On a Conic Through Twelve Notable Points

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Abstract: In this article we present a conic which passes through a twelve notable points and as a result of the conic we also study few Concurrency, Collinearity and Perspectivity results.

MSC: 97K30 • 68R10

Keywords: Circum Cevian Triangle • Bicevian Conic

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1. Introduction

Although Euler was apparently the first person (in 1763) to show that the midpoints of the sides of a triangle and the feet of the altitudes determine a unique circle it was not until 1820 that Brianchon an Poncelet showed that the three midpoints of the segments from the orthocenter to the vertices also lie on the same circle. Hence its name, the nine-point circle [3].

The concept of a nine point circle can be generalized to a nine point ellipse or a nine point hyperbola if we consider a general cevian instead of altitude. Consider three concurrent cevians with cevian point P locate mid-points E, F and G of the segments from cevian point to the vertices of the triangle. Also locate the feet of the cevian K, N and L. If we draw a conic through any five of the above points, we will get an ellipse and it will also pass through the sixth point [4].

Now locate the mid points of the three sides of the triangle, we will find that these points also fall on the ellipse constructed above. The conic remains an ellipse when the feet of cevians lie on the sides of the triangle and converts to a nine point hyperbola when the feet of the cevians lie on the extensions of the sides.

In our present article we study a special case of the nine point conic where cevian is replaced by internal angular bisector and cevian point as an incenter (I or X(1)). Hence it is clear that we can always construct a conic (ellipse or hyperbola) which passes through the traces of Incenter I((X(1)) and centroid G (X(2)) and it passes through the mid points of AI, BI and CI. Usually this conic is called as bicevian conic, may be this conic is well known in the literature but the way how we are dealing in this article is completely new. Unexpectedly, this nine point conic passes through three more notable points (point of intersections of the line formed by joining the apex of circum cevian triangle of I with the conic) In this short note we discuss how this twelve-point conic is useful in constructing the some notable triangle centers such as X(21), X(56), X(84), X(995), center of 12 point conic X(1125), X(2360), X(3616), X(19861), X(24471).

In the journey of this construction, with some modified configurations, unexpectedly we come across a new concurrency of special cevians, as a result we will meet one new triangle center say X(K) whose barycentric coordinates are (8s+a: 8s+b: 8s +c) which is not available in current edition of ETC(1-40000) [2].

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In addition to these we see few collinearity perspectivity results.

\[[1]6473,17825, {16844,19749}, {17290,25498}, {17304,24295}, {17400,24342}, \]

\[[2]7951,17527, {7982,11231}, {8040,11263}, {8056,24161}, {8185,11284}, {8252,18991}, {8253,18992}, \]

\[[3]6667,10609, {6691,25525}, {6701,15671}, {6707,17306}, {6713,15017}, \]

\[[4]946,3525, \] BI, CI) and V

\[[5]2478,18513, \]

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\[[72]2478,18513, \]

In addition to these we see few collinear-perspective perspectivity results.

2. Notation and Background

Let ABC be a non equilateral triangle. We denote its side-lengths by a, b, c, perimeter by 2s, its area by Δ and its circumradius by R, its inradius by r. We will homogenize barycentric coordinates with reference to ABC.

Lemma 2.1.

Let \( P = (u : v : w) \) be a point not on the side lines of triangle ABC. Then the equation of the conic through the traces of P and the midpoints of the three sides is \( \sum_{\text{cyclic}} vwx^2 - u(v + w)yz = 0 \) having the center as \( (2u + v + w : u + 2v + w : u + v + 2w) \) and also passes through the midpoints of \( AP(2u + v + w : v : w) \) and \( CP(2u + v + w : w) \) respectively [5].

Lemma 2.2.

Let \( P = (u : v : w) \). The lines AP, BR, CP intersect the circumcircle again at the points \( A(P) = \left( -a^2v^2w \over b^2w + c^2v : v : w \right) \), \( B(P) = \left( u : -b^2w^2u \over c^2u + a^2w : w \right) \) and \( C(P) = \left( u : v : -c^2u^2v \over a^2v + b^2u \right) \). These form the vertices of the Circumcevian triangle of P [5].

