrightarrow 0 & for \quad Y\longrightarrow\infty \end{cases} \tag{26}$$

The solutions of the equations (16), (17), (18), (23), (24) are obtained using the finite difference method. Equation (16) written in terms of finite difference form yields;

$$\frac{Re\,a^2}{Gr^{\frac{1}{2}}U_0^2}\left[\frac{U_{i,j}^{k+1}-U_{i,j}^{k}}{\Delta t}\right]+X_i\left[\frac{\left(U_{i+1,j}^{k}\right)^2-\left(U_{i-1,j}^{k}\right)^2}{2(\Delta X)}\right]+2\left[\frac{U_{i+1,j}^{k}+U_{i-1,j}^{k}}{2}\right]^2+$$
$$Gr^{\frac{1}{4}}\left\{\left[\frac{U_{i+1,j}^{k}+U_{i-1,j}^{k}}{2}\right]\left[\frac{V_{i,j+1}^{k}+V_{i,j-1}^{k}}{2}\right]+Y_j\left[\frac{V_{i,j+1}^{k}+V_{i,j-1}^{k}}{2}\right]\left[\frac{U_{i,j+1}^{k}-U_{i,j-1}^{k}}{2(\Delta Y)}\right]\right.$$
$$\left.+Y_j\left[\frac{U_{i+1,j}^{k}+U_{i-1,j}^{k}}{2}\right]\left[\frac{V_{i,j+1}^{k}-V_{i,j-1}^{k}}{2(\Delta Y)}\right]\right\}-2Er\frac{Y_j}{X_i}\left[\frac{V_{i,j+1}^{k}+V_{i,j-1}^{k}}{2}\right]=\gamma\left[\frac{\theta_{i,j+1}^{k}-\theta_{i,j-1}^{k}}{2(\Delta Y)}\right]$$
$$\left[\frac{U_{i,j+1}^{k}-U_{i,j-1}^{k}}{2(\Delta Y)}\right]+\left(1+\gamma\theta_{i,j}^{k}\right)\left[\frac{U_{i,j+1}^{k}-2U_{i,j}^{k}+U_{i,j-1}^{k}}{(\Delta Y)^2}\right]+\frac{1}{Gr^{\frac{1}{2}}}\left[\frac{U_{i+1,j}^{k}-2U_{i,j}^{k}+U_{i-1,j}^{k}}{(\Delta X)^2}\right]$$
$$+\frac{1}{Gr^{\frac{7}{16}}}\left[\frac{U_{i,j+1}^{k}-2U_{i,j}^{k}+U_{i,j-1}^{k}}{(\Delta Y)^2}\right]+\theta_{i,j}^{k}\left[\frac{Sin X_i}{X_i}\right]-MU_{i,j}^{k} \tag{27}$$

Let $w=\frac{Re\,a^2}{Gr^{\frac{1}{2}}U_0^2}$ and making $U_{i,j}^{k+1}$ the subject of the formula, the equation (27) can be rewritten as:

$$U_{i,j}^{k+1}=U_{i,j}^{k}+\frac{\Delta t}{w}\left\{\gamma\left[\frac{\theta_{i,j+1}^{k}-\theta_{i,j-1}^{k}}{2(\Delta Y)}\right]\left[\frac{U_{i,j+1}^{k}-U_{i,j-1}^{k}}{2(\Delta Y)}\right]+\left(1+\gamma\theta_{i,j}^{k}\right)\left[\frac{U_{i,j+1}^{k}-2U_{i,j}^{k}+U_{i,j-1}^{k}}{(\Delta Y)^2}\right]\right.$$
$$+\frac{1}{Gr^{\frac{1}{2}}}\left[\frac{U_{i+1,j}^{k}-2U_{i,j}^{k}+U_{i-1,j}^{k}}{(\Delta X)^2}\right]+\frac{1}{Gr^{\frac{7}{16}}}\left[\frac{U_{i,j+1}^{k}-2U_{i,j}^{k}+U_{i,j-1}^{k}}{(\Delta Y)^2}\right]+\theta_{i,j}^{k}\left[\frac{Sin X_i}{X_i}\right]$$
$$\left.-MU_{i,j}^{k}-X_i\left[\frac{\left(U_{i+1,j}^{k}\right)^2-\left(U_{i-1,j}^{k}\right)^2}{2(\Delta X)}\right]-2\left[\frac{U_{i+1,j}^{k}+U_{i-1,j}^{k}}{2}\right]^2+2Er\frac{Y_j}{X_i}\left[\frac{V_{i,j+1}^{k}+V_{i,j-1}^{k}}{2}\right]\right\} \tag{28}$$
$$-\frac{\Delta t\,Gr^{\frac{1}{4}}}{w}\left\{\left[\frac{U_{i+1,j}^{k}+U_{i-1,j}^{k}}{2}\right]\left[\frac{V_{i,j+1}^{k}+V_{i,j-1}^{k}}{2}\right]+Y_j\left[\frac{V_{i,j+1}^{k}+V_{i,j-1}^{k}}{2}\right]\left[\frac{U_{i,j+1}^{k}-U_{i,j-1}^{k}}{2(\Delta Y)}\right]\right.$$
$$\left.+Y_j\left[\frac{U_{i+1,j}^{k}+U_{i-1,j}^{k}}{2}\right]\left[\frac{V_{i,j+1}^{k}-V_{i,j-1}^{k}}{2(\Delta Y)}\right]\right\}$$

