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Effect of vibrations on convective instability using meshless method

Research Article

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Abstract: Influence of the vibrations on the critical conditions of convective instability in liquid to solid frontal polymerization is studied. The model includes the equation of heat, the equation of concentration and the Navier-Stokes equations under the Boussinesq approximation. The linear stability analysis of the problem is performed and the limit of convective instability is found as a function of the amplitude and the frequency of the gravitational vibrations. The problem is solved numerically using the multiquadric radial basis functions method.

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Keywords: Frontal polymerization • Linear stability analysis • Convective instability • MQ-RBF method • Gravitational vibrations

1. Introduction

There are three types of instabilities of reaction fronts. The first one is the thermo-diffusional instability which the boundary of stability are studied in [1–3] assuming that the density of the medium is constant. The second type is Darrieus-Landau instability which is studied in [4], this instability arises if the density of the medium is variable. Finally the convective instability is studied in [5–8] by using the Boussinesq approximation which means that the change of density is neglected in the model except for the term of buoyancy.

In the absence of vibrations, the problem of reaction front propagation is studied in [9], it is shown that the conditions of cellular instability is influenced by the Prandtl number and the front velocity, the conditions of oscillatory instability is determined, it is shown that if the Rayleigh number exceeds a certain critical value, the reaction front becomes unstable.

The influence of vibrations on convective instability in liquid to solid frontal polymerization has been studied in [10, 11]. The authors showed that the convection can be influenced by vibrations indeed if the amplitude of the vibrations is small the reaction front is stabilized moreover, in the opposite case the front loses its stability. The effect of vibrations on convective instability of reaction fronts in porous media has been studied in [12], the convective instability boundary was found and the front become more stable when the frequency of vibrations increases.

In this work, we study the effect of vertical gravitational modulation on the convective instability of the reaction fronts in liquid medium by using a meshless method (MQ-RBF) in the case where the reactant is liquid while the product is solid separated by a narrow reaction zone. To do so, we impose harmonic gravitational modulation with

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some given frequency σ and amplitude λ. Therefore the front is subjected to a modulated acceleration which is written in the form: $g(1+\lambda\sin(\sigma t))$, where g is the gravity acceleration.

The paper is organized as follows. In Section 2, we introduce the mathematical model. We perform an asymptotic analysis in Section 3 to obtain the interface problem. We fulfill a linear stability analysis around the stationary solutions in Section 4. In Section 5, we present the results. Finally, we give a conclusion in the Section 6.

2. Mathematical model

We consider the effect of periodic oscillations on the propagation of reaction fronts in liquid medium. The process is described by the following system of equations:

$$\frac{\partial T}{\partial t}+(v.\nabla)T=\kappa\Delta T+qK(T)\phi(\alpha), \tag{1}$$

$$\frac{\partial \alpha}{\partial t}+(v.\nabla)\alpha=K(T)\phi(\alpha), \tag{2}$$

$$\frac{\partial v}{\partial t}+(v.\nabla)v=-\frac{1}{\rho}\nabla p+\nu\Delta v+g(1+\lambda\sin(\sigma t))\beta(T-T_0)\gamma, \tag{3}$$

$$div(v)=0. \tag{4}$$

With the conditions to infinity:

$$z\to+\infty:\quad T=T_i,\quad \alpha=0,\quad and\quad v=0, \tag{5}$$

and

$$z\to-\infty:\quad T=T_b,\quad \alpha=1,\quad and\quad v=0. \tag{6}$$

Here T is the temperature, α depth of conversion, $v=(v_x,v_y,v_z)$ the velocity field, p pressure, κ the coefficient of thermal diffusivity, q is the heat produced in adiabatic conditions, ρ the density, ν kinematic viscosity, g gravity acceleration, λ and σ respectively represent the amplitude and the frequency of the vibrations, β the coefficient of thermal expansion, and γ the unit vector of the vertical direction, T_0 the average value of the temperature, T_i is the initial temperature and T_b is the adiabatic temperature, $T_b=T_i+q$. The reaction rate increases with temperature according to Arrhenius law: $K(T)=k_0exp(-E/R_0T)$, where E is the activation energy of the reaction, R_0 is the constant of perfect gases and k_0 is the pre-exponential factor. The chemical reaction is considered to be of zero order:

$$\phi(\alpha)=\begin{cases}1 & if\quad \alpha<1,\\ 0 & if\quad \alpha=0.\end{cases}$$

To obtain the dimensionless model, we define the following spatial variables:

$$x_1=\frac{xc_1}{\kappa},\quad y_1=\frac{yc_1}{\kappa},\quad z_1=\frac{zc_1}{\kappa},\quad t_1=\frac{tc_1^2}{\kappa},\quad p_1=\frac{p}{c_1^2\rho},\quad c_1=\frac{c}{\sqrt{2}},\quad v_1=\frac{v}{c_1},\quad \theta=\frac{T-T_b}{q},$$

where c correspond to the speed of propagation of the front of the reaction in the stationary case and defined by [13]:

$$c^2=\frac{2k_0\kappa R_0T_b^2}{qE}\exp\left(-\frac{E}{R_0T_b}\right).$$

Therefore the system (1)-(4) can be written in the following form (the index 1 is omitted):

$$\frac{\partial \theta}{\partial t}+(v.\nabla)\theta=\Delta\theta+Z\exp\left(\frac{\theta}{Z^{-1}+\delta\theta}\right)\phi(\alpha), \tag{7}$$

$$\frac{\partial \alpha}{\partial t}+(v.\nabla)\alpha=Z\exp\left(\frac{\theta}{Z^{-1}+\delta\theta}\right)\phi(\alpha), \tag{8}$$

$$\frac{\partial v}{\partial t}+(v.\nabla)v=-\nabla p+P\Delta v+PR(1+\lambda\sin(\mu t))(\theta-\theta_0)\gamma, \tag{9}$$

