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Fluid flow and heat transfer through a vertical cylindrical collapsible tube in the presence of magnetic field and an obstacle

Research Article

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Abstract: In this paper, fluid flow and heat transfer through a Collapsible tube that is vertical with a spherical obstacle and Magnetic fields applied perpendicularly to the main flow has been investigated. The governing equations for this flow are the equations of continuity, motion and energy. The fluid flow through a Collapsible tube present a complex phenomenon due to the interaction of flowing fluid and the tube, hence the equations governing the flow are nonlinear partial differential equations. These equations have been transformed into non-linear ordinary differential equations by introducing a similarity transformation. The resulting equation from the similarity transformation is solved numerically using the Collocation method. The collocation method was implemented in MATLAB by invoking th bvp4c function to obtain the profiles. The simulation results has been presented in form of tables and graphs and also discussed. The effects of varying the Reynolds number, Hartmann number, Eckert number, Unsteadiness parameter, Prandtl number, Grashof number and Thrust constant on fluid temperature , fluid velocity and the rate of heat transfer have been determined. Variation in the various parameters is observed to change the fluid primary velocity, temperature and the rate of heat transfer. This results finds application in physical, biological and applied sciences.

MSC: 80Axx • 92Bxx

Keywords: Collapsible-tube • Similarity-transformation • Unsteadiness • Transmural-pressure

1. Introduction

Cardiovascular diseases have been found to be the most causative of high mortality rates in both developed and developing countries, particularly in Africa. The study of fluid flowing in a human system is very vital in the quest of understanding the nature of complexity of blood movement. The mechanical characteristics of vessel walls are the causes of the many recorded cases of cardiovascular diseases. Collapsible tubes are easily deformed by the change in pressure between the inside of the vessel or tube and the outside.

Studies that are related to the transient fluid flows that are produced by simple wall motion have attracted lot of interest for several years due to its practical importance in understanding engineering and physiological flow problems. The entire tube in human body are flexible and also made of collapsible walls. Collapsing is caused by change in pressure referred to as transmural pressure. Mathematical modeling of this kind of flow provides important information

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to medical practitioners in order to understand the complexity of collapsible tube flows. Research done by Marzo and Luo[1] established that increase in external pressure up to a certain critical value makes the structure buckle. Makinde [2] investigated flow in Collapsible tubes, he used pertubation technique and Hermite-Pade approximations. It was established that temperature decreases with increase in prandtl number. Luo et al [3] investigated the unsteady behavior and linear stability of the flow in a Collapsible tube by using a fluid beam model. It was established that for high values of Reynolds number interesting cascade of instabilities is discovered because the wall stiffness is reduced. Varshney et al [4] investigated the effects of magnetic field on the blood flow in the artery having multiple stenoses. Radial transformation was used together with finite difference. It was established that the presence of stenoses and magnetic fields affected the flow. Marchandise et al [5] investigated an accurate modeling of unsteady flows in collapsible tubes. One-dimensional Runge-Kutta discontinuous Galerkin method coupled with lumped parameter models for the boundary conditions was used. Sankar et al[6] carried out Mathematical modeling of a complex system for MHD flow in Hemodynamics. Finite difference method was used to solve the pdes, it was established that velocity decreases with an increase in the magnetic field and pressure gradient. Prakash et al [7] investigated Radiative heat transfer to blood flow through a stenotic artery in the presence of magnetic field. It was established that magnetic properties of RBCs play vital roles in the increase of blood viscosity during exposure to magnetic field. Malekzadeh et al [8] established that pressure drop changes proportionally to the square of the product of the magnetic field and the sine of which the magnetic field was applied. Aminfar et al [9] established that the necessary drugs are bound with Ferrofluids that helps in concentrating the drug in a certain area by the magnificent application of magnetic fields. Siviglia and Toffolon [10] investigated multiple states for flow through a Collapsible tube with discontinuities. It was established that the complexity of the fluid-structure gives Collapsible tubes their specific dynamic features. The numerical solution was obtained by using finite volume method of the path conservative type. Kozlovsky et al [11] utilized a computational model for the evaluation of the geometry of the deformed cross-sectional area due to negative transmural pressure. Odejide [12] investigated unsteady fluid structure interactions in a collapsible wall micro-channel. The set of equations were coupled non linear partial differential equations which were solved numerically using a segregated approach with fully-implicit time stepping and second finite-difference discretization. The Reynolds number was varied independently to explore the unsteady behavior of the flow. It was established that nonlinear tension does not change the qualitative response but only affects pre-factors in the scaling. Anand and Christov [13] investigated steady low Reynolds number flow of a generalized Newtonian fluid through a slender elastic tube. Non-Newtonian effects of biofluids was captured by using the power-law rheological model. Perturbation approach yields analytical solutions for flow and deformation. Mehdari et al [14] investigated analytical model of an unsteady fluid flow through an elastic tube. The fluid was considered to be Newtonian and Incompressible, they took into consideration large Reynolds number and a small aspect ratio, the tube was assumed to be having a small shell, which they considered to be the source of asymmetric vibration. Ali et al [15]investigated flow of magnetic particles with isothermal heat in the the blood. They used Laplace and Hankel transformation techniques to solve the equations. It was noted that velocity profiles increase with an increase in Grashoff number.