2.1. Nine-point conic of Incenter \( (C_{I_B}) \)

Using Lemma 2.1 we can find the barycentric coordinates of the Nine-point conic of incenter \( (C_{I_B}) \) (Refer Fig. 1)

2.2. Circumcevian triangle of incenter

Using Lemma 2.2 we can find the barycentric coordinates of the circumcevian triangle of incenter (See Fig. 2)

Let us consider three points \( V_A, V_B \) and \( V_C \) whose barycentric coordinates as in Table 3

Theorem 2.1.

The Nine-point conic of incenter \( (C_{I_B}) \) and circumcevian triangle of incenter intersects at the points \( R, Q, R \) (midpoints of \( AI, BI, CI \)) and \( V_A, V_B \) and \( V_C \) such that \( C_{I_B} \cap A(1) = R \) and \( C_{I_B} \cap B(1) = P \) and \( V_A, C_{I_B} \cap C(1) = Q \) and \( V_B \). (Refer Fig. 3)
Table 1.

<table>
<thead>
<tr>
<th>Points</th>
<th>barycentric coordinates</th>
</tr>
</thead>
</table>
| Traces of Incenter on the sides BC, CA, AB are X, Y, Z respectively | X = (0 : b : c)  
Y = (a : 0 : c)  
Z = (a : b : 0) |
| Traces of centroid G on the sides BC, CA, AB are D, E, F respectively | D = (0 : 1 :1)  
E = (1 : 0 : 1)  
F = (1 : 1 : 0) |
| Midpoints of AI, BI, CI are P, Q, R respectively | P = (2s+a : b :c)  
Q = (a : 2s+b : c)  
R = (a : b : 2s+c) |

Equation of nine-point conic \( (C_{I_9}) \) of incenter I
\[
bcx^2 + cay^2 + abz^2 - a(b+c)yz - b(c+a)zx - c(a+b)xy = 0
\] (1)

Equation of nine-point conic \( (C_{I_9}) \) of incentre I
\[
\text{center of } C_{I_9} = (2s+a : 2s+b : 2s+c)
\]
\[
= X(1125) = \text{complement of } X(10)
\]

Proof. The equation of nine-point conic of incenter \( (C_{I_9}) \) is given by
\[
bcx^2 + cay^2 + abz^2 - a(b+c)yz - b(c+a)zx - c(a+b)xy = 0
\] (1)

Equation of \( A(I) \) \( B(I) \) is given by
\[
b(b+c)x + a(c+a)y - abz = 0
\] (2)

By eliminating \( abz^2 \) from the Eqs. (1),(2) and by regrouping, we can factorize the expression as
\[
(x : y : z) = (a : b : a + b + 2c)
\]

Hence
\[
C_{I_9} \cap A(I) \cap B(I) = R \text{ and } V_C = (a : b : 2s + c) \text{ and } (a^2(s-b) : b^2(s-a) : sc(a+b) - abc).
\]

Similarly we can prove the remaining relations.

Remark:

That is the nine-point conic of incenter passes through other 3 notable points \( V_A, V_B \) and \( V_C \).
Table 2.

<table>
<thead>
<tr>
<th>Circum cevian triangle of incenter $I(a:b:c)$</th>
<th>Barycentric Coordinates</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vertices $A^{(I)}$, $B^{(I)}$ and $C^{(I)}$</td>
<td>$A^{(I)} = (-a^2 : b(b + c) : c(b + c))$, $B^{(I)} = (a(c + a) : -b^2 : c(c + a))$, $C^{(I)} = (a(a + b) : b(a + b) : -c^2)$</td>
</tr>
<tr>
<td>Sides $A^{(I)}B^{(I)}$, $B^{(I)}C^{(I)}$ and $C^{(I)}A^{(I)}$</td>
<td>$A^{(I)}B^{(I)} : b(b + c)x + a(c + a)y - abz = 0$, $B^{(I)}C^{(I)} : -bcx + c(c + a)y + b(a + b)z = 0$, $C^{(I)}A^{(I)} : c(b + c)x - cay + a(a + b)z = 0$</td>
</tr>
</tbody>
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Fig. 2.