$$div(v)=0, \tag{10}$$

with the following boundary conditions:

$$z\to+\infty:\quad \theta=-1,\quad \alpha=0,\quad and\quad v=0, \tag{11}$$

$$z\to-\infty:\quad \theta=0,\quad \alpha=1,\quad and\quad v=0. \tag{12}$$

Where $Z=\dfrac{qE}{R_0T_b^2}$, $P=\dfrac{\nu}{\kappa}$, $R=\dfrac{g\beta q\kappa^2}{\nu c^3}$ denote respectively the Zeldovich number, the Prandtl number and the Rayleigh number, $\delta=\dfrac{R_0T_b}{E}$, $\theta_0=\dfrac{(T_b-T_0)}{q}$ and $\mu=\dfrac{2\kappa\sigma}{c^2}$.

3. Asymptotic analysis of the model

In this section, we fulfill an asymptotic analysis with $\varepsilon = Z^{-1}$ taken as small parameter and we obtain a closed interface problem.

Denote by $\zeta(x, y, t)$ the location of the reaction zone in the laboratory frame reference. The new independent variable is given by $z_1 = z - \zeta(x, y, t)$. The new functions θ_1, α_1, v_1, p_1 is introducing as follow:

$$\theta(x, y, z, t) = \theta_1(x, y, z_1, t), \quad \alpha(x, y, z, t) = \alpha_1(x, y, z_1, t),$$

$$v(x, y, z, t) = v_1(x, y, z_1, t), \quad p(x, y, z, t) = p_1(x, y, z_1, t).$$

The system of the equations (7) - (10) can be written in the form (the index 1 is omitted):

$$\frac{\partial \theta}{\partial t} - \frac{\partial \theta}{\partial z_1}\frac{\partial \zeta}{\partial t} + (v\tilde{\nabla})\theta = \tilde{\Delta}\theta + Z\exp\left(\frac{\theta}{Z^{-1}+\delta\theta}\right)\phi(\alpha), \tag{13}$$

$$\frac{\partial \alpha}{\partial t} - \frac{\partial \alpha}{\partial z_1}\frac{\partial \zeta}{\partial t} + (v\tilde{\nabla})\alpha = Z\exp\left(\frac{\theta}{Z^{-1}+\delta\theta}\right)\phi(\alpha), \tag{14}$$

$$\frac{\partial v}{\partial t} - \frac{\partial v}{\partial z_1}\frac{\partial \zeta}{\partial t} + (v.\nabla)v = -\tilde{\nabla}p + P\tilde{\Delta}v + PR(1+\lambda\sin(\mu t))(\theta-\theta_0)\gamma, \tag{15}$$

$$\frac{\partial v_x}{\partial x} - \frac{\partial v_x}{\partial z_1}\frac{\partial \zeta}{\partial x} + \frac{\partial v_y}{\partial y} - \frac{\partial v_y}{\partial z_1}\frac{\partial \zeta}{\partial y} + \frac{\partial v_z}{\partial z_1} = 0, \tag{16}$$

where:

$$\tilde{\Delta} = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z_1^2} - 2\frac{\partial^2}{\partial x \partial z_1}\frac{\partial \zeta}{\partial x} - 2\frac{\partial^2}{\partial y \partial z_1}\frac{\partial \zeta}{\partial y} + \frac{\partial^2}{\partial z_1^2}\left(\left(\frac{\partial \zeta}{\partial x}\right)^2 + \left(\frac{\partial \zeta}{\partial y}\right)^2\right) - \frac{\partial}{\partial z_1}\left(\frac{\partial^2 \zeta}{\partial x^2} + \frac{\partial^2 \zeta}{\partial y^2}\right),$$

$$\tilde{\nabla} = \left(\frac{\partial}{\partial x} - \frac{\partial}{\partial z_1}\frac{\partial \zeta}{\partial x}, \frac{\partial}{\partial y} - \frac{\partial}{\partial z_1}\frac{\partial \zeta}{\partial y}, \frac{\partial}{\partial z_1}\right).$$

We use the asymptotic expansion method and seek the external and inner solution of the problem (13) - (16) then we find the matching conditions as the same method using in [14]. We obtain the following jump conditions:

$$\left(\frac{\partial \tilde{\theta}^1}{\partial \eta}\right)^2\Bigg|_{+\infty} - \left(\frac{\partial \tilde{\theta}^1}{\partial \eta}\right)^2\Bigg|_{-\infty} = -2A^{-1}\exp\left(\theta^1\right), \tag{17}$$

$$\frac{\partial \tilde{\theta}^1}{\partial \eta}\Bigg|_{+\infty} - \frac{\partial \tilde{\theta}^1}{\partial \eta}\Bigg|_{-\infty} = -A^{-1}\left(\frac{\partial \xi^0}{\partial t} + s\right). \tag{18}$$

where

$$A = 1 + \left(\frac{\partial \zeta^0}{\partial x}\right)^2 + \left(\frac{\partial \zeta^0}{\partial y}\right)^2, \quad s = \tilde{v}_x^0\frac{\partial \zeta^0}{\partial x} + \tilde{v}_y^0\frac{\partial \zeta^0}{\partial y} - \tilde{v}_\eta^0.$$

Using the matching conditions and truncating the development as follows:

$$\theta^0 \approx \theta, \theta^1|_{z_1=-0} \approx Z\theta|_{z_1=0}, \zeta^0 \approx \zeta, v^0 \approx v.$$