2. Mathematical Model

The study considers a two-dimensional unsteady flow in a collapsible tube. The fluid considered is Newtonian and incompressible. The flow is taken to be along the z and r direction with velocity u and v respectively. Where z is taken to be along the axial of the main flow while r is taken to be along the radial.

3. Governing Equations

We consider a cylindrical coordinate system (r,θ,z) where θ is the azimuthal angle and r is the radial distance as shown in Fig. 1. We take a case where u_z and u_r are velocity components in z and r directions respectively. The tube's wall is at $r = a_0\sqrt{(1-\alpha t)}$ Makinde (2005) , where α is a constant of dimension $[T^{-1}]$ which characterizes unsteadiness in the flow field, a_0 is the characteristic radius of the tube at time t = 0. The governing equations are the equations of continuity, mass and energy which are given respectively as:

$$\frac{\partial(u_z)}{\partial z} = 0. \tag{1}$$

$$\rho(\frac{\partial\vec{V}}{\partial t} + \vec{V}\cdot\vec{\nabla}\vec{V}) = -\vec{\nabla}P + \vec{\nabla}\cdot\vec{\tau} + \vec{F} \tag{2}$$

$$\frac{\partial T}{\partial t} = \frac{K}{\rho c_p}\left(\frac{1}{r}\frac{\partial T}{\partial r} + \frac{\partial^2 T}{\partial r^2}\right) + \frac{\mu}{\rho c_p}\left[\left(\frac{\partial u_z}{\partial r}\right)^2\right] \tag{3}$$

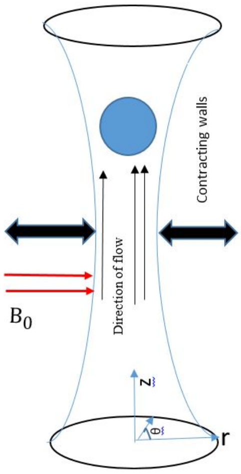

Fig. 1. Geometry of the Collapsible tube

Boussinesq's approximation for the Navier-stokes equation is used to solve non-isothermal flow, it is derived from the Navier stokes equation which states that density is constant in the Navier-stokes equation except in terms that involves gravitational acceleration. In this approximation density variation only pertains to buoyancy term ρg. It assumes that variations in density have no effect on the flow field, except that they give rise to buoyancy forces.

The buoyancy term $\Delta\rho g = (\rho-\rho_0)g$, which can further be written as $(\rho-\rho_0)g = -\rho_0\beta(T-T_w)g$. Where β is the co-efficient of thermal expansion and ρ_0 is the initial density.

But since the flow is taken past the sphere then the Boussinesq's approximation becomes,

$$(\rho-\rho_0)g = -\rho_0\beta(T-T_w)g\, sin\left(\frac{z^*}{b}\right)$$

Where b is taken to be the radius of the sphere , which is the obstacle being considered in this study.
Incorporating the obstacle we have:

$$\rho\left[\frac{\partial u_z}{\partial t}+u_r\frac{\partial u_z}{\partial r}+u_z\frac{\partial u_z}{\partial z}\right] = -\frac{\partial p}{\partial z}+\rho g_z+\mu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u_z}{\partial r}\right)+\frac{\partial^2 u_z}{\partial z^2}\right]+\vec{J}\times\vec{B}+\rho_1 g_z$$

which can be written as;

$$\frac{\partial u_z}{\partial t}+u_r\frac{\partial u_z}{\partial r}+u_z\frac{\partial u_z}{\partial z} = -\frac{1}{\rho}\frac{\partial p}{\partial z}+g_z+\frac{\mu}{\rho}\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u_z}{\partial r}\right)+\frac{\partial^2 u_z}{\partial z^2}\right]+\frac{\vec{J}\times\vec{B}}{\rho}+\frac{\rho_1 g_z}{\rho}$$

