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Generating Chaotic Sounds of 1-D Maps with its Application

Research Article

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Abstract: Dynamical System is a branch of mathematics and worldwide research interested field in mathematics. There are major five tool kits of dynamical systems and chaos is one of them interesting in the present world. In all branches of science, chaos is present. So it is the center of the researchers in mathematics all over the world. In this research, We have discussed the one-dimensional chaotic maps like tent map, cubic map, and sine map. We tried to generate the chaotic sound using these maps and these sounds are huge applications in our real life. To perform this work we have used the MATHEMATICA software.

MSC: 37A10 • 37D45

Keywords: Dynamical System • Chaos • 1-D Maps • Scratching Sound • Stick-slip Motion

1. Introduction

Recent research on annoying sounds generated by scratching on a chalkboard with fingernails has been presented from the viewpoint of psychoacoustics [3, 4]. In this research, We propose some mathematical models for generating a scratching sound. Such a sound can be generated from the stick-slip motion of a frictional system, and it is shown here that this mechanism is closely related to the tent map, cubic map, and sine map. These present mathematical models are generating annoying scratching sounds. Such sounds are generated by frictional motion and are attributed to the chaotic nature of the frequency spectrum thereby produced [9]. The proposed models are based on the tent map, cubic map, and sine map and are modified to have the stick-slip property of a frictional vibration.

2. Basic Preliminaries

2.1. Dynamical Systems

Dynamical Systems is a branch of mathematics that attempts to understand processes in motion. For instance, the motion of the stars and galaxies in heaven is a dynamical system. The world's weather is another system that changes in time as is the stock market.

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2.2. Iteration

Iterate means to repeat a process over and over again. To iterate a function means to apply the function over and over again, using the output of the previous evaluation as the input for the next. For a function $f(x)$, $f^2(x)$ is the second iterate of f,namely $f(f(x))$, $f^3(x)$ is the third iterate $f(f(f(x)))$, and in general, $f^n(x) = f \circ f \circ f \circ \cdots \circ f(x)$ is the n-fold composition of f with itself.

2.3. Linear iteration

An iteration rule of the form $x \to Ax + B$ (A, B constants) is called a linear iteration rule.

2.4. Piece-wise linear iteration

An iteration rule of the form $x \to A|x| + B$ (A, B : constants) is called a piecewise linear iteration rule since the rule comes in two linear pieces for $x > 0$ and $x < 0$.

2.5. Graphical iteration

Graphical iteration means to display the orbit of initial seed x_0 geometrically under the given function $f(x)$, at first we draw a vertical line from the diagonal $y = x$ to the graph $f(x)$, then draw a horizontal line from the graph $f(x)$ back to the diagonal $y = x$.

2.6. Non-linear iteration

An iteration rule of the form $x \to Ax^2 + Bx + C$ $(A, B, C$ constants) is called a Nonlinear iteration rule.

2.7. Orbit

Given Given $x_0 \in R$ (x_0 is called the seed or initial value of the orbit), we define the orbit of under f to be the sequence $\{x_0,\ x_1 = f(x_0),\ x_2 = f^2(x_0), \cdots, x_n = f^n(x_0), \cdots\}$.

2.8. Fixed Points

Let $f : \mathbf{R} \to \mathbf{R}$ be a map. The point x_0 is called fixed point if $f(x_0) = x_0$. Note that and in general $f^n(x_0) = x_0$.

2.9. Attracting fixed point

The point x_0 is called an attracting fixed point if $\left|f'(x_0)\right| < 1$.

2.10. Repelling fixed point

The point x_0 is called repelling fixed point if $\left|f'(x_0)\right| > 1$.

2.11. Neutral fixed point

The point x_0 is called a neutral fixed point if $\left|f'(x_0)\right| = 1$. Sometimes a neutral fixed point is called indifferent or hyperbolic fixed point.

2.12. Periodic orbit or cycle

The point x_0 is called periodic if $f^n(x_0) = x_0$ for some $n > 0$, where n is called the prime period of the orbit.

2.13. Chaotic Orbits

Over the last twenty-five years, one of the major developments in mathematics is that many simple functions like the quadratic function of real variable exhibits orbits of incredible complexity known as "sensitivity to initial conditions" and also called chaotic behavior. This is our first idea of chaotic behavior, the foremost important tool of dynamical activities.