1. Point of intersection of Nine-point conic of incenter $(C_I)$ with the circumcevian triangle of incenter $(C_{A_1})$

<table>
<thead>
<tr>
<th>Name of the Point of intersections</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C_{I_a} \cap A^{(I)}B^{(I)}$</td>
</tr>
<tr>
<td>$C_{I_b} \cap B^{(I)}C^{(I)}$</td>
</tr>
<tr>
<td>$C_{I_c} \cap C^{(I)}A^{(I)}$</td>
</tr>
</tbody>
</table>

2. It is easy to verify that the set of lines $\{A^{(I)}B^{(I)}, C_I\}$, $\{B^{(I)}C^{(I)}, AI\}$ and $\{C^{(I)}A^{(I)}, BI\}$ are perpendicular at their point of intersections $R, P$ and $Q$ respectively.

3. It is clear that the circumcevian triangle of $I$ is similar to the excentral triangle of $ABC$.

4. Theorem 2.1 also gives the construction of the points $V_A, V_B$ and $V_C$.


6. This twelve point bicevian conic contains the Feuerbach point $X(11)$.

7. The twelve point conic $C_{I_{12}}$ which we discussed in Theorem 2.1 has center $(C_{I_{12}})$ and the coordinates of center $= (2s + a : 2s + b : 2s + c) = \text{complement of } X(10) = X(1125)$.

### Table 3.

<table>
<thead>
<tr>
<th>Points</th>
<th>Barycentric Coordinates</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V_A$</td>
<td>$(sa(b + c) - abc : b^2(s - c) : c^2(s - b))$</td>
</tr>
<tr>
<td>$V_B$</td>
<td>$(a^2(s - c) : sb(c + a) - abc : c^2(s - a))$</td>
</tr>
<tr>
<td>$V_C$</td>
<td>$(a^2(s - b) : b^2(s - a) : sc(a + b) - abc)$</td>
</tr>
</tbody>
</table>

### 3. Some Collinearity and Perspectivity Results

**Proposition 3.1.**
The lines $AV_A, BV_B$ and $CV_C$ are concur at $\text{EXSIMILICENTER} X(56)$. That is the triangles $ABC$ and $V_AV_BV_C$ are perspective and the perspector is $X(56)$.

**Proof.** The lines $AV_A, BV_B$ and $CV_C$ have barycentric equations

1. $(0)x + c^2(s - b)y - b^2(s - c)z = 0$
2. $c^2(s - a)x + (0)y - a^2(s - c)z = 0$
3. $b^2(s - a)x - a^2(s - b)y - (0)z = 0$

It is clear that,

$$
\begin{vmatrix}
0 & c^2(s - b) & -b^2(s - c) \\
-c^2(s - a) & 0 & -a^2(s - c) \\
b^2(s - a) & -a^2(s - b) & 0
\end{vmatrix} = 0
$$

Hence (3), (4) and (5) are concur. And they intersect at \( \left( \frac{a^2}{s - a} : \frac{b^2}{s - b} : \frac{c^2}{s - c} \right) = \text{EXSIMILICENTER} = X(56) \).

**Remark:** This article gives a new way of constructing the point $X(56)$. (Refer Fig. 5)

**Proposition 3.2.**
The lines $A^{(I)}V_A, B^{(I)}V_B$ and $C^{(I)}V_C$ are concur at $X(995)$. That is the triangles $A^{(I)}B^{(I)}C^{(I)}$ and $V_AV_BV_C$ are perspective and the perspector is $X(995)$. 
Proof: The lines \(A^{(I)}V_A, B^{(I)}V_B\) and \(C^{(I)}V_C\) have barycentric equations

\[
bc(b^2 - c^2)x - ca(b^2 + c^2 + ca)y + ab(b^2 + c^2 + ab)z = 0 \tag{6}
\]

\[
bc(c^2 + a^2 + bc)x + ca(c^2 - a^2)y - ab(c^2 + a^2 + ab)z = 0 \tag{7}
\]

\[
-bc(a^2 + b^2 + bc)x + ca(a^2 + b^2 + ca)y + ab(a^2 - b^2)z = 0 \tag{8}
\]