Incorporating the Boussinesq's approximation in the equation above due to flow over the sphere,

$$\frac{\partial u_z}{\partial t}+u_r\frac{\partial u_z}{\partial r}+u_z\frac{\partial u_z}{\partial z} = -\frac{1}{\rho}\frac{\partial p}{\partial z}-\beta(T-T_w)g\, sin\left(\frac{z^*}{b}\right)+\frac{\mu}{\rho}\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u_z}{\partial r}\right)+\frac{\partial^2 u_z}{\partial z^2}\right]+\frac{\vec{J}\times\vec{B}}{\rho}+\frac{\rho_1 g_z}{\rho}$$

$$\frac{\partial u_z}{\partial t} = -\frac{1}{\rho}\frac{\partial p}{\partial z}-\beta(T-T_w)g\, sin\left(\frac{z^*}{b}\right)+\frac{\mu}{\rho}\left[\frac{1}{r}\frac{\partial u_z}{\partial r}+\frac{\partial^2 u_z}{\partial r^2}\right]-\frac{\sigma B_0^2 u_z}{\rho}+\frac{\rho_1 g_z}{\rho} \tag{4}$$

$$\frac{\partial T}{\partial t} = \frac{K}{\rho c_p}\left(\frac{1}{r}\frac{\partial T}{\partial r}+\frac{\partial^2 T}{\partial r^2}\right)+\frac{\mu}{\rho c_p}\left[\left(\frac{\partial u_z}{\partial r}\right)^2\right] \tag{5}$$

4. Boundary Conditions

The boundary conditions governing flow through a vertical Collapsible tube at the center and wall of the tube are:

$$u_z = U_\infty, \frac{\partial u_z}{\partial r} = 0,\ \ T = T_\infty,\ \ at\ \ r = 0; \tag{6}$$

$$u_z = 0,\ \ T = T_w,\ \ r = 1\ \ and\ \ u_z = 0,\ \ T = T_\infty,\ \ r = b \tag{7}$$

Where T_w is the wall temperature and T_∞ is centerline temperature and b is the radius of sphere.

5. Similarity Transformation

The partial differential equations are transformed into ordinary differential equations . The model of this flow consists of non linear partial differential equations as depicted by equations (4) and (5) , to use bvp4c Collocation method, its a requirement that the equations are transformed into ordinary differential equations. Continuity equation is expected to be satisfied by the introduction of the stream function $\psi(r,z,t)$.

Since the stream function satisfies the equation of continuity. Then, there is need to look for a similarity transformation. The goal of similarity transformation is that it looks for the type of potential flows for which similar solutions exists.

The similarity solutions to the unsteady two dimensional boundary layer equation is much complex due to the fact that the three variables r,z,t need to be reduced to a single variable say η .

The assumption taken into account is that the velocity is purely along z axis and depends on r , z and t.

The following transformations are used to transform the equations to obtain the non-dimensional numbers. The transformation for velocity and temperature are given as:

$$u_z(r,z,t) = U(z,t)f(\eta) \tag{8}$$

Here $U(z,t)$ is the potential flow velocity and $\eta = \frac{r}{g(z,t)}$ where $g(z,t)$ is the scale factor of the ordinate and in this case since the z- coordinate is mainly concerned with the boundary layer growth, then it is referred to as the dimensionless scale factor.

$$u_z = -\frac{Q}{z}\frac{1}{\delta^{(m+1)}}f(\eta) \tag{9}$$

and,

$$\frac{\omega(\eta)}{\delta^{(m+1)}} = \frac{T-T_w}{T_\infty - T_w} \tag{10}$$

This transformations were in line with the work done by [16], [17], [18] and [19].

Applying the transformations given by equations (9) and (10) on (4) and (5) the following is obtained.

$$f''(\eta) + \frac{1}{\eta}f'(\eta) + \frac{(m+1){a_0}^{1+m}}{\delta^{m+1}}\lambda f(\eta) - Ha^2 f(\eta) + \frac{{a_0}^2 z\delta^{m+1}}{\mu_0 Q}\frac{\partial p}{\partial z} +$$

$$\frac{z}{v}Gr\omega(\eta)\, sin(z) - ReTc = 0 \tag{11}$$

$$\frac{1}{Pr}\left[\omega''(\eta) + \frac{1}{\eta}\omega'(\eta)\right] + \frac{(m+1)a_0^{m+1}}{\delta^{(m+1)}}\lambda\omega(\eta) + \frac{Eca_0^2}{z^2\delta^{(m+1)}}f'^2(\eta) = 0 \tag{12}$$

6. Boundary Conditions Transformation

The boundary conditions governing flow through a vertical Collapsible tube are as (6) and (7).