2.14. Sensitivity on Initial Conditions

Mathematically, consider the metric space X equipped with the metric d and the continuous map $f : X \to X$ .We says that the map f exhibits sensitive dependence on initial conditions if $\exists \delta > 0$ called the sensitivity constant of f

such that for any $x \in X$ and any open neighborhood $N_\delta(x)$ of x for some $\varepsilon > 0$ there exists a point $y \in N_\delta(x)$ and $n \geqslant 0$ such that $d\left(f^n(x), f^n(y)\right) \geqslant \delta$.

2.15. Devaney's Definition of Chaos [2]

Let X be a metric space. A continuous function $f : X \to X$ is said to be chaotic on X if f has the following three properties:

(C-1) Periodic points are dense within the space X

(C-2) f is topologically transitive

(C-3) f has sensitive dependence on initial conditions

Mathematically,

(C-1) $P_k(f) = \{x \in X : f^k(x) = x \, (\exists k \in \mathbf{N})\}$ is dense in X.

(C-2) For $\forall\, U, V : non-empty\, open\, sets$ of X, such that

(C-3) $\exists\delta > 0$ (sensitive constant) which satisfies: $\forall\, x \in X\; and\; \forall N(x,\varepsilon)$, $\exists\, y \in N(x,\varepsilon)$ and $\exists\, k \leqslant 0$ such that $d(f^k(x), f^k(y)) > \delta$.

3. Tent Map

In mathematics, the tent map with parameter μ is the real-valued function f_μ defined by $T_\mu := \mu \min\{x,\, 1-x\}$, the name being due to the tent-like shape of the graph of f_μ . For the values of the parameter within 0 and 2, $T_\mu$maps the unit interval $[0, 1]$ into itself, thus defining a discrete-time dynamical system on it. In particular, iterating a point x_0 in $[0,\ 1]$gives rise to a sequence x_n:

$$x_{n+1} = T_\mu(x_n) = \begin{cases} 2\mu x_n \text{ for } x_n < \dfrac{1}{2} \\ 2\mu(1-x_n) \text{ for } \dfrac{1}{2} \leqslant x_n \end{cases}$$

where μ is a positive real constant.

3.1. Frictional Motion and the Tent Map

Particle motion in one dimension under the influence of periodic impulsive forces was described in [5]. The Tent map is shown below of the system

$$x_{n+1} = \begin{cases} 2\lambda x_n\, ; 0 \leqslant x_n < {}^1/_2 \\ 2\lambda\,(x_n - 1)\, ; {}^1/_2 \leqslant x_n \leqslant 1 \end{cases} \tag{1}$$

Where the parameter $\lambda, 0 \leqslant \lambda \leqslant 1$, determines the range of x_n.

3.2. Chaotic Behavior of the Tent Map

The Tent map is a classic example of chaotic behavior, and it is found to be strongly related to frictional vibration. The sound generated by the Tent map, with the parameter λ in equation (1) varying from 0.5 to 1, is considered [6].

Fig. 1. Original sound. Sample rate: 2 kHz: $0.5 \leqslant \lambda \leqslant 1.0$.

This following slight modification of the program features a frequency spectrum almost like to a scratching sound.

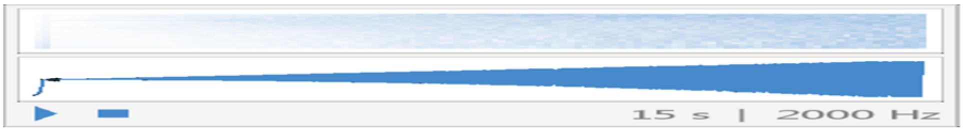


Fig. 2. High sample-rate sound. Sample rate: 2 kHz: $0.5 \leqslant \lambda \leqslant 1.0$.

One of the major modifications is that the sample rate is increased to 2kHz. An annoying chilling reaction appears to coincide with the Tent system entering or leaving a chaotic phase.

3.3. Representation of Stick-Slip Motion

The scratching model described above may be a periodic slip model. Frictional motions generally are described as stick-slip phenomena. One of the foremost beautiful sounds to several people is that produced by bowing a violin or other musical instrument. On the opposite hand, the sound produced by scratching a chalkboard with the fingernails is typically considered highly annoying. In both cases, however, the mechanism for producing sounds is that the same, that is, the stick-slip phenomenon of a frictional system. This comprised of a repetition of a stick stage and a slip stage. Fig. 3 shows the sound for the actual stick-slip motion of a bowed string. Periodic motions have been observed, with each period showing a high rising motion followed by an interval of decrease. The preceding corresponds to the slip stage which is the motion of short duration and the latter equivalents to the stick stage, where the string moves at a continuing speed alongside with the bow and continues until the slip occurs.
To import the stick-slip property into the Tent map, we introduce the following rules:

(a) If $x_{n+1} \geqslant x_n$ in equation (1) then transition occurs immediately and we called this the slip motion.