Equation (17) written in terms of finite difference form yields;

$$\frac{Re\,a^2}{Gr^{\frac{1}{2}}U_0^2}\left[\frac{V_{i,j}^{k+1}-V_{i,j}^{k}}{(\Delta t)}\right]+\left[\frac{U_{i+1,j}^{k}+U_{i-1,j}^{k}}{2}\right]\left[\frac{V_{i,j+1}^{k}+V_{i,j-1}^{k}}{2}\right]+X_i\left\{\left[\frac{V_{i,j+1}^{k}+V_{i,j+1}^{k}}{2}\right]\left[\frac{U_{i+1,j}^{k}-U_{i-1,j}^{k}}{2(\Delta X)}\right]\right.$$
$$\left.+\left[\frac{U_{i+1,j}^{k}+U_{i-1,j}^{k}}{2}\right]\left[\frac{V_{i+1,j}^{k}-V_{i-1,j}^{k}}{2(\Delta X)}\right]\right\}+Gr^{\frac{1}{4}}\left\{2\left[\frac{V_{i,j+1}^{k}+V_{i,j-1}^{k}}{2}\right]^2+Y_j\left[\frac{\left(V_{i,j+1}^{k}\right)^2-\left(V_{i,j-1}^{k}\right)^2}{2(\Delta Y)}\right]\right\}$$
$$+2Er\frac{X_i}{Y_j}\left[\frac{U_{i+1,j}^{k}+U_{i-1,j}^{k}}{2}\right]=\frac{1}{Gr^{\frac{1}{2}}}\left[\frac{V_{i+1,j}^{k}-2V_{i,j}^{k}+V_{i-1,j}^{k}}{(\Delta X)^2}\right]+\frac{1}{Gr^{\frac{7}{16}}}\left[\frac{V_{i,j+1}^{k}-2V_{i,j}^{k}+V_{i,j-1}^{k}}{(\Delta Y)^2}\right]-MV_{i,j}^{k} \tag{29}$$

Making $V_{i,j}^{k+1}$ the subject of the formula in equation (29), then

$$V_{i,j}^{k+1}=V_{i,j}^{k}+\frac{\Delta t}{w}\left\{\frac{1}{Gr^{\frac{1}{2}}}\left[\frac{V_{i+1,j}^{k}-2V_{i,j}^{k}+V_{i-1,j}^{k}}{(\Delta X)^2}\right]+\frac{1}{Gr^{\frac{7}{16}}}\left[\frac{V_{i,j+1}^{k}-2V_{i,j}^{k}+V_{i,j-1}^{k}}{(\Delta Y)^2}\right]-MV_{i,j}^{k}\right.$$
$$\left.-\left[\frac{U_{i+1,j}^{k}+U_{i-1,j}^{k}}{2}\right]\left[\frac{V_{i,j+1}^{k}+V_{i,j-1}^{k}}{2}\right]-2Er\frac{X_i}{Y_j}\left[\frac{U_{i+1,j}^{k}+U_{i-1,j}^{k}}{2}\right]\right\}$$
$$-\frac{X_i\,\Delta t}{w}\left\{\left[\frac{V_{i,j+1}^{k}+V_{i,j+1}^{k}}{2}\right]\left[\frac{U_{i+1,j}^{k}-U_{i-1,j}^{k}}{2(\Delta X)}\right]+\left[\frac{U_{i+1,j}^{k}+U_{i-1,j}^{k}}{2}\right]\left[\frac{V_{i+1,j}^{k}-V_{i-1,j}^{k}}{2(\Delta X)}\right]\right\} \tag{30}$$
$$-\frac{Gr^{\frac{1}{4}}\,\Delta t}{w}\left\{2\left[\frac{V_{i,j+1}^{k}+V_{i,j-1}^{k}}{2}\right]^2+Y_j\left[\frac{\left(V_{i,j+1}^{k}\right)^2-\left(V_{i,j-1}^{k}\right)^2}{2(\Delta Y)}\right]\right\}$$

Equation (18) written in terms of finite difference form yields;

$$\frac{Re\,a^2}{Gr^{\frac{1}{2}}U_0^2}\left[\frac{\theta_{i,j}^{k+1}-\theta_{i,j}^{k}}{2(\Delta t)}\right]+X_i\left[\frac{U_{i+1,j}^{k}+U_{i-1,j}^{k}}{2}\right]\left[\frac{\theta_{i+1,j}^{k}-\theta_{i-1,j}^{k}}{2(\Delta X)}\right]+\theta_{i,j}^{k}\left[\frac{U_{i+1,j}^{k}+U_{i-1,j}^{k}}{2}\right]$$

$$+X_i\theta_{i,j}^{k}\left[\frac{U_{i+1,j}^{k}-U_{i-1,j}^{k}}{2(\Delta X)}\right]+Gr^{\frac{1}{4}}\left\{Y_j\left[\frac{V_{i,j+1}^{k}+V_{i,j-1}^{k}}{2}\right]\left[\frac{\theta_{i,j+1}^{k}-\theta_{i,j-1}^{k}}{2(\Delta Y)}\right]\right.$$

$$\left.+\theta_{i,j}^{k}\left[\frac{V_{i,j+1}^{k}+V_{i,j-1}^{k}}{2}\right]+Y_j\theta_{i,j}^{k}\left[\frac{V_{i,j+1}^{k}-V_{i,j-1}^{k}}{2(\Delta Y)}\right]\right\}=\frac{1}{Pr\,Gr^{\frac{1}{4}}}\left[\frac{\theta_{i,j+1}^{k}-2\theta_{i,j}^{k}+\theta_{i,j-1}^{k}}{(\Delta Y)^2}\right] \tag{31}$$