By using (9) and (10) on (6) and (7) it becomes,

$$f = U_\infty,\ f'(0) = 0,\ \omega(0) = \delta^{m+1}\ \ if\ \ \eta = 0 \tag{13}$$

$$f(1) = 0,\ \omega(1) = 0\ \ if\ \ \eta = 1\ \ and\ \ f(b) = 0,\ \omega(b) = 0\ \ if\ \ \eta = \text{b} \tag{14}$$

The non dimensional numbers obtained from the above equations are: $Re = \frac{Q\rho}{\mu_0}$ which is the Reynolds number, $Pr = \frac{c_p\mu_0}{k}$ which is the Prandtl number, $Ha = B_0 a_0\sqrt{\frac{\sigma}{\mu_0}}$ which represent the hartmann number, $Ec = \frac{Q^2/a_0^2}{c_p(T_\infty - T_w)}$ is the Eckert number, $\lambda = \frac{\rho\delta^m}{\mu_0 a_0^{m-1}}\frac{d\delta}{dt}$ is the unsteadiness parameter, $Gr = \frac{g\beta(T_\infty - T_w)b^2}{Q}$ is the Grashof number and $Tc = \frac{{a_0}^2 z\rho_1\delta^{m+1}g_z}{\mu_0\rho Q}$.

Here δ is the time-dependent length scale, m is a constant related to collapsibility, f is dimensionless velocity while ω is dimensionless temperature.

7. Numerical Solution

Equation 11 and 12 are solved numerically using collocation method and implemented in MATLAB using bvp4c function.

The bvp4c is a MATLAB solver that is based on the Collocation method that provides a continuous solution with a 4^{th} order accuracy. The method utilizes a mesh of points to divide the interval of integration into sub-intervals, where each sub-interval is solved based on the system of algebraic equations and the boundary equations provided. The solver estimates the error of the numerical solution on each subinterval. The solver adapts the mesh to repeat the process once the equation does not satisfy the tolerance criteria. Since BVPs can have more than one solution, BVP codes require users to supply an initial guess for the solution desired towards predicting the right solution. The codes then adapt the mesh so as to obtain an accurate numerical solution with the most number of mesh points. Coming up with a sufficiently good guess is often the hardest part of solving a BVP. The bvp4c takes an unusual approach to the control of error that helps it deal with poor guesses.

Prior to the execution of the bvp4c, the higher order non linear ODEs are reduced to first order non linear ODEs. Reduction of order is achieved by introduction of:

$$y_1 = f, \quad y_2 = f' \quad , y_3 = \omega \quad , y_4 = \omega'$$

, which reduces equations (11) and (12) and the transformed boundaries to be the following systems:

$$\begin{cases} y_1' = y_2 \\ y_2' = Ha^2 y_1 - \frac{1}{\eta} y_2 - \frac{(m+1)\ a_0^{m+1}}{\delta^{(m+1)}} \lambda y_1 - \frac{z a_0^2 \delta^{(m+1)}}{\mu_0 Q} p_z - \frac{zGr}{v} y_3\ sin(z) + ReTc \\ y_3' = y_4 \\ y_4' = -\frac{1}{\eta} y_4 - Pr \frac{(m+1)\ a_0^{m+1}}{\delta^{(m+1)}} \lambda y_3 - \frac{Pr\ Ec\ a_0^2}{z^2\ \delta^{(m+1)}} y_2^2 \end{cases} \tag{15}$$

The combined boundary conditions for the model are,

$$\begin{cases} y_1 = \frac{U_\infty z \delta^(m+1)}{Q}, \quad y_2 = 0\ ,\ y_3 = \delta^{(m+1)} \ \text{ at the center} \\ y_1 = 0\ ,\ y_3 = 0 \ \text{ at the wall } \ and \ \ y_1 = 0\ ,\ y_3 = 0 \ \text{ at r=b} \end{cases} \tag{16}$$

8. Results and Discussions

The above system (15) and (16) is inputed in MATLAB bvp4c function to obtain the results.

It is observed from Fig. 2 Increase in Reynolds number results in an increase in velocity since increase in Reynolds number means reduction of viscous forces and this leads to increase in temperature. From Fig. 3, It is noted that