(b) If $x_{n+1} < x_n$ then the value of x within (x_{n+1}, x_n)decreases linearly at a constant speed Δx and we called this the stick motion. The value determines the speed of the bow.

(c) Thus we obtain the following modified Tent map representing the stick-slip phenomenon.

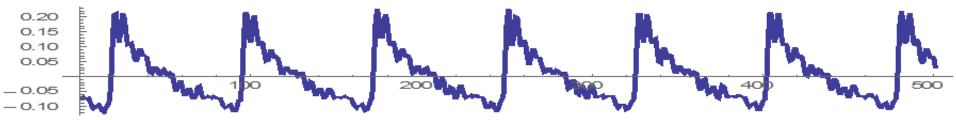


Fig. 3. Waveform of the bowed string, sampling frequency 50 kHz. Number of points: 500.

3.4. The Scratching-Sound Generator

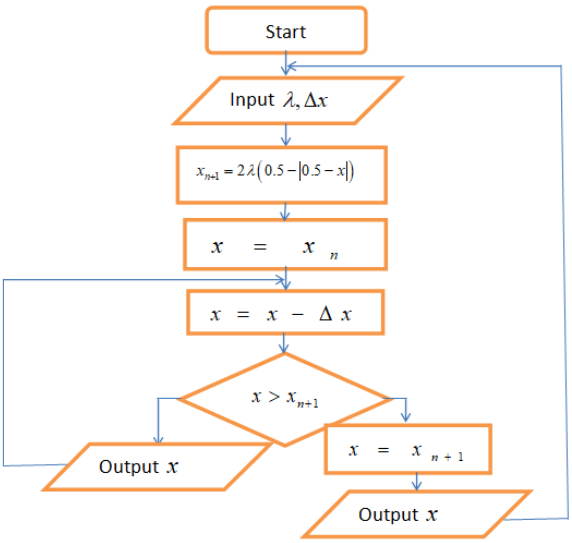


Fig. 4. Flow chart of the modified Tent map.

The flow chart of the new Tent map discussed above is shown in Fig. 4. The left branch describes the stick stage and therefore the right branch describes the slip stage. The velocity parameter Δx gives the bowing speed. Here we show the behavior of the Tent map. The value λ gives the height of the triangle and can be varied from 0.5 to 1, while x_0 is the initial value of x [7]

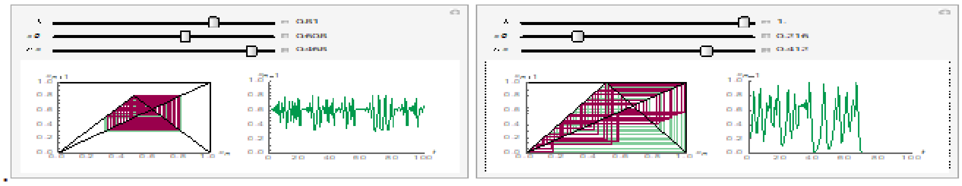
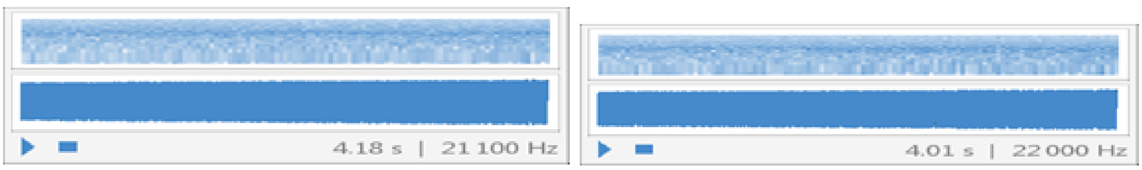


Fig. 5. Tent map (left) and the generated waveform (right). λ: the height of the triangle. x_0: the initial value. Δx: the decreasing speed.

3.5. Examples of Scratching Sound

3.5.1. Linearly Modulated Model

The value $\lambda_0 = 0.90, 0.95$ and is modulated linearly sound is given below also between 0.9 and 0.95.