The jump conditions can be written in the following form:

$$\left(\frac{\partial \theta}{\partial z_1}\right)^2\Bigg|_{+0} - \left(\frac{\partial \theta}{\partial z_1}\right)^2\Bigg|_{-0} = 2Z\left(1 + \left(\frac{\partial \zeta}{\partial x}\right)^2 + \left(\frac{\partial \zeta}{\partial y}\right)^2\right)^{-1}\exp\left(Z\,\theta|_0\right), \tag{19}$$

$$\frac{\partial \theta}{\partial z_1}\Bigg|_{+0} - \frac{\partial \theta}{\partial z_1}\Bigg|_{-0} = -\left(1 + \left(\frac{\partial \zeta}{\partial x}\right)^2 + \left(\frac{\partial \zeta}{\partial y}\right)^2\right)^{-1}\left(\frac{\partial \zeta}{\partial t} + \left(v_x\frac{\partial \zeta}{\partial x} + v_y\frac{\partial \zeta}{\partial y} - v_z\right)\Bigg|_{z_1=0}\right). \tag{20}$$

Therefore the interface problem are:

For $z_1 > \zeta$:

$$\frac{\partial \theta}{\partial t} + v\nabla\theta = \Delta\theta, \tag{21}$$

$$\alpha = 0, \tag{22}$$

$$\frac{\partial v}{\partial t} + v\nabla v = -\nabla p + P\Delta v + PR(1+\lambda sin(\mu t))(\theta+\theta_0)\gamma, \tag{23}$$

$$div(v) = 0. \tag{24}$$

For $z_1 < \zeta$:

$$\frac{\partial \theta}{\partial t} = \Delta\theta, \tag{25}$$

$$\alpha = 1, \quad v = 0. \tag{26}$$

For $z_1 = \zeta$:

$$\theta|_{\zeta-0} = \theta|_{\zeta+0}, \tag{27}$$

$$\left.\frac{\partial \theta}{\partial z_1}\right|_{\zeta-0} = \left.\frac{\partial \theta}{\partial z_1}\right|_{\zeta+0} = \left(1 + (\zeta'_x)^2 + \left(\zeta'_y\right)^2\right)^{-1}, \tag{28}$$

$$\left.\left(\frac{\partial \theta}{\partial z_1}\right)^2\right|_{\zeta-0} = \left.\left(\frac{\partial \theta}{\partial z_1}\right)^2\right|_{\zeta+0} = -2Z\left(1 + (\zeta'_x)^2 + \left(\zeta'_y\right)^2\right)^{-1} \exp\left(Z\,\theta|_{\zeta}\right), \tag{29}$$

$$v_x = v_y = v_z = 0. \tag{30}$$

The boundary conditions are

$$z_1 = -\infty : \theta = 0, v = 0; \quad z_1 = +\infty : \theta = -1, v = 0. \tag{31}$$

4. Linear stability analysis

To study the linear analysis stability of solution, we seek the solution in this form:

$$\theta = \theta_s + \tilde{\theta}, \quad p = p_s + \tilde{p}, \quad v = v_s + \tilde{v}, \tag{32}$$

where:

$$(\theta_s(z_2), \alpha_s(z_2)) = \begin{cases} (0,1), & if \quad z_2 < 0 \\ (\exp(-uz_2) - 1, 0), & if \quad z_2 > 0 \end{cases} \tag{33}$$

with $z_2 = z_1 - ut$, and $\tilde{\theta}$, $\tilde{p}$ and $\tilde{v}$ are the perturbations.
Substituting (32) in (25) and (21) - (24), we obtain for the terms of the first order:

for $z_2 > \xi$:

$$\frac{\partial \tilde{\theta}}{\partial t} = \Delta\tilde{\theta} + u\frac{\partial \tilde{\theta}}{\partial z_2} - \tilde{v}_z\theta'_s, \tag{34}$$

$$\frac{\partial \tilde{v}}{\partial t} = -\nabla\tilde{p} + P\Delta\tilde{v} + u\frac{\partial \tilde{v}}{\partial z_2} + PR\left(1+\lambda\sin(\mu t)\right)\tilde{\theta}\gamma, \tag{35}$$

$$div(\tilde{v}) = 0, \quad (36)$$

for $z_2 < \xi$:

$$\frac{\partial \tilde{\theta}}{\partial t} = \Delta\tilde{\theta} + u\frac{\partial \tilde{\theta}}{\partial z_2}, \quad (37)$$

where: $\xi = \zeta - ut$

We note: $\tilde{\theta} = \hat{\theta}_1$ for $z_2 < \xi$ and $\tilde{\theta} = \hat{\theta}_2$ for $z_2 > \xi$.

Now we linearize the jump conditions by taking account that:

$$\theta_{\xi\pm 0} = \theta_s(0) + \xi\theta_s'(\pm 0) + \tilde{\theta}(\pm 0),$$

and

$$\left.\frac{\partial \theta}{\partial z_2}\right|_{(\xi\pm 0)} = \theta_s'(\pm 0) + \xi\theta_s''(\pm 0) + \left.\frac{\partial \tilde{\theta}}{\partial z_2}\right|_{(\xi\pm 0)},$$

we obtain for the higher order terms

$$\hat{\theta}_2(0) - \hat{\theta}_1(0) = u\xi, \quad (38)$$

$$\hat{\theta}_2'(0) - \hat{\theta}_1'(0) = -u^2\xi - \frac{d\xi}{dt}, \quad (39)$$

$$-u\left(u^2\xi + \hat{\theta}_2'(0)\right) = Z\hat{\theta}_1(0). \quad (40)$$

Using the equations (30) and (24), the boundary conditions can be written in the following form:

$$\hat{v}_z = 0, \frac{\partial \hat{v}_z}{\partial z} = 0. \quad (41)$$

We take the perturbations in the following form:

$$\hat{\theta}_j = \theta(z_2, t)\exp\left(i\left(k_1x + k_2y\right)\right), \qquad j = 1, 2, \quad (42)$$