Making $\theta_{i,j}^{k+1}$ the subject of the formula in equation (31), then

$$\theta_{i,j}^{k+1}=\theta_{i,j}^{k}+\frac{\Delta t}{w}\left\{\frac{1}{Pr\,Gr^{\frac{1}{4}}}\left[\frac{\theta_{i,j+1}^{k}-2\theta_{i,j}^{k}+\theta_{i,j-1}^{k}}{(\Delta Y)^2}\right]-X_i\left[\frac{U_{i+1,j}^{k}+U_{i-1,j}^{k}}{2}\right]\left[\frac{\theta_{i+1,j}^{k}-\theta_{i-1,j}^{k}}{2(\Delta X)}\right]-\theta_{i,j}^{k}\left[\frac{U_{i+1,j}^{k}+U_{i-1,j}^{k}}{2}\right]\right.$$
$$\left.-X_i\theta_{i,j}^{k}\left[\frac{U_{i+1,j}^{k}-U_{i-1,j}^{k}}{2(\Delta X)}\right]\right\}-\frac{\Delta t\,Gr^{\frac{1}{4}}}{w}\left\{Y_j\left[\frac{V_{i,j+1}^{k}+V_{i,j-1}^{k}}{2}\right]\left[\frac{\theta_{i,j+1}^{k}-\theta_{i,j-1}^{k}}{2(\Delta Y)}\right]+\theta_{i,j}^{k}\left[\frac{V_{i,j+1}^{k}+V_{i,j-1}^{k}}{2}\right]\right.$$
$$\left.+Y_j\theta_{i,j}^{k}\left[\frac{V_{i,j+1}^{k}-V_{i,j-1}^{k}}{2(\Delta Y)}\right]\right\} \tag{32}$$

The finite difference corresponding to the surface skin friction coefficient and the rate of heat transfer are defined by;

$$C_f=2(1+\gamma)Gr^{\frac{1}{4}}X_i\left(\frac{U_{i,j+1}^{k}-U_{i,j-1}^{k}}{2(\Delta Y)}\right)_{Y=0} \tag{33}$$

$$Nu=-Gr^{\frac{1}{4}}\left(\frac{\theta_{i,j+1}^{k}-\theta_{i,j-1}^{k}}{2(\Delta Y)}\right)_{Y=0} \tag{34}$$

Therefore, equations (28), (30), (32), (33) and (34) are the final equations.

3. Results and discussions

To understand the physical effects of variable viscosity on an isothermal sphere in a rotating system, we analyzed the effect of dimensionless parameters on velocity and temperature profiles of the fluid flow, heat transfer and skin friction on the sphere surface. The numerical results of equations (28), (30), (32), (33) and (34) are implemented in the MATLAB computer program. The program has displayed graphs of the effects of viscous variation parameter (γ), Grashof number (Gr), Magnetic parameter (M) and rotational parameter (Er) on primary and secondary velocity profiles, temperature profiles, surface skin friction and heat transfer. It is noticed that the secondary velocity values are consistently negative indicating that back flow are generated in the secondary flow for all values of dimensionless parameters of interest. These graphs are represented and are discussed below.

3.1. Effects of viscous variation parameter

From Fig. 2, it is observed that an increase in viscous variation parameter leads to a decrease in primary velocity profiles (U). Near the sphere surface, the velocity profiles increase up to a certain point representing the local maximum velocity and then decrease gradually and finally approach to zero. The reason the velocity of the fluid at a sphere boundary has to be the same as that of the boundary itself and the layer of the fluid which can slip due to the presence of the viscous force away from the boundary surface undergoes a retardation with the adjacent layers. Increase in viscous variation parameter means an increase fluid viscosity which tends to retard the motion of the fluid, consequently, the primary velocity profiles (U) of the fluid decrease.

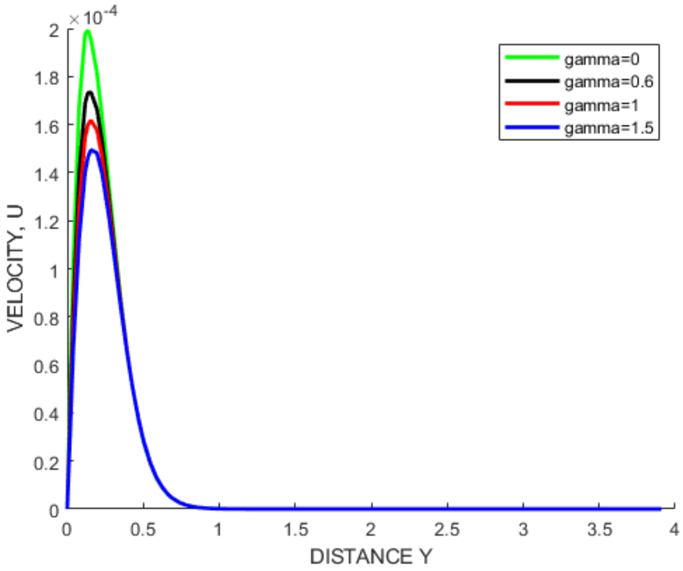

Fig. 2. Primary velocity (U) varying viscous variation parameter, γ

From Fig. 3, it is observed that an increase in viscous variation parameter reveals the existence of back flow that does not change in profiles due to effect of buoyancy forces and the pressure that reacts to the surrounding environment on the sphere boundary.