It is clear that,

\[
\begin{vmatrix}
bc(b^2 - c^2) & -ca(b^2 + c^2 + ca) & ab(b^2 + c^2 + ab) \\
bc(c^2 + a^2 + bc) & ca(c^2 - a^2) & -ab(c^2 + a^2 + ab) \\
-bc(a^2 + b^2 + bc) & ca(a^2 + b^2 + ca) & ab(a^2 - b^2)
\end{vmatrix} = 0.
\]

Hence (6), (7) and (8) are concur. And they concur at \((a^2(4s^2 - 2sa - 3bc) : b^2(4s^2 - 2sb - 3ca) : c^2(4s^2 - 2sc - 3ab)) = X(995)\).

Remark:

1. This article gives a new way of constructing the point \(X(995)\). (Refer Fig. 6).
2. The four points incenter \(X(1)\), centroid \(X(2)\), \(X(995)\), \(X(K)\) are collinear. (Refer Fig. 6).
Proof. Consider a line in barycentric coordinates which contains the points $X(1)$ and $X(2)$ is

$$(b - c)x + (c - a)y + (a - b)z = 0 \quad (9)$$

Now it is easy to verify that center $X(K) = (8s + a : 8s + b : 8s + c)$ lies on (9) since $(8s + a)(b - c) + (8s + b)(c - a) + (8s + c)(a - b) = 0$.

Similarly we can verify that

$$\sum_{a,b,c} a^2(4s^2 - 2sa - 3bc)(b - c) = 4s^2 \sum_{a,b,c} a^2(b - c) - 2s \sum_{a,b,c} a^3(b - c) - 3abc \sum_{a,b,c} a(b - c)$$
$$= -4s^2(a - b)(b - c)(c - a) + 2s(a - b)(b - c)(c - a)(a + b + c) - 0 = 0$$

So the point X(995) lies on (9). Hence proved.

Fig. 6.

**Proposition 3.3.**
The points $V_1 = BV_C \cap CV_B$, $V_2 = CV_A \cap AV_C$ and $V_3 = AV_B \cap BV_A$ and the triangles $ABC$ and $V_1V_2V_3$ are perspective. (Refer Fig. 7)

**Proof.** The lines $BV_C$ and $CV_B$ have barycentric equations

$$(sc(a + b) - abc)x + (0)y - a^2(s - b)z = 0 \quad (10)$$
$$(sb(c + a) - abc)x - a^2(s - c)y + (0)z = 0 \quad (11)$$

Lines (6), (7) intersect at the point $V_1 = \left(\frac{sa(b + c) - abc}{s - c} : \frac{sb(a + c) - abc}{s - a} : \frac{sc(a + b) - abc}{s - b}\right)$.

Similarly

$$V_2 = CV_A \cap AV_C = \left(\frac{sa(b + c) - abc}{s - c} : b^2 : \frac{sc(a + b) - abc}{s - a}\right)$$

and

$$V_3 = AV_B \cap BV_A = \left(\frac{sa(b + c) - abc}{s - b} : \frac{sb(a + b) - abc}{s - a} : \frac{sc(a + b) - abc}{s - c}\right)$$

From these coordinates, it is clear that triangles $V_1V_2V_3$ is perspective with $ABC$ at

$$\Lambda = AV_1 \cap BV_2 \cap CV_3 = \left(\frac{sa(b + c) - abc}{s - a} : \frac{sb(c + a) - abc}{s - b} : \frac{sc(a + b) - abc}{s - c}\right)$$

Fig. 7.
Remark: The perspector \( \Lambda = \left( \frac{sa(b + c) - abc}{s - a} : \frac{sb(c + a) - abc}{s - b} : \frac{sc(a + b) - abc}{s - c} \right) \) given in (12) is the triangle center \( X(24471) = \Lambda = X(6) \cap X(7) \cap X(8) \).