$$\hat{v}_j = v_2(z_2, t)\exp\left(i\left(k_1x + k_2y\right)\right), \qquad z_2 > \xi, \quad (43)$$

$$\xi = \varepsilon_1(t)\exp\left(i\left(k_1x + k_2y\right)\right). \quad (44)$$

where ε_1 is the amplitude, k_1 and k_2 are the wave numbers according to the two horizontal directions. To eliminate the pressure p and the components of velocity v_x and v_y in the system of equations (34)-(36), we apply the operator rotational to equation (35). Therefore the system of equations (34)-(36) can be written in the following form:

$$\frac{\partial \tilde{\theta}}{\partial t} - u\frac{\partial \tilde{\theta}}{\partial z_2} + \tilde{v}_z\theta_s' = \Delta\tilde{\theta}, \quad (45)$$

$$\frac{\partial}{\partial t}(\Delta\tilde{v}_z) - u\frac{\partial}{\partial z_2}(\Delta\tilde{v}_z) = P\Delta\Delta\tilde{v}_z + PR\left(\frac{\partial^2\tilde{\theta}}{\partial x^2} + \frac{\partial^2\tilde{\theta}}{\partial y^2}\right)\left(1 + \lambda\sin(\mu t)\right). \quad (46)$$

Replacing (42), (43) in (45) and (46), we obtain the following system

$$\frac{\partial \theta}{\partial t} = \theta'' + u\theta' - k^2\theta + u\exp(-uz_2)\,v, \quad (47)$$

$$\frac{\partial}{\partial t}\left(v'' - k^2v\right) = P\left(v'''' - 2k^2v'' + k^4v\right) + u\left(v''' - k^2v'\right) - PRk^2\left(1 + \lambda\sin\left(\mu t\right)\right)\theta, \quad (48)$$

with the boundary conditions:

$$v(0, t) = v'(0, t) = 0, \quad (49)$$

$$\theta'(0, t) = -u\theta(0, t), \quad (50)$$

where $k = \sqrt{k_1^2 + k_2^2}$.

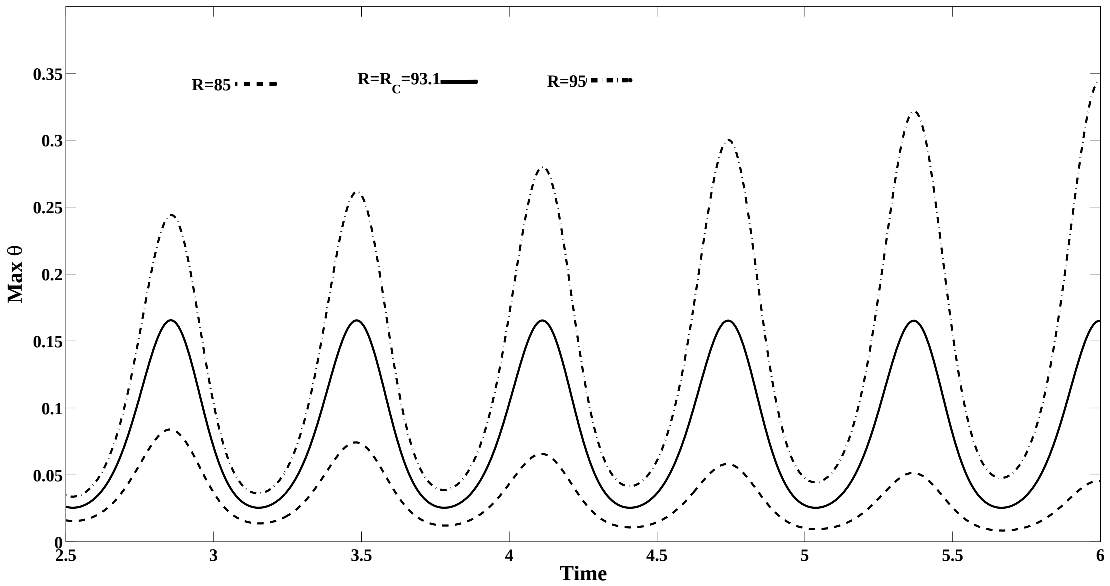

Fig. 1. The evolution of the maximum of temperature as a function of time for $k = 1.5$, $P = 10$, $\mu = 10$, and for $R = 85$, $R = 93.1$ and $R = 95$.

5. Numerical results

We study in this work the cellular instability which means that the eigenvalue crosses the imaginary axis across zero. To find the boundary of convective instability, we begin by reducing the system (47)-(50) in the following form:

$$\frac{\partial\theta}{\partial t} = \theta'' + u\theta' - k^2\theta + u\exp(-uz)\,v, \tag{51}$$

$$\frac{\partial\omega}{\partial t} = P\omega'' + u\omega' - Pk^2\omega + PRk^2\left(1 + \lambda sin\left(\mu t\right)\right)\theta, \tag{52}$$

$$v'' - k^2 v + \omega = 0, \tag{53}$$

in the interval $0 \le z \le L$, with the boundary conditions:

$$z = 0: \quad \theta' = -u\theta, \quad v = v' = 0, \tag{54}$$

$$z = L: \quad \theta = v = \omega = 0. \tag{55}$$

We outlined briefly the multiquadric radial basis function method (MQ-RBF) [15–17] to solve problem (51) – (55), let's discretize the time $t_n = n\Delta t,\ n = 0, ..., nt$ and the space $z_i = i\Delta z,\ i = 0, ..., nz$ where nz and nt are integers and $\Delta z,\ \Delta t$ are receptively the space and time step. We denote $\theta_i^n = \theta(z_i, t_n)$, $\omega_i^n = \omega(z_i, t_n)$ and $v_i^n = v(z_i, t_n)$, and we approximate θ, ω and v respectively by $\widehat{\theta}$, $\widehat{\omega}$ and $\widehat{v}$ as the following:

$$\widehat{\theta}(z, t) = \sum_{j=1}^{N} \chi_j(t)\Phi_j(\|z - z_j\|_2, \varepsilon),$$

$$\widehat{\omega}(z, t) = \sum_{j=1}^{N} \Lambda_j(t)\Phi_j(\|z - z_j\|_2, \varepsilon),$$

$$\widehat{v}(z, t) = \sum_{j=1}^{N} \Gamma_j(t)\Phi_j(\|z - z_j\|_2, \varepsilon),$$