(a)Final value of $\lambda = 0.999995$ (b)Final value of $\lambda = .949997$

Fig. 6. Sound generated with the Tent map. Sample rate: 21.1 kHz and 22kHz. λ: linearly driven from 0.9 to 0.95; $\Delta x = 0.08$.

3.5.2. Sinusoidally Modulated Model

The value λ is modulated sinusoidally between 0.901 and 0.911. $\lambda_{min} = .901, \lambda_{max} = .911$

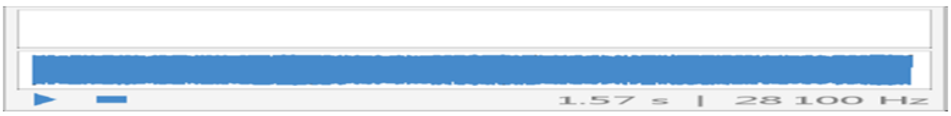


Fig. 7. Sound generated with the Tent map; Sample rate: 28.1 kHz. λ: Sinusoidally driven. Δx=0.08.

3.5.3. Randomly Modulated Model

The value λ is modulated with a low pass-filtered random noise [5]. $\lambda_{min} = .896863$, $\lambda_{max} = .92398$

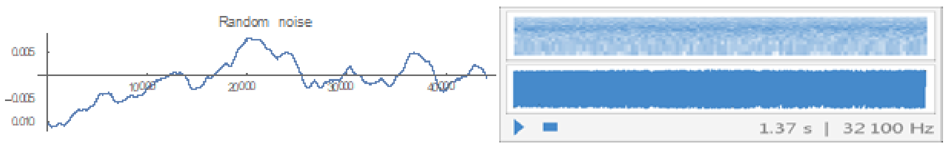


Fig. 8. Sound generated with the Tent map. Sample rate: 32.1 kHz. λ: randomly driven. Δx=0.08.

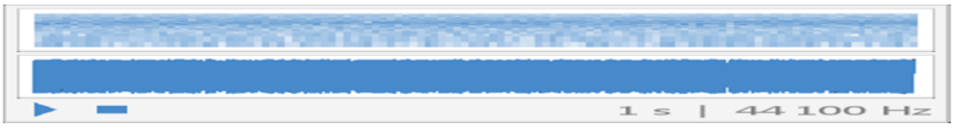


Fig. 9. Sound generated with the value λ fixed to 0.8

Fig. 9 shows the sound with the value λ set to 0.8, which sounds like stationary random noise. This result implies that chaos itself does not necessarily cause chills, but rather that chills are due to the transition in and out of chaos." (This has been added in the finally edited manuscript.

4. Cubic Map

The Cubic map $f : \mathbb{R} \to \mathbb{R}$ is defined by $f(x) = x^3 - rx$. Where r is the parameter, which is one of the simplest polynomial maps of the desired type. If r is restricted to the range $0 \leqslant r \leqslant 3$ then f maps the interval $x \in [-1, 1]$ into itself and we will study the family $f = f_r$ for these parameter values.

4.1. The Scratching-Sound Generator

The flow chart of the new Cubic map discussed in Fig. 10. The left branch describes the stick stage and the right branch describes the slip stage [7]. The velocity parameter Δx gives the bowing speed.

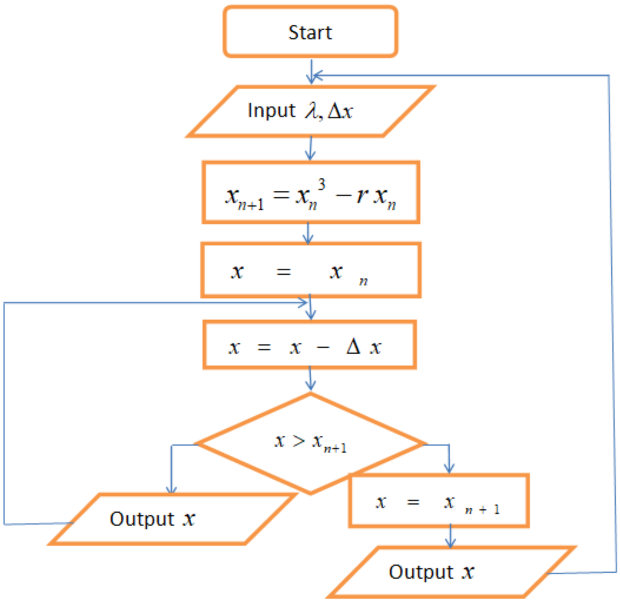


Fig. 10. Flow chart of the modified Cubic map.