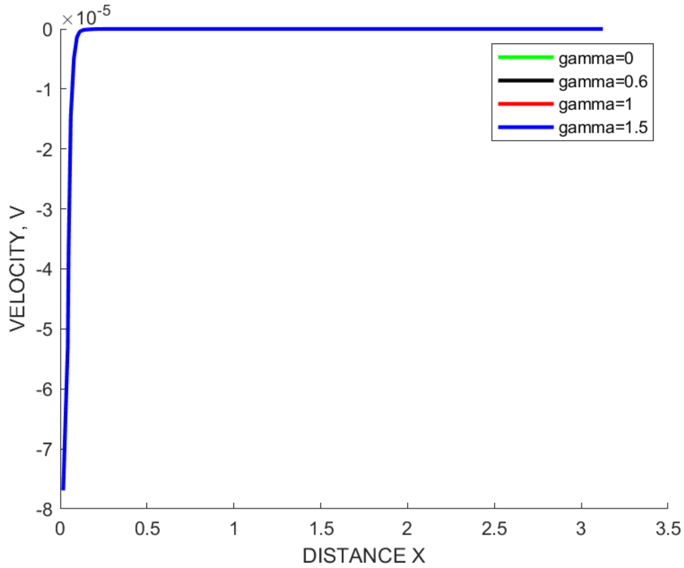

Fig. 3. Secondary velocity (V) varying viscous variation parameter, γ

Fig. 4 shows the effects of viscous variation parameter on the surface skin friction of the sphere. As the skin friction varies proportionally to the viscous variation parameter, therefore, increase in viscous variation parameter leads to an increase in surface skin friction as shown in figure below.

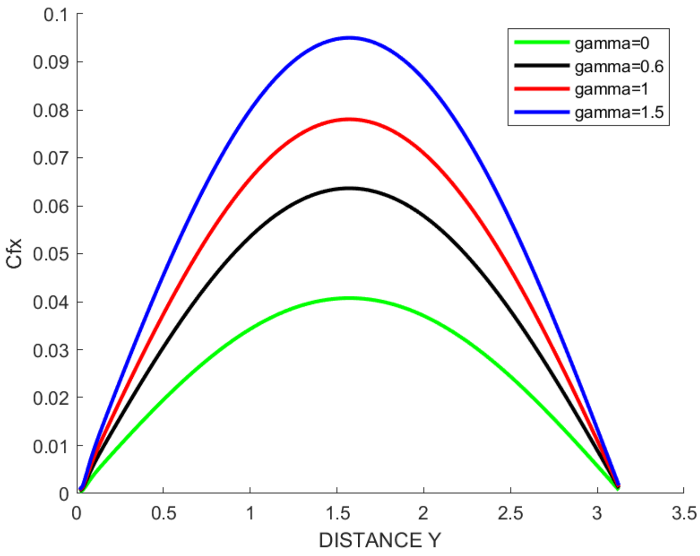

Fig. 4. Surface skin friction varying viscous variation parameter, γ

From Fig. 5, it is observed that an increase in viscous variation parameter leads to the decrease in temperature profile which drops to zero according to the boundary condition. Because an increase in viscous variation parameter leads to an increase in viscous force which results to a decrease in fluid temperature. Therefore, an increase in viscous variation parameter leads to a decrease in temperature profile.

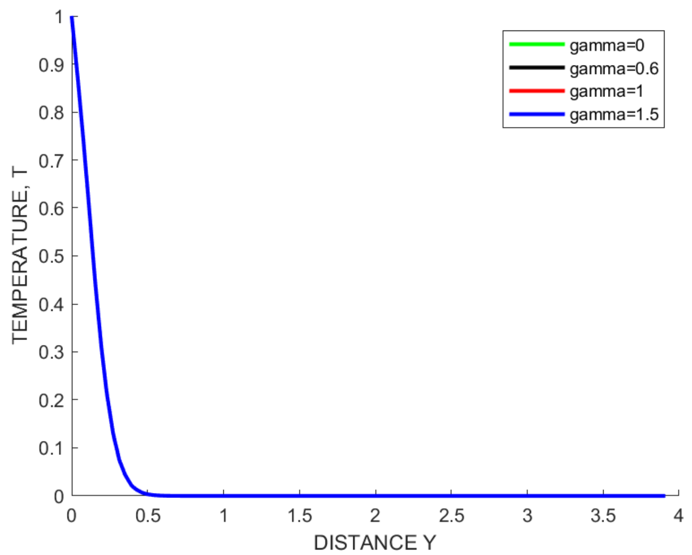

Fig. 5. Temperature profile (T) varying viscous variation parameter, γ

From Fig. 6, it is noticed that an increase in viscous variation parameter leads to a decrease in heat transfer. As an increase in viscous variation parameter leads to a decrease in fluid temperature, then a decrease in fluid temperature leads to a decrease in temperature gradient which reduces in rate of heat transfer. Consequently, the heat transfer decreases when the viscous variation parameter increases.

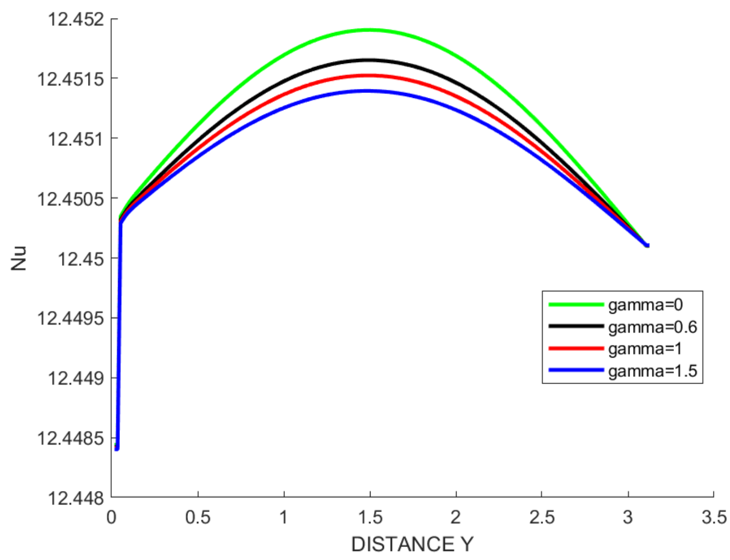

Fig. 6. Heat transfer varying viscosity variation parameter, γ

3.2. Effects of Grashof number

From Fig. 7, it is noticed that an increase in Grashof number leads to an increase in primary velocity profiles (U). As Grashof number gives relative importance of buoyancy force to viscous force, then an increase in Grashof number leads to a decrease in viscous force, consequently leading to an increase in the fluid velocity.