Proposition 3.4.
The lines \( A^{(I)}V_1, B^{(I)}V_2 \) and \( C^{(I)}V_3 \) are concur at \( X(2360) \) That is the triangles \( A^{(I)}B^{(I)}C^{(I)} \) and \( V_1V_2V_3 \) are perspective and the perspector is \( X(2360) \).

Proof. The lines \( A^{(I)}V_1, B^{(I)}V_2 \) and \( C^{(I)}V_3 \) have barycentric equations

\[
\begin{align*}
\text{(13)} & & sbc(b^2 - c^2)x + a^2c(c + a)(s - c)y - a^2b(a + b)(s - b)z = 0 \\
\text{(14)} & & -b^2c(b + c)(s - c)x + sca(c^2 - a^2)y + b^2a(a + b)(s - a)z = 0 \\
\text{(15)} & & c^2b(b + c)(s - b)x - c^2a(c + a)(s - a)y + sab(a^2 - b^2)z = 0
\end{align*}
\]

It is clear that

\[
\begin{vmatrix}
sb(c^2 - a^2) & a^2c(c + a)(s - c) & -a^2b(a + b)(s - b) \\
-b^2c(b + c)(s - c) & sca(c^2 - a^2) & b^2a(a + b)(s - a) \\
c^2b(b + c)(s - b) & -c^2a(c + a)(s - a) & sab(a^2 - b^2)
\end{vmatrix} = 0.
\]

Hence (13), (14) and (15) are concur. And they concur at

\[
\left( \frac{a^2}{b + c}, \frac{b^2}{c + a}, \frac{c^2}{a + b} \right) = X(2360)
\]

Hence the triangles \( A^{(I)}B^{(I)}C^{(I)} \) and \( V_1V_2V_3 \) are perspective and the perspector is \( X(2360) \).

Remark:

1. This article gives a new way of constructing the point \( X(2360) \)(Refer Fig. 8).
2. The points \( X(56), X(995), X(2360) \) are collinear. (Refer Fig. 9).

Proposition 3.5.
The lines \( V_AV_1, V_BV_2 \) and \( V_CV_3 \) are concur at \( X(19861) \). That is the triangles \( V_AV_BV_C \) and \( V_1V_2V_3 \) are perspective and the perspector is \( X(19861) \).
Proof. The lines $V_AV_1$, $V_BV_2$ and $V.CV_3$ have barycentric equations

$$(b-c)(2s^3 - 2s^2a + s(a^2 - 2bc) + abc)x - a(s-c)(2s^2 - 2sb + b^2)y + a(s-b)(2s^2 - 2sc + c^2)z = 0$$

$b(s-c)(2s^2 - 2sa + a^2)x + (c-a)(2s^3 - 2s^2b + sb - 2ca) + abc)y - b(s-a)(2s^2 - 2sc + c^2)z = 0$

$-c(s-b)(2s^2 - 2sa + a^2)x + c(s-a)(2s^3 - 2sb + b^2)y + (a-b)(2s^3 - 2sc + c^2 - 2ab + abc)z = 0$

After transformations using

$$k_a = 2s^2 - 2sa + a^2 = s^2 + (s-a)^2$$
$$k_b = 2s^2 - 2sb + b^2 = s^2 + (s-b)^2$$
$$k_c = 2s^2 - 2sc + c^2 = s^2 + (s-c)^2$$

$$- \left( \frac{k_a - k_b}{c} \right) = a - b, - \left( \frac{k_b - k_c}{a} \right) = b - c \text{ and } - \left( \frac{k_c - k_a}{b} \right) = c - a$$

and

$$(2s^3 - 2s^2a + s(a^2 - 2bc) + abc) = s(2s^3 - 2sa + a^2 - bc(b + c) = s(k_a) - bc(b + c)$$

$$(2s^3 - 2s^2b + s(b^2 - 2ca) + abc) = s(2s^3 - 2sb + b^2 - ca(c + a) = s(k_b) - ca(c + a)$$