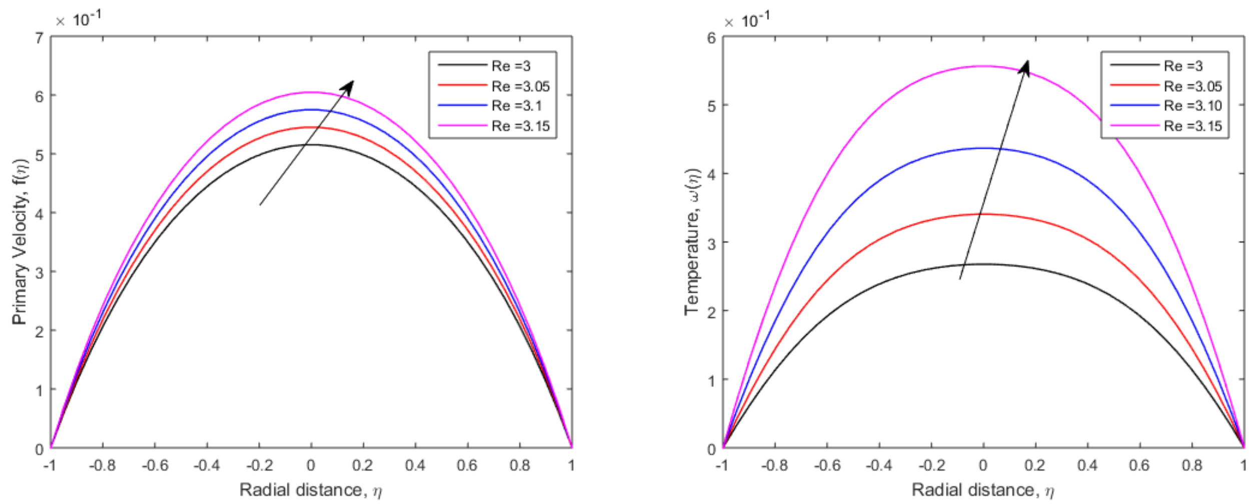

Fig. 2. Graph of dimensionless velocity and Temperature profiles for varying Reynolds number, Re

increase in Hartmann number leads to a decrease in velocity. This is because the lorentz force is known to act against the direction of flow of the fluid which in turn reduces the magnitude of velocity.Increase of Hartmann number creates an opposite flow towards the main flow which increase temperature due to thermal conduction.

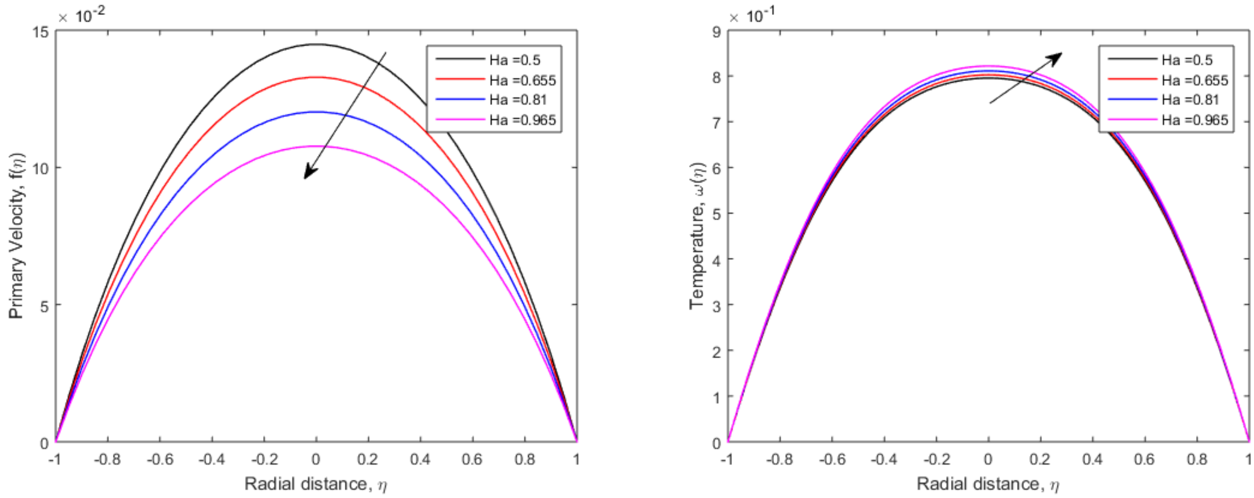

Fig. 3. Graph of dimensionless velocity and Temperature profiles for varying hartman number, Ha