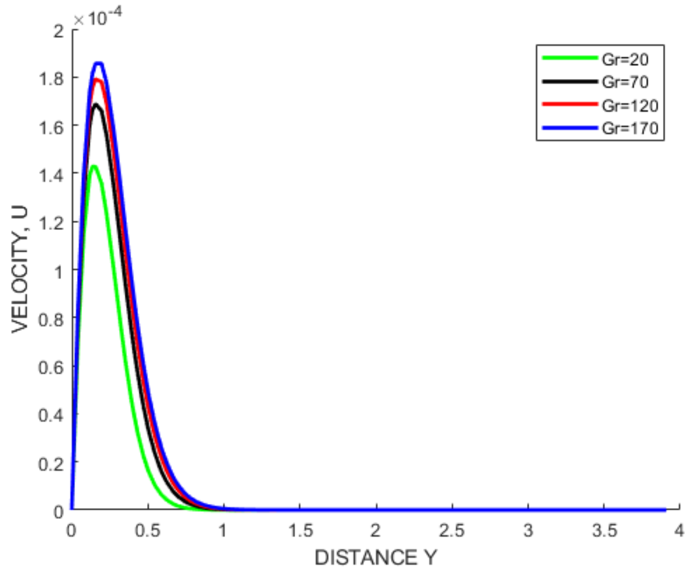

Fig. 7. Primary velocity (U) varying Grashof number, Gr

Fig. 8 shows the secondary velocity profiles (V) decrease slightly in magnitude with an increase in Grashof number due to the circular motion displayed by the fluid, the buoyancy forces and the change in pressure gradient on the boundary of the sphere.

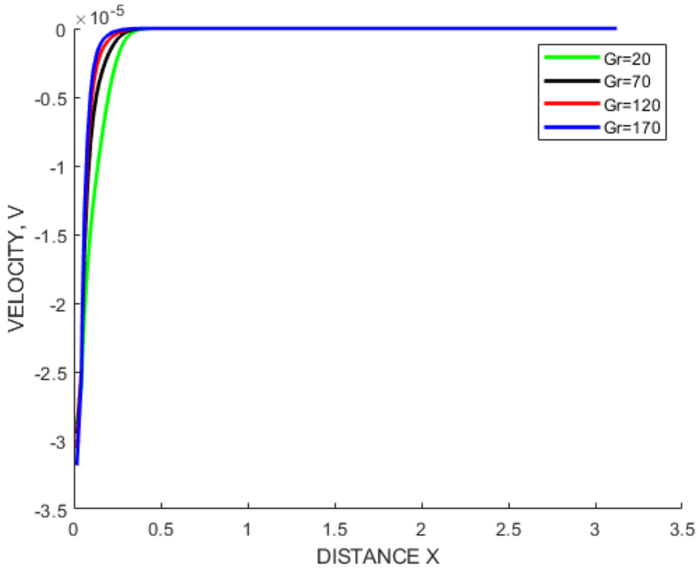

Fig. 8. Secondary velocity (V) varying Grashof number, Gr

Fig. 9 shows a slight increase in the temperature profiles when Grashof number increases, that leads to a decrease in the viscous force.

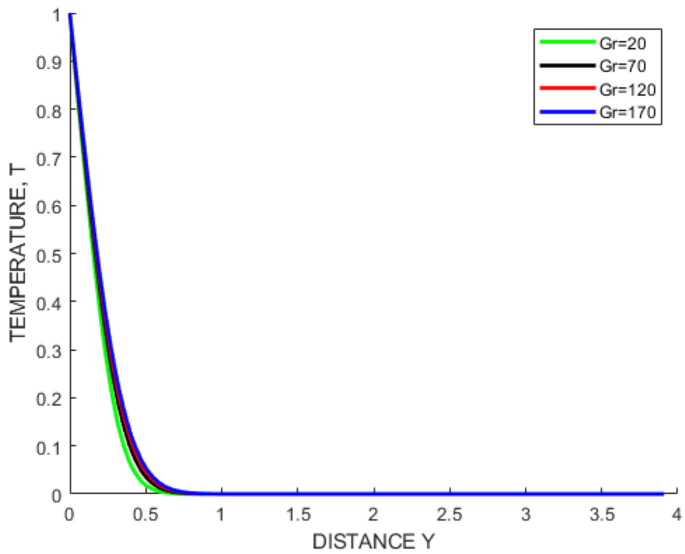

Fig. 9. Temperature profiles (T) varying Grashof number, Gr

Fig. 10 shows the surface skin friction increases with an increase in Grashof number resulting to an increase in fluid velocity and in fluid velocity gradient.