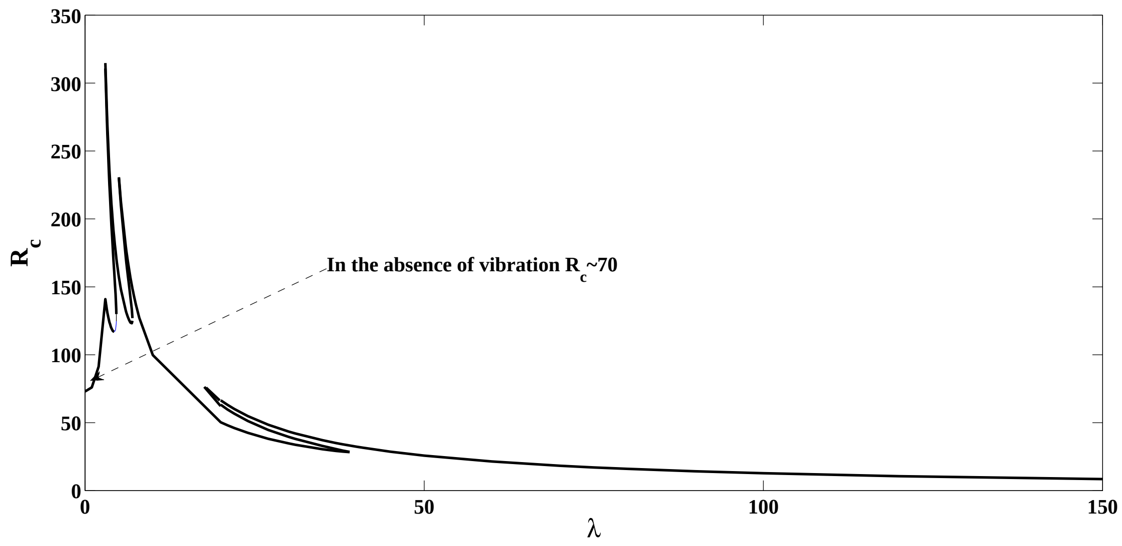

Fig. 2. Convective stability limit curve: critical Rayleigh number as a function of the amplitude of the vibrations for $k = 1.5$, $P = 10$ and $\mu = 10$.

where $\Phi_j(\|z - z_j\|_2, \varepsilon)$ is the multiquadric radial basis function given by $\Phi_j(\|z - z_j\|_2, \varepsilon) = \sqrt{(z - z_j)^2 + \varepsilon^2}$, ε is a shape parameter.

By approximating time by the Euler explicit method and using the MQ-RBF with the implicit schema, the system (51) – (53) can be written as follows:

$$\widehat{\theta}^{n+1} = \widehat{\theta}^n + \Delta t \left(\Phi_{zz} \times \chi^{n+1} + u\Phi_z \times \chi^{n+1} - k^2 \Phi \times \chi^{n+1} + u \exp(-u z(i)) \Phi \times \Gamma^n\right),$$

$$\widehat{\omega}^{n+1} = \widehat{\omega}^n + \Delta t \left(P\Phi_{zz} \times \Lambda^{n+1} + u\Phi_z \times \Lambda^{n+1} - Pk^2 \Phi \times \Lambda^{n+1} + PRk^2(1 + \lambda \sin(\mu t^n)) \Phi \times \chi^n\right),$$

$$\widehat{v}^{n+1} = \frac{1}{k^2}\left(\Phi_{zz} \times \Gamma^{n+1} + \Phi \times \Lambda^{n+1}\right),$$

where $\times$ is a matrix-vector multiplication.

To avoid that there is no boundary condition for vorticity ω at $z = 0$, we introduce a second order artificial boundary condition for vorticity in these form (see [18]):

$$2\omega_1 + \omega_2 = -\frac{6v_2}{(\Delta z)^2}, \tag{56}$$

we use the MQ-RBF with implicit schema to solve the problem (51)-(55) except for the velocity v which is taken from the previous time step for the boundary condition (56). We check that the results don't depend on the length L of the interval.

In Fig. 1, we have represented the evolution of maximum of temperature as a function of time for the values $\lambda = 2$, $k = 1.5$, $P = 10$ and $\mu = 10$. The critical value obtained is $R_c = 93.1$. Below this value, the oscillation amplitude remains bounded. When R exceeds its critical value the amplitude is unbounded as function of time.

The critical value of the Rayleigh number as a function of the amplitude for different frequencies are shown in Figs. 2 and 3. If $\lambda = 0$, we obtain the same critical value $R_c \approx 70$ that found by Garbey et al. [9], [19] in the case without vibrations. For small vibration amplitudes, we observe a stabilization of reaction front: $R_c(\lambda)$ is an increasing function of λ. For λ large, there is a decreasing branch in the stability curve (see Figs. 2 and 3), it corresponds to parametric instability where high amplitude vibrations destabilize the front.