$$(2s^3 - 2s^2c + s(c^2 - 2ab) + abc) = s(2s^3 - 2sc + c^2 - ab(a + b) = s(k_c) - ab(a + b)$$
Hence the triangles $V_1V_2$ and $V_1V_3$ as

$$
(k_b - k_c)(s_k - bc(b + c))x + a^2(s - c)(k_b)y - a^2(s - b)(k_c)z = 0
$$

$$
-b^2(s - c)(k_a)x + (k_c - k_a)(sk_b - ca(c + a))y + b^2(s - a)(k_c)z = 0
$$

$$
c^2(s - b)(k_a)x - c^2(s - a)(k_b)y + (k_a - k_b)(sk_c - ab(a + b))z = 0
$$

It is clear that,

$$
\begin{vmatrix}
(k_b - k_c)(s_k - bc(b + c)) & a^2(s - c)(k_b) & -a^2(s - b)(k_c) \\
-b^2(s - c)(k_a) & (k_c - k_a)(sk_b - ca(c + a)) & b^2(s - a)(k_c) \\
c^2(s - b)(k_a) & -c^2(s - a)(k_b) & (k_a - k_b)(sk_c - ab(a + b))
\end{vmatrix} = 0
$$

Hence (16), (17) and (18) are concyclic, and they concur at

$$
(aT + abc(4s - a) : bT + abc(4s - b) : cT + abc(4s - c))
$$

$$
= \left(\left(\sum_{a,b,c} a^2(a - b - c) + 2bc(b + c) + 4abc\right) : \left(\sum_{a,b,c} b^2(b - c - a) + 2ca(c + a) + 4abc\right) : \left(\sum_{a,b,c} c^2(c - a - b) + 2ab(a + b) + 4abc\right)\right)
$$

$$
= X \left(19861\right)
$$

Hence the triangles $V_1V_2V_3$ and $V_1V_2V_3$ are perspective and the perspector is $X(19861)$. (Refer Fig. 10)

**Remark:** $X(56), X(19861)$ and $X(24471)$ are collinear. (Refer Fig. 11)

**Proposition 3.6.**

The points $P_1 = BR \cap CQ$, $Q_1 = CP \cap AR$ and $R_1 = AQ \cap BP$ and the triangles $ABC$ and $P_1Q_1R_1$ are perspective. (Refer Fig. 12)

**Proof.** The lines $BR$ and $CQ$ have barycentric equations

$$
(2s + c)x + (0)y - az = 0
$$

$$
(2s + b)x - ay + (0)z = 0
$$

Lines (19), (20) intersect at the point $P_1 = (a : 2s + b : 2s + c)$.

Similarly $Q_1 = CP \cap AR = (2s + a : b : 2s + c)$ and $R_1 = AQ \cap BP = (2s + a : 2s + b : c)$

From these coordinates, it is clear that triangles $P_1Q_1R_1$ is perspective with $ABC$ at

$$
\lambda = AP_1 \cap BQ_1 \cap CR_1 = (2s + a : 2s + b : 2s + c)
$$

(21)

**Remark:**

1. The perspector $\lambda = (2s + a : 2s + b : 2s + c)$ given in (21) is the triangle center $X(1125)[1]$.

2. It is clear that $P_1, Q_1$ and $R_1$ are the centroids of the triangles $BCI, CAI$ and $ABI$. The triangle $P_1Q_1R_1$ is homothetic to $ABC$, and the center of homothety is $\lambda = AP_1 \cap BQ_1 \cap CR_1 = (2s + a : 2s + b : 2s + c)$ = $X(1125)$ = complement of $X(10)$. 


3. The point $X(1125)$ is also the center of twelve point conic of incenter $C_{I12}$. This article gives a new way of constructing the point $X(1125)$.