Here we show the behavior of the Cubic map. The value λ gives the height of the staircase and can be varied from 0.0 to 1.0, while x_0 is the initial value of x.

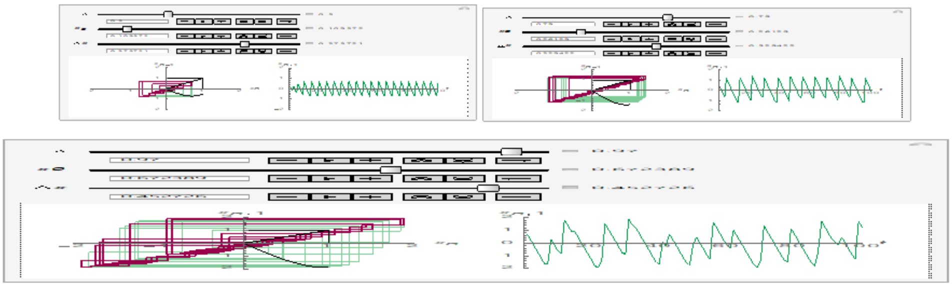
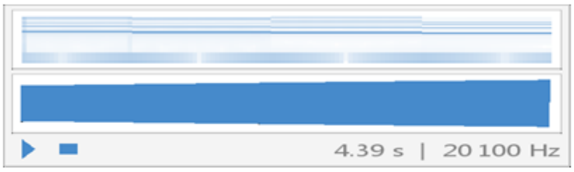


Fig. 11. Cubic map (Left) and the generated waveform (right). λ: the height of the staircase. x_0: the initial value. Δx: the decreasing speed.

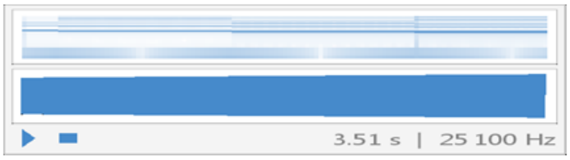
4.2. Examples of Scratching Sound

4.2.1. Linearly Modulated Model

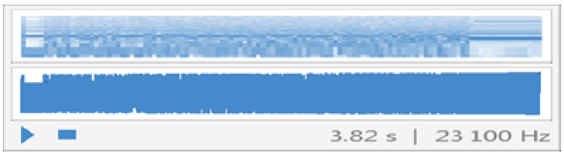
The value λ is modulated linearly between 0.40 to 0.45 (Fig. 12(a), Fig. 12(b)), and 0.9 and 0.95 (Fig. 12(c), Fig. 12(d)), modulated linearly sound is periodic which is given in Fig. 12.



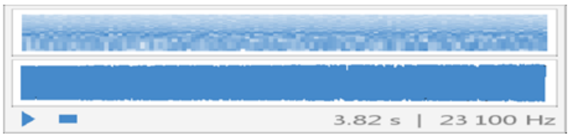
(a) Final value $\lambda = 0.45$



(b) Final value $\lambda = 0.499991$



(c) Final value $\lambda = 0.949997$



(d) Final value $\lambda = 0.949997$

Fig. 12. Sound generated with the Cubic map. Sample rate: 23.1 kHz.

4.2.2. Sinusoidally Modulated Model

The value λ is modulated sinusoidally between 0.901 and 0.911 [5].

$$\lambda_{\min} = .901, \ \lambda_{\max} = .911$$

Fig. 13. Sound generated with the Cubic map.Sample rate: 28.1 kHz. λ: Sinusoidally driven. Δx=0.08

4.2.3. Randomly Modulated Model

The value λ is modulated with a low pass-filtered random noise [6].

$$\lambda_{\min} = 0.918106, \ \lambda_{\max} = 0.947146$$

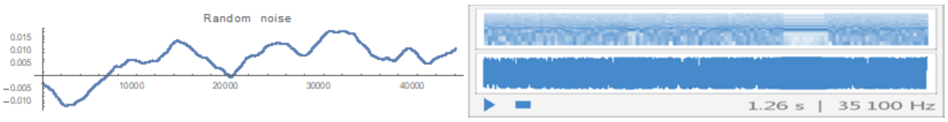


Fig. 14. Sound generated with the Cubic map. Sample rate: 32.1 kHz. λ: randomly driven. Δx=0.08.