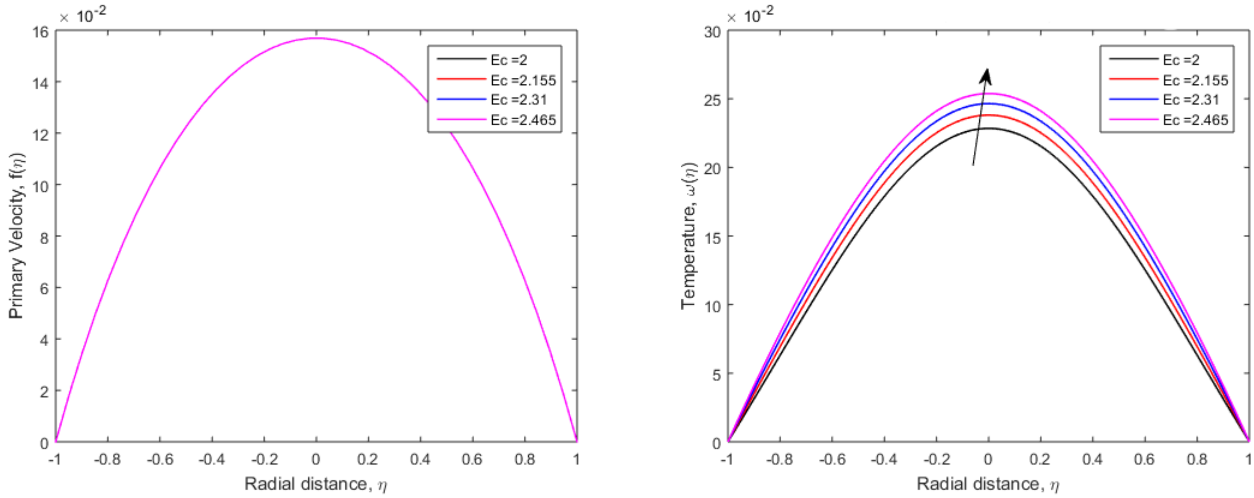

Fig. 4. Graph of dimensionless velocity and Temperature profiles for varying Eckert number, Ec

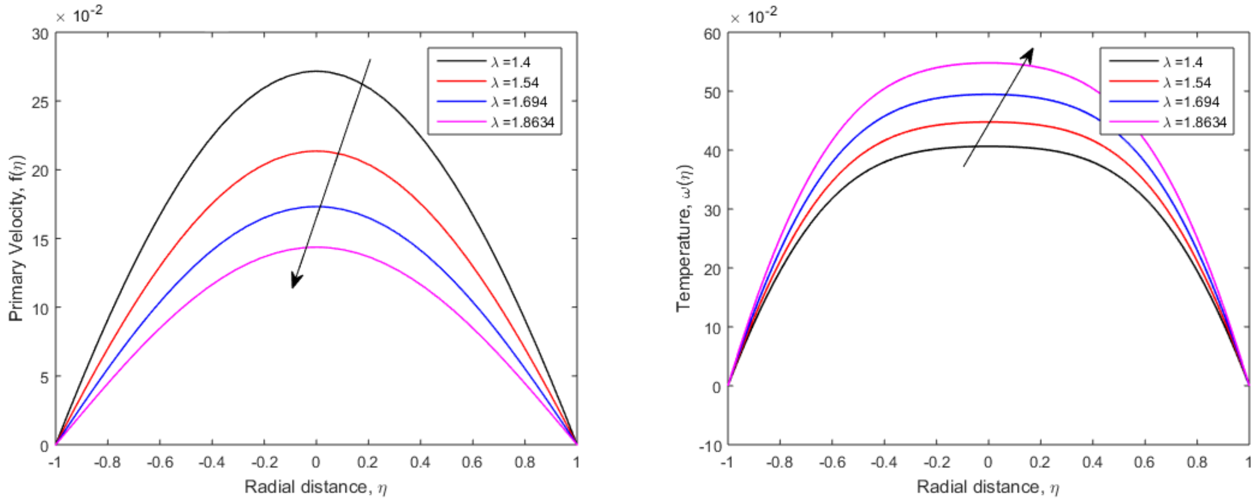

Fig. 5. Graph of dimensionless velocity and Temperature profiles for varying Unsteadiness Parameter, λ

From Fig. 4, increase in Eckert number has no significant effect on velocity profile. Increase in Ec means increase in kinetic energy which leads to rapid collision of particles which results in dissipation of heat and hence increase in temperature. From Fig. 5, as the unsteadiness parameter increases the velocity of the fluid flow tends to reduce. The increase in unsteadiness parameter leads to the thickening of the boundary layer. Also due to no slip condition and the viscous drag on the fluid molecules velocity decreases. Temperature increases with time as observed.

From Fig. 6, there is no significant change in velocity profiles when there is an increase in prandtl number. it is also observed that an increase in the Prandtl number causes a decrease in temperature profiles. From Fig. 7, velocity of