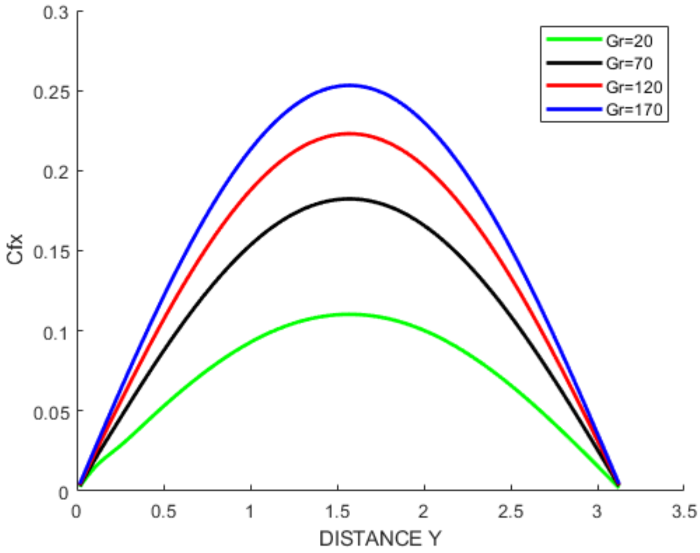

Fig. 10. Surface skin-friction varying Grashof number, *Gr*

It is noticed from Fig. 11 that whenever Grashof number increases, then the rate of heat transfer is increased but the thermal boundary layers thickness is the same. An increase in Grashof number results to an increase in fluid temperature and an increase in fluid temperature leads to an increase in temperature gradient of the fluid which implies to an increase in the heat transfer. Consequently, an increase in Grashof number leads to an increase in heat transfer profiles.

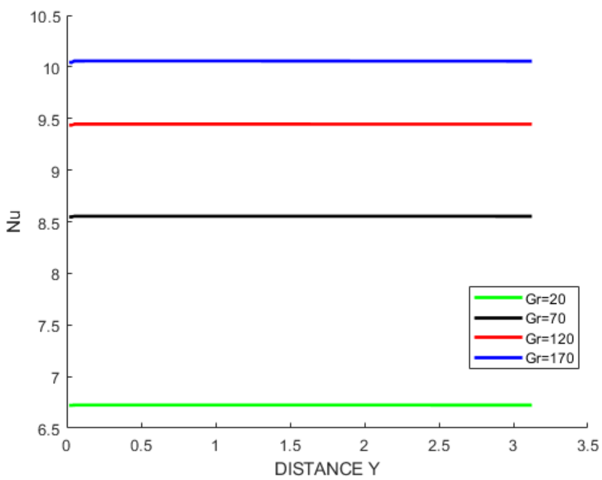

Fig. 11. Heat transfer varying Grashof number, *Gr*

3.3. Effects of Magnetic parameter

From Fig. 12, an increase in Magnetic parameter leads to a decrease in primary velocity profiles (U). Since transverse uniform magnetic field is applied, then the electrical currents are induced in the fluid. The interaction between these induced currents and applied magnetic field produces the Lorentz force that tends to delay the fluid motion. Therefore, an increase in Magnetic parameter leads to a decrease in primary velocity profiles (U).

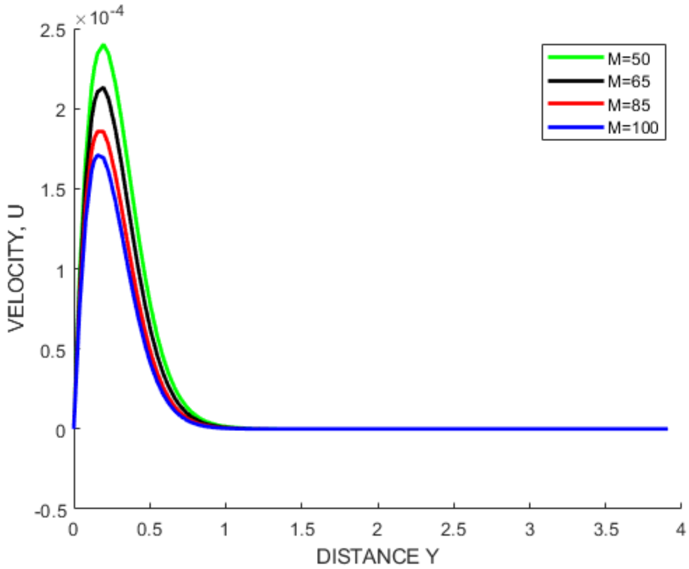

Fig. 12. Primary velocity (U) varying Magnetic parameter, M

Fig. 13 shows that the secondary velocity profiles (V) decrease in magnitude when the values of Magnetic parameter increase. This is due to the presence of Lorentz forces which undergo a retardation of the fluid motion.

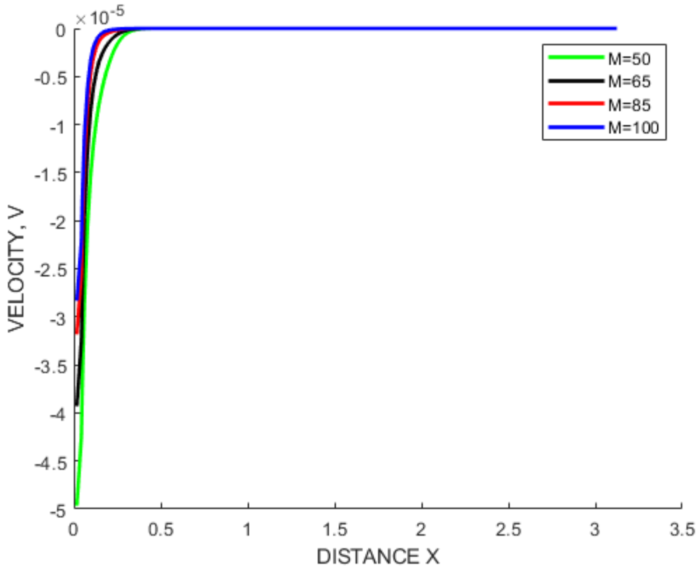

Fig. 13. Secondary velocity (V) varying Magnetic parameter, M

In Fig. 14, it is observed that an increase in Magnetic parameter leads to a decrease in surface skin friction. This is due to an increase in Magnetic parameter results to a decrease in fluid velocity which leads to a decrease in velocity gradient.