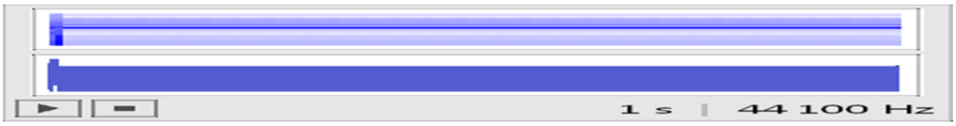


Fig. 15. Sound generated with the value λ fixed to 0.8

5. Sine Map

The Sine map is defined by $f(x) = \lambda \sin(\pi x); x \in [0,1]$, $\lambda \in [0,1]$, here λ is a parameter whose value lies between 0 and 1.

5.1. The Scratching-Sound Generator

The flow chart of the new Sine map is shown in Fig. 16. The left branch describes the stick stage and the right branch describes the slip stage [7]. The velocity parameter Δx gives the bowing speed.

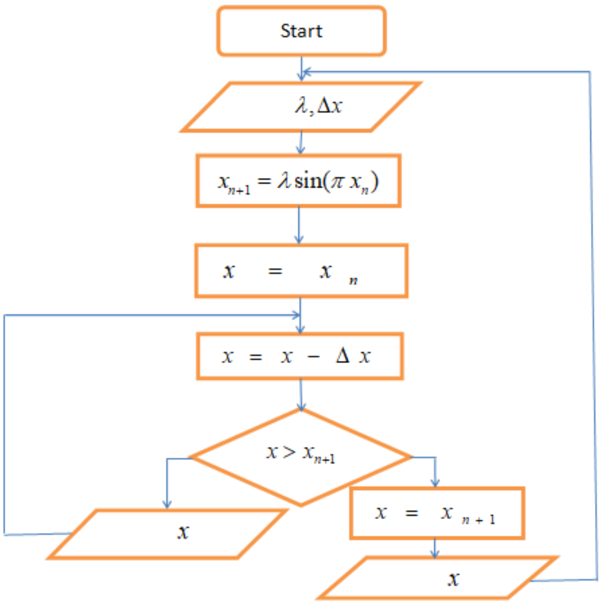


Fig. 16. Flow chart of the Sine map

Here we show the behavior of the Sine map. The value λ gives the height of the parabola and can be varied from 0 to 1, while x0 is the initial value of x.

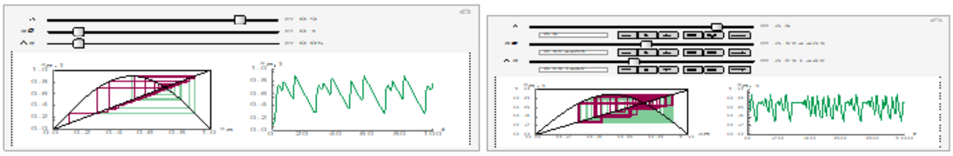


Fig. 17. Sine map (left) and the generated waveform (right). λ: the height of the parabola. x_0: the initial value. Δx: the decreasing speed.

5.2. Examples of Scratching Sound

5.2.1. Linearly Modulated Model

The value λ is modulated linearly between 0.8 and 0.95. Final value of $\lambda = 0.999995$

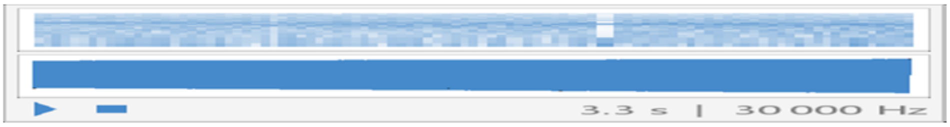


Fig. 18. Sound generated with the Sine map. Sample rate: 30 kHz. λ: linearly driven from 0.9 to 0.95; Δx=0.08.

5.2.2. Sinusoidally Modulated Model

The value λ is modulated sinusoidally between 0.901 and 0.911 [6].

$$\lambda_{\min} = .901, \lambda_{\max} = .911$$

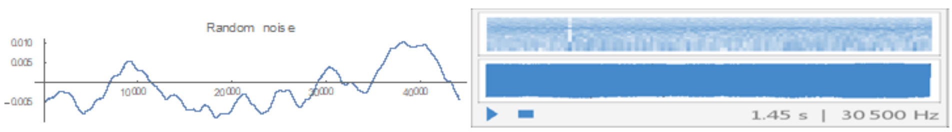
Fig. 19. Sound generated with the Sine map. Sample rate: 28 kHz. λ: Sinusoidally driven. Δx=0.08.