**Proposition 3.7.**
The lines $A^{(I)}P_1$, $B^{(I)}Q_1$, and $C^{(I)}R_1$ are concurrent at Schiffler point. That is the triangles $A^{(I)}B^{(I)}C^{(I)}$ and $P_1Q_1R_1$ are perspective and the perspector is Schiffler point.
On a Conic Through Twelve Notable Points

Proof. The lines \( A^{[1]} P_1, B^{[1]} Q_1 \) and \( C^{[1]} R_1 \) have barycentric equations
\[
\begin{align*}
(b^2 - c^2)x + a(c + a)y - a(a + b)z &= 0 \quad (22) \\
b(b + c)x + (c^2 - a^2)y - a(a + b)z &= 0 \quad (23) \\
c(b + c)x - a(a + b)y + (a^2 - b^2)z &= 0 \quad (24)
\end{align*}
\]
It is clear that
\[
\begin{vmatrix}
(b^2 - c^2) & a(c + a) & -a(a + b) \\
b(b + c) & (c^2 - a^2) & -a(a + b) \\
c(b + c) & -a(a + b) & (a^2 - b^2)
\end{vmatrix} = 0
\]
Hence (22), (23) and (24) are concur. And they concur at
\[
\left( \frac{s}{b+c} : \frac{s}{c+a} : \frac{c^2}{a+b} : s-c \right) = \text{Schiffler point} = X(21)
\]
Hence the triangles \( A^{[1]} B^{[1]} C^{[1]} \) and \( P_1 Q_1 R_1 \) are perspective and the perspector is Schiffler point.

Remark:
1. This article gives a new way of constructing the point X(21). (Refer Fig. 13).
2. The points X(19861), X(21), X(2360) are collinear (Refer Fig. 14).

Proposition 3.8.
The lines \( PP_1, QQ_1 \) and \( RR_1 \) are concur at X(3616). That is the triangles \( PQR \) and \( P_1 Q_1 R_1 \) are perspective and the perspector is X(3616).

Proof. The lines \( PP_1, QQ_1 \) and \( RR_1 \) have barycentric equations
\[
\begin{align*}
(b - c)x - (4s - b)y + (4s - c)z &= 0 \quad (25) \\
(4s - a)x + (c - a)y - (4s - c)z &= 0 \quad (26) \\
-(4s - a)x + (4s - b)y + (a - b)z &= 0 \quad (27)
\end{align*}
\]
It is clear that
\[
\begin{vmatrix}
(b - c) & -(4s - b) & (4s - c) \\
(4s - a) & (c - a) & -(4s - c) \\
-(4s - a) & (4s - b) & (a - b)
\end{vmatrix} = 0
\]
Hence (25), (26) and (27) are concur. And they concur at
\[
\left( \frac{s + a}{s + b} : \frac{s + b}{s + c} : X(3616) \right)
\]
Hence the triangles \( PQR \) and \( P_1 Q_1 R_1 \) are perspective and the perspector is X(3616). (Refer Fig. 15)
Remark:
1. This article gives a new way of constructing the point X(3616).
2. X(1), X(2), X(995), center of the conic $C_{I_{12}}$, X(1125), X(3616), X(19861) and X(K).

Proof. Consider a line in barycentric coordinates which contains the points X(1) and X(2) is [see Eq. (9)]

$$(b - c)x + (c - a)y + (a - b)z = 0$$

In remark-2 of Proposition 3.2 we proved that X(995), X(K) lies on (9) and now it is clear that the points center of the conic $C_{I_{12}}$, X(1125), X(3616) also lies on (9). That is all the six points X(1), X(2), X(995), center of the conic X(1125), X(3616), X(19861) and X(K) are collinear.

Proposition 3.9.
The diagonal points of the quadrangle $BCV_BV_C$ are X(56), $V_1$ and $W_1$. The points $W_2$, $W_3$ are similarly defined. Then The lines $A^{(I)}W_1$, $B^{(I)}W_2$ and $C^{(I)}W_3$ form a perspective triangle $A_WB_WC_W$ with $ABC$ with perspector X(84) = Isogonal conjugate of Bevan point. (Refer ??)
4. Conclusion

This article has shown new and more elegant approach to construct some notable triangle centers such as $X(21), X(56), X(84), X(995)$, center of the conic $C_{12}, X(1125), X(2360), X(3616), X(19861), X(24471)$ and $X(K)$ using a
bicevian conic (12 point conic).

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