In Fig. 3 (for a larger frequency of excitation $\mu = 15$), it is observed that the stability curve which separates the two stable and unstable zones has two additional branches thus increasing the area of the surface in the parameter diagram where the solution exists and remains bounded. These extra branches are connected to the main stability region for large values of λ. If we fix $\lambda = 50$ and increase the value of R. For $R < 19.5$, the solution is stationary corresponding to negative real eigenvalues and this solution loses its stability when R crosses for the first time the stability limit $R = 19.5$. For this critical Rayleigh number, the eigenvalue with the largest real part exceeds 0. For larger values of R, the dominant eigenvalue crosses again 0, but this time from the right half-plane to the left half-plane. The solution

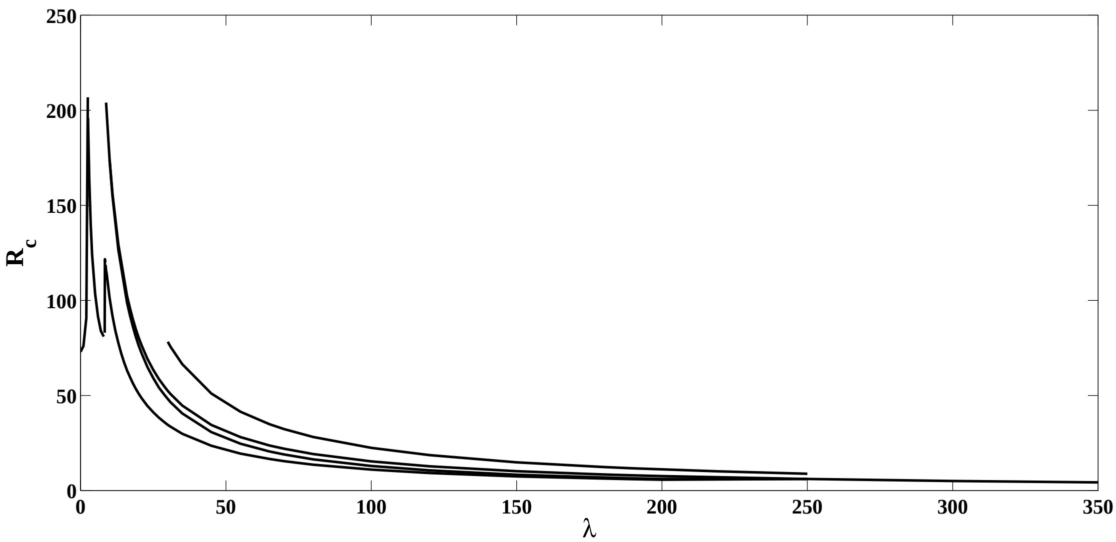

Fig. 3. Limit curve of convective stability: critical Rayleigh number as a function of the amplitude of the vibrations for $k = 1.5$, $P = 10$ and $\mu = 15$.

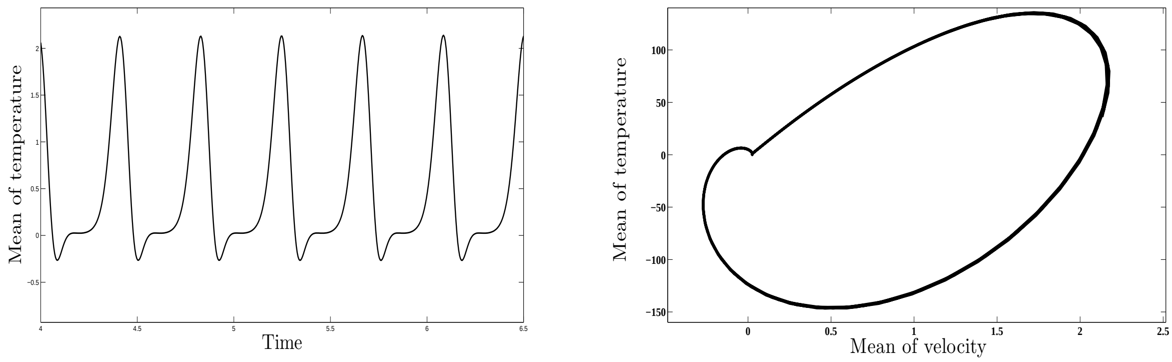

Fig. 4. Mean of temperature as a function of time (left), Phase portrait (right), for $k = 1.5$, $P = 10$, $\mu = 15$ and $R = 19.5$.

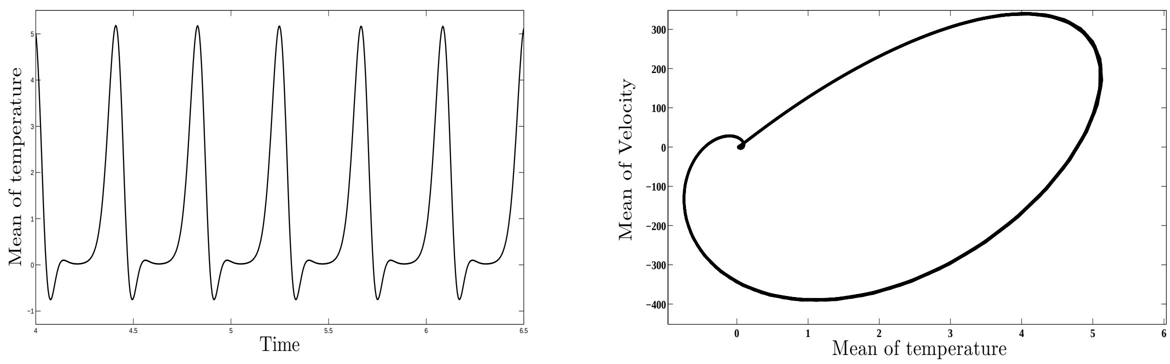

Fig. 5. Mean of temperature as a function of time (left), Phase portrait (right), for $k = 1.5$, $P = 10$, $\mu = 15$ and $R = 24.69$.

becomes stable again. If we increase R again, it pass to 0 for the third time, which makes the solution unstable again. In Fig. 3 and by using the method MQ-RBF another branch was detected for larger values of R and was not detected by the implicit finite difference method used in [14].