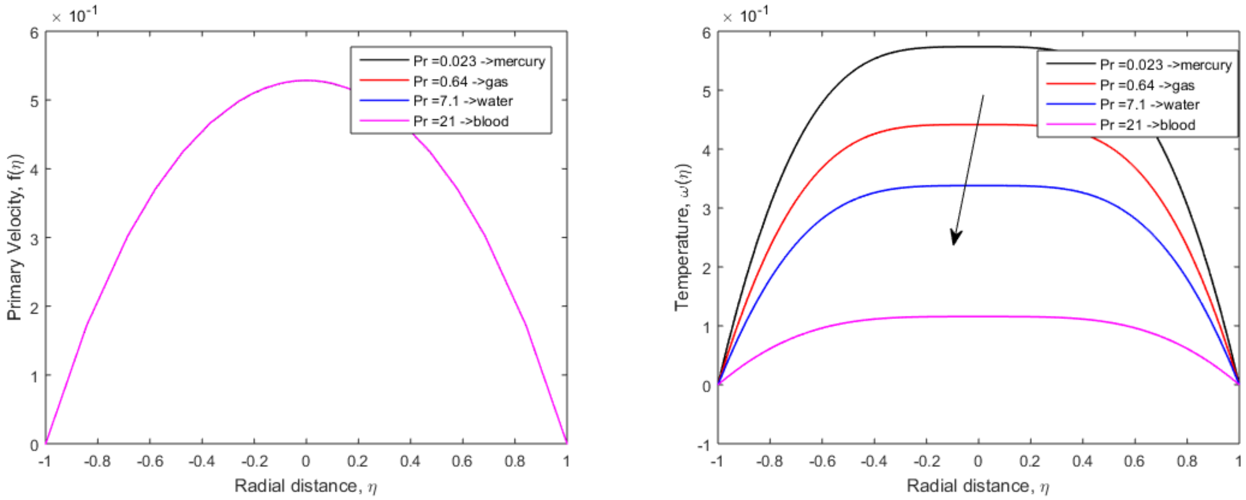

Fig. 6. Graph of dimensionless velocity and Temperature profiles for varying Prandtl number, Pr

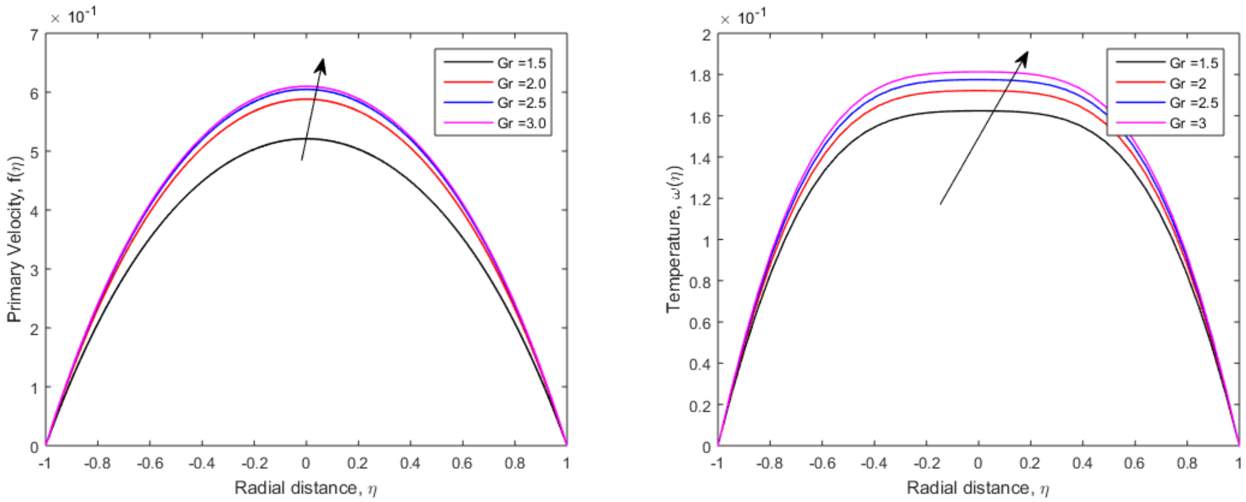

Fig. 7. Graph of dimensionless velocity and Temperature profiles for varying Grashof number, ρ_1

the fluid increases with an increase in thermal Grashof number due to convection which corresponds to an increase in the Kinetic energy as the fluid is flowing, which results to rise in internal energy.

From Fig. 8, increase in Thrust constant results in a decrease in velocity of the fluid flow. Increase in Thrust constant implies an increase in forces acting against the fluid flow as a result of the weight exerted on the fluid by the sphere, which results to the decrease in velocity. The retardation of velocity by the Thrust constant creates an increase in internal energy due to collision of particles and hence leads to the observed increase in temperature of the fluid.

8.1. Heat transfer

The Local Nusselt number is proportional to change in temperature $\omega'(0)$ is computed and its numerical values tabulated. From Table 1, an increase in the Reynolds number results to an increase in the rate of heat transfer. Increase in Prandtl number results to a decrease in Nusselt number Nu. Increase in Prandtl number leads to an increase in the thickness of the thermal boundary which results in reduction in heat transfer rate. Increase in hartmann number Ha results in an increase in the rate of heat transfer. Thermal boundary layer thickness decreases with increase in