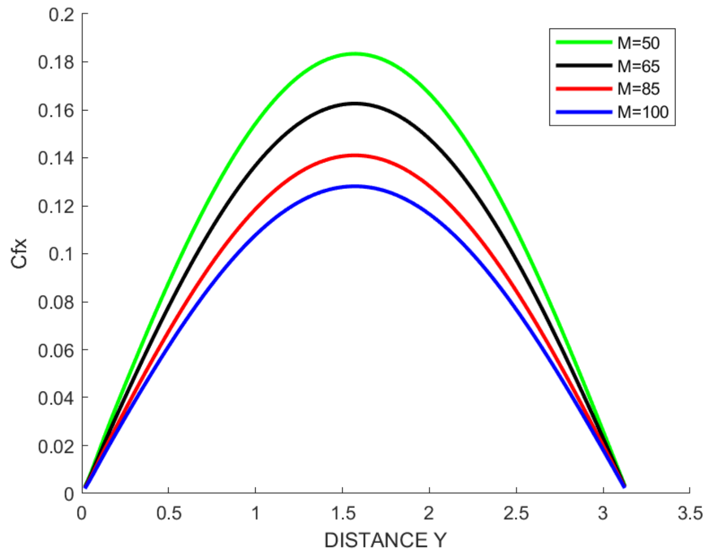

Fig. 14. Surface skin friction varying Magnetic parameter, M

From Fig. 15, an increase in Magnetic parameter leads to a decrease in heat transfer. Increase in Magnetic parameter results to an increase in the Lorentz force which implies to a decrease in the temperature gradient of the fluid. Therefore, leading to a decrease in heat transfer when the Magnetic parameter increases.

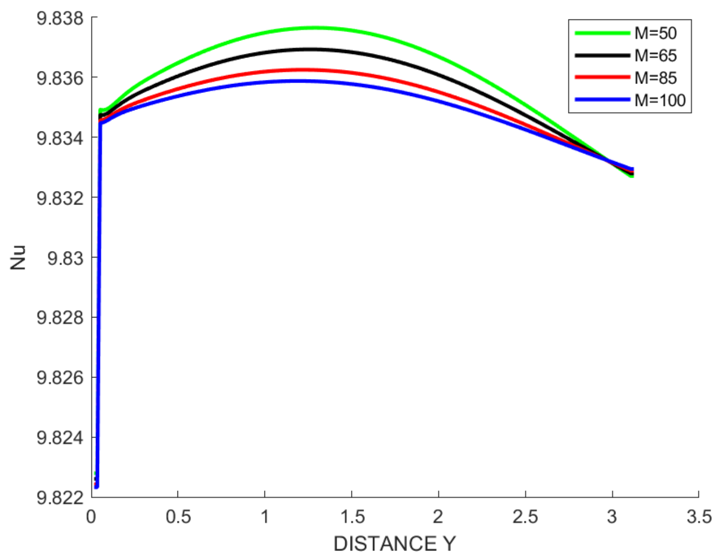

Fig. 15. Heat transfer varying Magnetic parameter, M

3.4. Effects of rotational parameter

From Fig. 16, it is observed that an increase in rotational parameter leads to a decrease in primary velocity profiles (U) while in Fig. 17, it leads to an increase in magnitude of the secondary velocity profiles (V). The deceleration of primary velocity profiles (U) observed during the rotating system has been influenced by a high fluid viscosity due to the laminar characteristics of the fluid within the boundary layer and the resistance of the viscous force provided from the fluid layers which may slip in the fluid. In addition, it is noticed that the secondary flow is an order of magnitude greater than the primary flow. Thus, the secondary flow plays a compensation role in the rotating system and reduces the effects of viscous force during the rotating system process in order to enhance the fluid motion.