To study the influence of convection on the stability of the front of the polymerization reaction, we consider the parameter diagram of Fig. 3 ($k = 1.5$, $P = 10$ and $\mu = 15$), we fix λ to value 50 and we increase R, when R reaches its first critical value $R_c = 19.5$ the stationary solution loses its stability and the stable solution becomes oscillatory and periodic of period $T \sim 0.4$ (see Fig. 4 to the left) following a Hopf bifurcation. Indeed, the dominant eigenvalue crosses for the first time the imaginary axis, at this first critical value the phase portrait presents a closed curve (a limit cycle) (see Fig. 4 on the right).

If we continue to increase R, the periodic solution loses its stability and diverges when R reaches its second critical value $R_c = 24.69$, the solution stabilizes again and oscillates with the same previous period $T \sim 0.4$ (see Fig. 5). In fact, at this critical value of R, the system returns to the stability region.

For critical values $R_c = 28.16$ and $R_c = 41.15$, the solution is periodic but of period $2T \sim 0.8$ (see Figs. 6 and 7). There is thus a doubling of period and the corresponding cycles are split into two symmetrical parts. When the value of R reaches the critical value $R_c = 41.49$, the evanescent solution becomes periodic of period T (see Fig. 8) and if R exceeds this critical value the solution diverges. Note that the amplitude of periodic oscillations increases with the value of R_c.

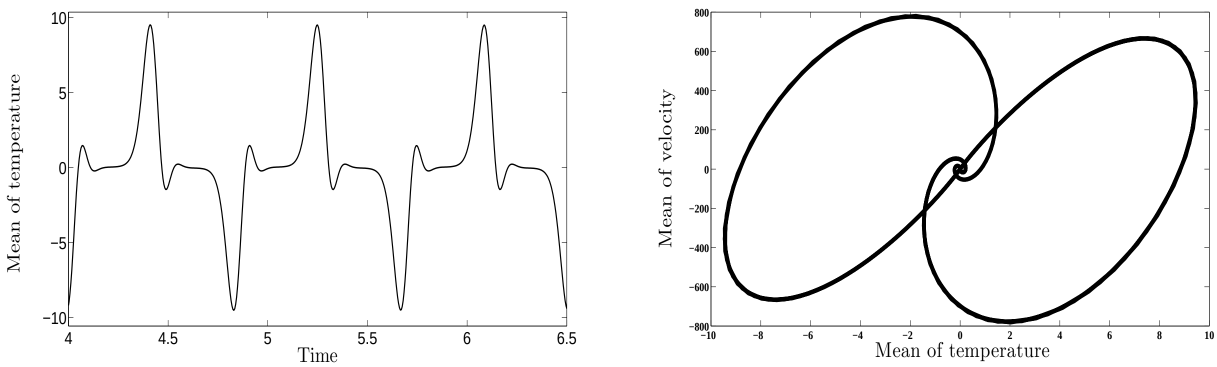

Fig. 6. Mean of temperature as a function of time (left), Phase portrait (right), for $k = 1.5$, $P = 10$, $\mu = 15$ and $R = 28.16$.

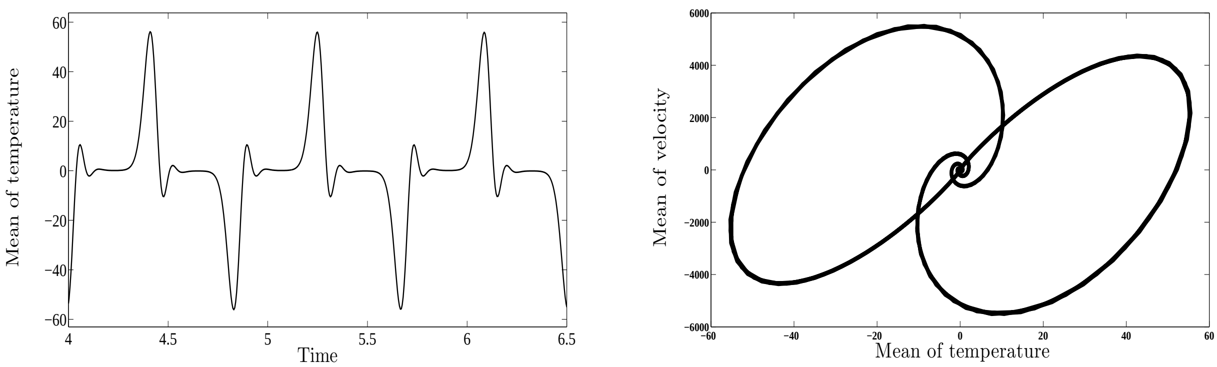

Fig. 7. Mean of temperature as a function of time (left), Phase portrait (right), for $k = 1.5$, $P = 10$, $\mu = 15$ and $R = 41.15$.

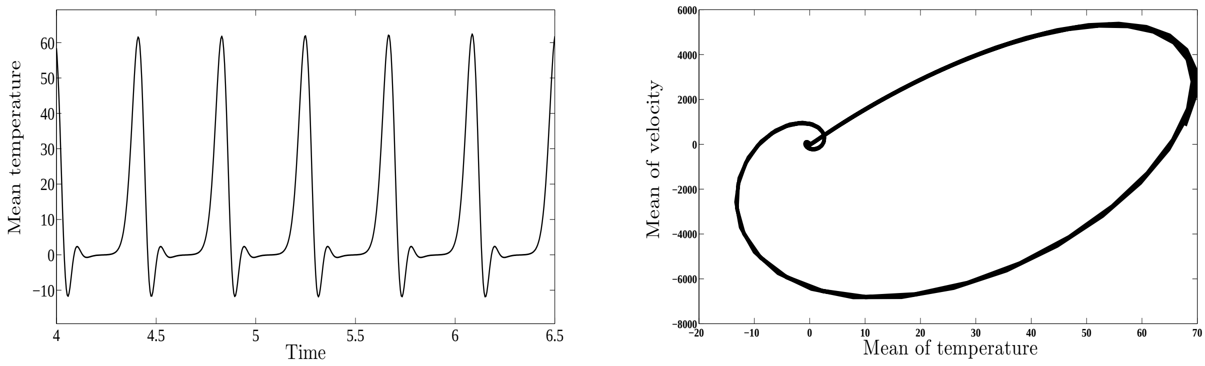

Fig. 8. Mean of temperature as a function of time (left), Phase portrait (right), for $k = 1.5$, $P = 10$, $\mu = 15$ and $R = 41.49$.