5.2.3. Randomly Modulated Model

The valueλ_0=0.98 is modulated with random noise is given below:

$$\lambda_{\min} = 0.971219, \lambda_{\max} = 0.990103, \quad \lambda_{\min} = 0.880111, \lambda_{\max} = 0.905987$$

The value λ is modulated with a low pass-filtered random noise [2].



$$\lambda_{\min} = .896863, \quad \lambda_{\max} = .92398$$

Fig. 20. Shows the sound with the value λ set to 0.8, which sounds like stationary random noise, and the chilling reaction is less than that of the sound in Fig. 17.

The values λ_0=0.8, the sound is periodic. The figure is given below:

$$\lambda_{\text{min}} = 0.795, \lambda_{\text{max}} = 0.805, \lambda_{\text{min}} = 0.850566, \lambda_{\text{max}} = 0.860566$$

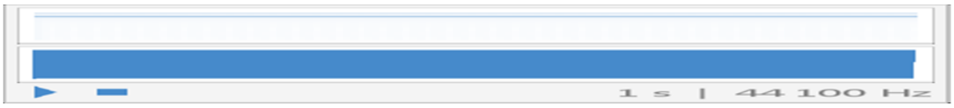


Fig. 21. Sound generated with the value λ fixed to 0.8

Finally, we observe that Figs. 9, 15 and 21 show that the sound with the value λ set to 0.8, which sounds like stationary random noise. This result shows that chaos itself doesn't necessarily cause chills; rather chills are due to the transition in and out of chaos.

6. Results and Discussion

The real example of chaotic sound is thunderstorms sound which is an amazing display of the Creator's might. An average Thunderstorm pours down several hundred million gallons of water like quantity which is equivalent to water flows over Niagara Falls every 6 minutes. Similarly, a storm releases 10 million Kilowatt-hours of energy which is equivalent to a 20 Kiloton nuclear warhead. And large, severe thunderstorms can be 10, or even 100 times more energetic. At any given moment, hundreds of storms are occurring somewhere around the world. This amounts to about 16 million thunderstorms each year. The accompanying lightning illuminates an entire skyline. A bolt may reach over 5 miles in length, contain over 100 million electrical Volts and soar to temperatures approaching 50,000 degrees Fahrenheit in a split second, hotter than the surface of the sun. On average 100lightning strikes the earthevery second. Several million bolts reach earth each day. The tremendous power, the incredible speed, and therefore the glaring flash are clear manifestations of our Creator's majestic power.

Larry Vardiman (Ph.D., Atmospheric Scientist, and Directorof Research at ICR) said, "Thunderstorms are an incredible phenomenon in the atmosphere. They are awesome. They are powerful. They are frightening. They have all kinds of energy releases.That's basically what's happening maybe a thunderstorm is releasing large quantities of energy in our atmosphere by vertical motions. Thunderstorms contribute a huge amount to the water cycle. The hydrologic cycle water is evaporated from the oceans. The vapor is drifted over the continents and then it falls as rain and then flows back into the sea. Thunderstorms are a major part of that cycle, where it converts that water vapor back into liquid water or rain and falls to the earth. When the lightning stroke goes through the air, it releases the nitrogen in the vapor from and absorbs it into the water and it falls down and fertilizer the ground. So our crops are fertilized by the nitrogen produced by thunderstorms."

Dr. Garry Parker (Biologist, Author of the Creation: Facts of Life) said, Lightning and thunderstorms in Scripture are often used as a symbol of God's wrath against rebellious people. But everybody has been stuck by the awesome beauty of lightning and storm the smell of the fresh air. The lightning itself puts together two gases in the air- nitrogen, and hydrogen to make fertilizer [8].

7. Conclusion

I have presented here to create chaotic sound using one-dimensional chaotic maps such as tent map, cubic map, and sine map. These mathematical models produce such sounds as are generated by scratching a chalkboard or glass plate with fingernails. These exhibits chaotic properties and unpleasant chilling reactions are found to be strongly related to chaotic behavior. However, it still remains to understand the psychoacoustic explanation of reactions to such sounds. In the near future, We will try to extend the high frequencies of these sounds and also try to the others one and higher dimensional chaotic maps so that we use these sounds in our real life.

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