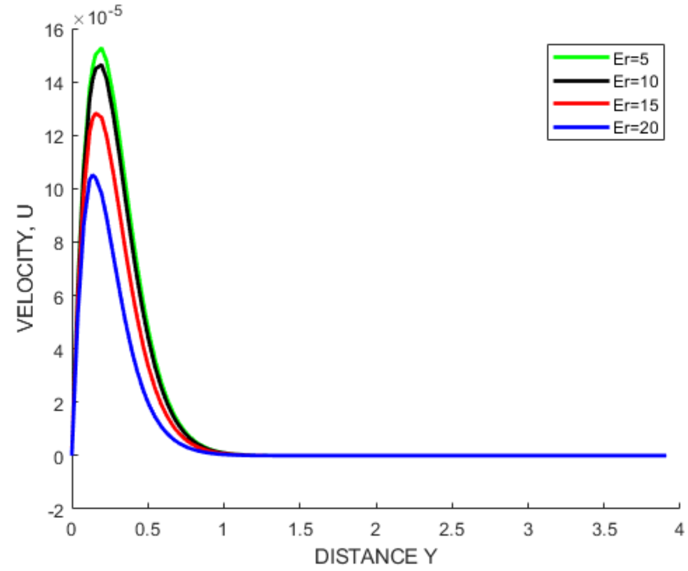

Fig. 16. Primary velocity (U) varying rotational parameter, Er

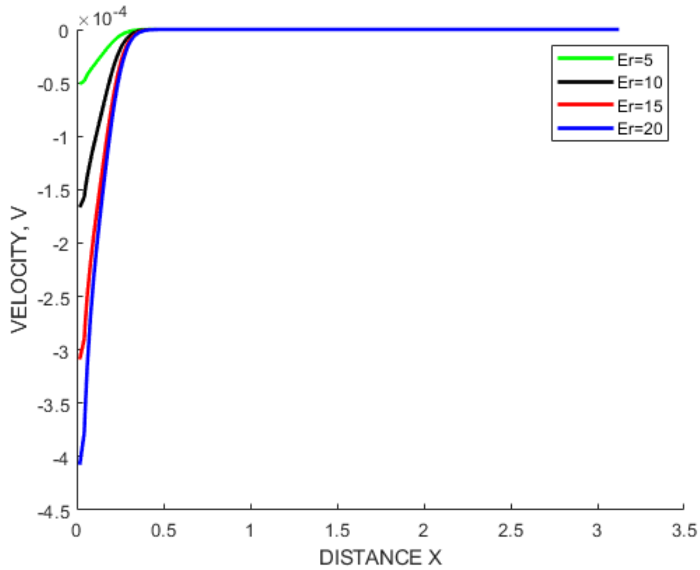

Fig. 17. Secondary velocity (V) varying rotational parameter, Er

Fig. 18 shows an increase in rotational parameter leads to a decrease in surface skin friction. Since the surface skin friction is evaluated along x-axis, it is obvious that the surface skin friction be in a relationship with respect to the primary velocity profiles which decrease along x-axis. As a result, the velocity gradient decreases which results to a decrease in surface skin friction.

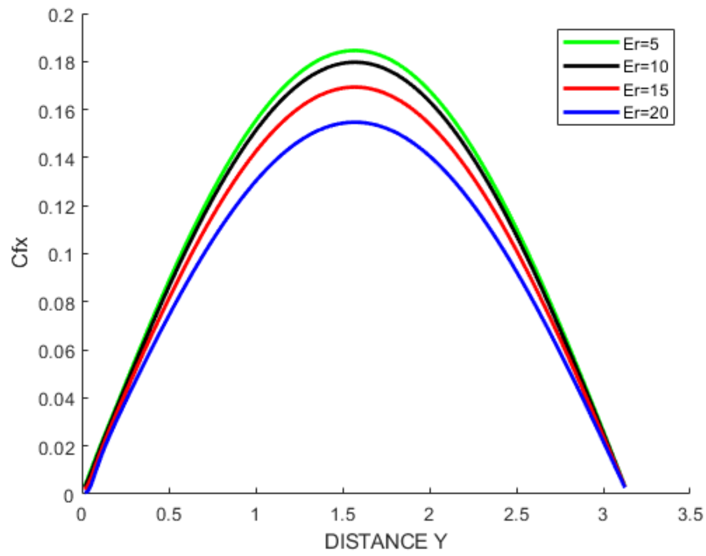

Fig. 18. Surface skin friction varying rotational parameter, Er

From Fig. 19, it is observed that an increase in rotational parameter leads to an increase in heat transfer. The velocity distribution of the rotating system increases, leading to a rise in fluid temperature. Therefore, an increase in temperature of the fluid leads to an increase in temperature gradient which increases in heat transfer as shown in the figure below.

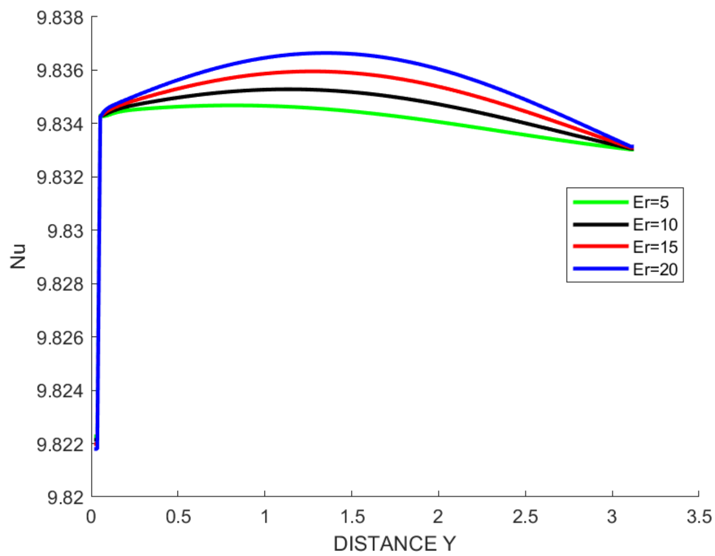

Fig. 19. Heat transfer varying rotational parameter, Er

4. Conclusions

The effects of variable viscosity dependent of temperature on unsteady natural convection hydromagnetic fluid flow over an isothermal sphere in a rotating system was numerically investigated. First of all, we found that an increase in viscous variation parameter led to an increase in surface skin friction which consequently led to a decrease in primary velocity profiles, temperature profile, heat transfer and revealed the existence in back flow. Secondly, an increase in Grashof number led to an increase in primary velocity profiles, surface skin friction, heat transfer, a slight change in temperature but led to a decrease in magnitude of secondary velocity profiles. Thirdly, an increase in Magnetic parameter led to a decrease in magnitude of secondary velocity profiles, primary velocity profiles, surface skin friction and heat transfer. Finally, an increase in rotational parameter led to an increase in magnitude of secondary

velocity profiles, heat transfer but led to a decrease in primary velocity profiles and surface skin friction.
This study can find applications in epidemiology by playing a vital role for instance in the basic understanding, diagnosis and treatment of many diseases such as cardiovascular, renal, diabetic, etc. Because the blood laminar flow is the normal situation for blood flow throughout most of the circulatory system and blood flow in arteries is dominated by unsteady flow phenomena. This can also help in understanding the complex flow in the design of power generators, geophysical fluids dynamics due to their wide and possible applications.

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