6. Conclusion

In this work, we have studied the effect of vertical harmonic gravitational modulation on the convective stability of the reaction fronts in liquid media. The reagent is supposed to be liquid while the product is solid. The liquid reagent is heated from below in contact with the thin layer where the exothermic reaction separating the reagent takes place from the product called the reaction front. Under certain conditions, this convection can destabilize the front. To approach the threshold of convective instability, the problem of the original reaction-diffusion is first reduced to a singular perturbation using an asymptotic development. Thus, the linear stability analysis of the stationary solution for the interface problem is performed. The reduced interface problem obtained is then solved numerically by using to the meshless method MQ-RBF. It has been shown that for relatively small values of amplitudes λ, the periodic vibration has a stabilizing effect (Figs. 2 and 3). The results also revealed the existence of other branches of marginal stability in the parameter plane (λ, R). For a fixed value of the vibration amplitude λ, if we increase the value of the critical Rayleigh number R_c, we successively find a stationary oscillating solution with a period T then with a double period $2T$ and to oscillate back into the last limit of stability with a period of oscillation T (Figs. 4-8). The results of this work show that in the presence of a vibration, the instability of convection of the reaction fronts in liquid media can be controlled and the reaction fronts can remain stable in certain regions of the parameter diagram and particularly for vibrations of small amplitudes.

References

[1] A.P. Aldushin, S.G. Kasparyan, Thermodiffusional instability of a combustion front, Soviet Physics–Doklady 24 (1979) 29–31.

[2] K.G. Shkadinsky, B.I. Khaikin, A.G. Merzhanov, Propagation of a pulsating exothermic reaction front in the condensed phase, Combustion, Explosion, and Shock Waves 7 (1971) 15–22.

[3] R. Chand, A. Kumar, Thermal instability of rotating Maxwell visco-elastic fluid with variable gravity in porous medium, International Journal of Advances in Applied Mathematics and Mechanics 1(2)(2013) 30–38.

[4] P. Clavin, Dynamic behavior of premixed flame fronts in laminar and turbulent flows, Progr. Energy Comb. Sci. 11 (1985) 1–59.

[5] B.J. Matkowsky, G.I. Sivashinsky, Propagation of a pulsating reaction front in solid fuel combustion, SIAM J. Appl. Math. 35 (1978) 465–478.

[6] B.J. Matkowsky, G.I. Sivashinsky, An asymptotic derivation of two models in flame theory associated with the constant density approximation, SIAM J. Appl. Math. 37 (1979) 686–699.

[7] K. Allali, O. Khalfi, On convective instability of reaction fronts in porous media using the Darcy-Brinkman formulation, International Journal of Advances in Applied Mathematics and Mechanics 2(2)(2014) 7–20.

[8] L. Salhi, A. Taik, Reaction fronts in porous media, influence of Lewis number, linear stability analysis, International Journal of Advances in Applied Mathematics and Mechanics 5(1)(2017) 15–27.

[9] M. Garbey, A. Taik, V. Volpert, Linear stability analysis of reaction fronts in liquids, Quart. Appl. Math. 54 (1996) 225–247.

[10] A. Taik, K. Allali, F. Bikany, V. Volpert, Influence of vibrations on convective instability of reaction fronts in liquids, Mathematical Modelling of Natural Phenomena 5(7)(2010) 35–41.

[11] K. Allali, F. Bikany, A. Taik, V. Volpert, Linear stability analysis of reaction fronts propagation in liquids with vibra-

tions, Int Electron J Pure Appl Math. 1(2)(2010) 196–215.

[12] H. Aatif, K. Allali, K. El Karouni, Influence of vibrations on convective instability of reaction fronts in porous media, Mathematical Modelling of Natural Phenomena 5(5)(2010) 123–137.

[13] B.V. Novozhilov, The rate of propagation of the front of an exothermic reaction in a condensed phase, J. Dokl. Phys. Chem. 141 (1961) 836–838.

[14] K. Allali, V. Volpert, J.A. Pojman, Influence of vibrations on convective instability of polymerization, Journal of Engineering Mathematics 41 (2001) 13–31.

[15] F. Benkhaldoun, A. Halassi, D. Ouazar, M. Seaid, A. Taik, A stabilized meshless method for time-dependent convection-dominated flow problems, Mathematics and Computers in Simulation 137 (2017) 159–176.

[16] Y. Alhuri, F. Benkhaldoun, D. Ouazar, M. Seaid, A. Taik, A meshless method for numerical simulation of depth–averaged turbulence flows using a $k--\epsilon$ model, International Journal for Numerical Methods in Fluids 80 (1)(2016) 3–22.

[17] K. Allali, Y. Joundy, A. Taik, V. Volpert, Dynamics of convective thermal explosion in porous media, To apear in International Journal of Bifurcation and Chaos (2020).

[18] L.G. Loitsyanskii, Mechanics of Liquids and Gases, Pergamon Press, pp. 802, New York, 1966.

[19] M. Garbey, A. Taik, V. Volpert, Influence of natural convection on stability of reaction fronts in liquids, Quart. Appl. Math. 53 (1998) 1–35.