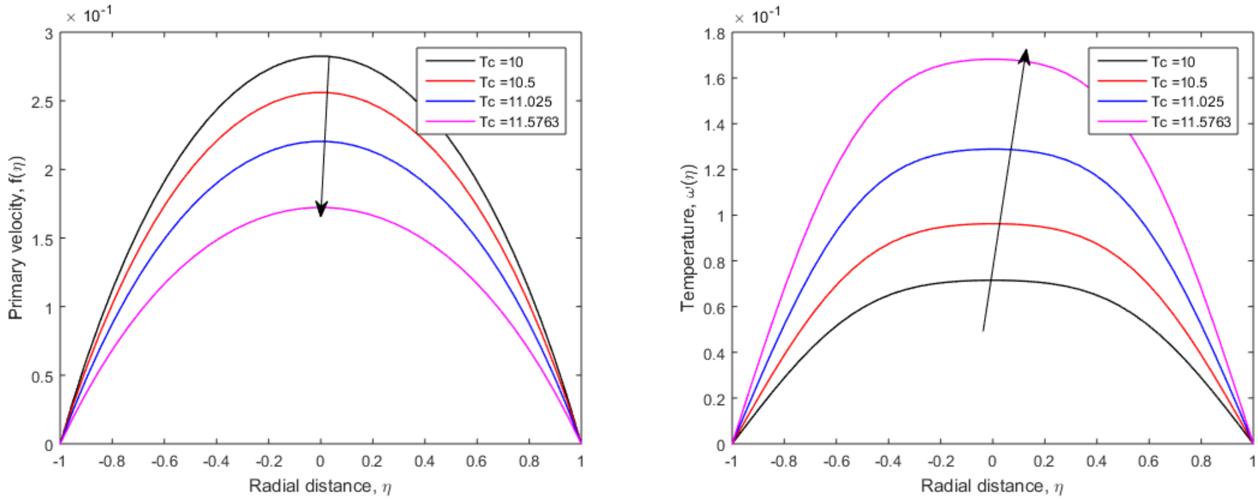

Fig. 8. Graph of dimensionless velocity and Temperature profiles for varying Thrust Constant, Tc

Table 1. Heat transfer values for various values of the parameter of Re, Ha, Pr, Ec and λ

Re	Ha	Pr	Ec	λ	Nu
25	0.2	1	0.2	0.35	0.3130
25	0.2	1.42	0.2	0.35	0.2821
25	0.2	2.84	0.2	0.35	0.2313
25	0.2	5.68	0.2	0.35	0.1588
4	0.2	0.71	0.2	0.35	0.1236
8	0.2	0.71	0.2	0.35	0.1753
16	0.2	0.71	0.2	0.35	0.2492
32	0.2	0.71	0.2	0.35	0.3535
25	0.2	0.71	0.2	0.35	0.3130
25	0.2	0.71	0.4	0.35	0.3111
25	0.2	0.71	0.8	0.35	0.3074
25	0.2	0.71	1.6	0.35	0.3000
25	0.2	0.71	0.2	0.35	0.3130
25	0.4	0.71	0.2	0.35	0.3130
25	0.8	0.71	0.2	0.35	0.3133
25	1.6	0.71	0.2	0.35	0.3139
25	0.2	0.71	0.2	0.35	0.3130
25	0.2	0.71	0.2	0.70	0.2835
25	0.2	0.71	0.2	1.40	0.2343
25	0.2	0.71	0.2	2.80	0.1158

Ha resulting in the observed increase in heat transfer. Increase in Unsteadiness parameter λ causes a decrease in Nusselt number. This implies that Nusselt number decreases with time dependent length scale. This is expected since the velocity and temperature decreases gradually with time, gradually equalizing with their respective free stream values.

9. Validation

Comparing the results in the present study with the study done by Odejide(2015), it is seen that there exist a lot of similarities as tabulated in Table 2,

Table 2. Heat transfer rate for various values of Prandtl number Pr

Pr	Odejide [12]	Present study
0.71	0.3105	0.3130
1.42	0.2726	0.2821
2.84	0.2267	0.2313
5.68	0.1493	0.1588

10. Conclusions

The results obtained show that with an increase in Reynolds number, the fluid velocity increases. However, an increase in Hartmann number, density of the obstacle and Unsteadiness parameter decreases the velocity profiles. Imposing a transverse magnetic field to a flow slows down the velocity of the fluid. Increasing Thrust constant causes an increased forced retarding the motion of the fluid.

The fluid temperature increases with an increase in Reynolds number, Hartmann number, Unsteadiness parameter, Eckert number, Thrust constant and Grashof number. However, an increase in Prandtl number causes the temperature to decrease.

Heat transfer rate increases with rise in Reynolds number and also by an increase in the Hartmann number. The heat transfer rate reduces with an increase in Prandtl number, Eckert number and Unsteadiness parameter.
The results obtained from this study are useful in the medical field for instance in the treatment of cancer. The study of flow through Collapsible tubes has vital applications in Bioengineering and medical fields. The study is important in gaining an understanding of the complexity of the flow.

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