Int. J. Adv. Appl. Math. and Mech. 7(3) (2020) 96 – 101 (ISSN: 2347-2529)
Journal homepage: www.ijaamm.com

International Journal of Advances in Applied Mathematics and Mechanics

A Novel Approach for the Solution of a Love's Integral Equations Using Chebyshev Polynomials

Research Article

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Received 17 December 2019; accepted (in revised version) 03 February 2020

Abstract: In this paper a novel technique implementing Chebyshev polynomials is introduced for the numerical solution of a Love's integral equations. The Love's integral equation is a class of second kind Fredholm integral equations, and it can be used to describe the capacitance of the parallel plate capacitor (PPC) in the electrostatic field. This numerical technique developed by Huabsomboon et al. bases on using Taylor-series expansion [1–3] and [4]. We compare the numerical solution using Chebyshev technique with the numerical solution obtained by using Chebyshev expansion. It is shown that the numerical results are excellent.

MSC: Love's integral equation • Fredholm integral equation • Chebyshev polynomials method • Approximate solution

Keywords: 45B05 • 45B99

1. Introduction

The topics of integral equations have been an increasing interest in the past years, because these kinds of equations appear in various fields of applied science and engineering. So, getting solutions with a high level of accuracy for the integral equations is a very important task. Considering that many real-world mathematical problems, especially in the area of applied mathematics are too complicated to be solved in exact terms, the using of numerical methods has been swiftly developed recently. Love's integral equation (Fredholm equation of the second kind) [5–14] has shown a big interest in the several applied physics fields such as polymer structures, aerodynamics, fracture mechanics hydrodynamics and elasticity engineering. For literature related to the numerical solutions of singular integral equations of the deterministic type, a survey of different analytical methods for the solution of random integral equations has been proposed by Bharucha-Reid [5] and Christensen et al. [6]. Love's equations were early established by Love [10–12, 15–18] for solving some magnetic and electrical fields problems. The actual study is concerned with calculation of the normalized field created conjointly by two similar plates of radius R, separated by a distance k×R, where k is a positive real parameter, and at equal or opposite potential, with zero potential at infinity, is the solution of the Love's [16, 17] second kind integral equation:

$$u(x) = g(x) + \int_{-2}^{+2} K(x,t)\, u(t)\, dt \tag{1}$$

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Where u is the normalized ield to be determined, $g(x)$ is a given function and $K(x,t)$ is the rational kernel function with values in $[-1;1]$ and defined by:

$$K(x,t) = \pm\frac{1}{\pi}\frac{k}{k^2+(x-t)^2} \tag{2}$$

Where the sign ± codes for equal or opposite potential cases.

The solution of Love's integral equations using Bernstein polynomials is described in [23]. The solution of Urysohn type of integral equations using Bernstein polynomials, Chebyshev polynomials and Hermite polynomials investigated in [24],[25] and [26] respectively.

2. Chebyshev polynomials

The Chebyshev polynomials, named after Pafnuty Chebyshev, are a sequence of orthogonal polynomials which are related to de Moivre's formula and which can be defined recursively. The general form of the Chebyshev polynomials [6] of nth degree is defined by

$$T_n(x) = \sum_{m=0}^{[n/2]} (-1)^m \frac{n!}{(2m)!(n-2m)!}\left(1-x^2\right)^m x^{n-2m} \tag{3}$$

where

$$[n/2] = \begin{cases} n/2 & \text{if } n \text{ is even} \\ (n+1)/2 & \text{if } n \text{ is odd} \end{cases}$$

The first few Chebyshev polynomials from the equation (2) are given below:

$$T_0(x) = 1, \qquad T_1(x) = x,$$
$$T_2(x) = 2x^2 - 1, \qquad T_3(x) = 4x^3 - 3x,$$
$$T_4(x) = 8x^4 - 8x^2 + 1, \qquad T_5(x) = 16x^5 - 20x^3 + 5x,$$
$$T_6(x) = 32x^6 - 48x^4 + 18x^2 - 1$$

Now the first six Chebyshev polynomials over the interval [-1, 1] are shown in Fig. 1

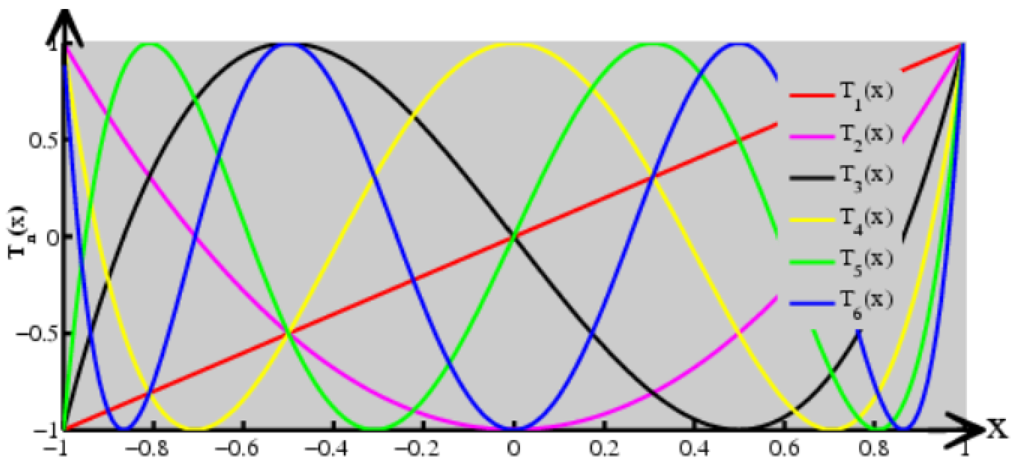

Fig. 1. Graph of first 6 Chebyshev polynomials over the interval $[-1,1]$

3. Solution of a Love's integral equations

In this section, first we consider the Love's integral equation (LIE) of the second kind given by [20, 21]:

$$u(x) \pm \frac{1}{\pi}\int_{-1}^{1} K(x,t)\,u(t)\,dt = 1, \quad -1 \leqslant x \leqslant 1 \tag{4}$$

Where $u(x)$ is the unknown functions to be determined, $(x,t) = \frac{k}{k^2+(x-t)^2}$, the kernel is a continuous function.

The function $u(x)$ may be expanded by a finite series of Chebyshev polynomial as follows:

$$u(x) = \sum_{n=0}^{\infty} c_n T_n(x) \tag{5}$$

where $c_n = (u(x), T_n(x))$. We conceder a truncated series eq. (5) as:

$$u_N(x) = \sum_{n=0}^{\infty} c_n T_n(x) = C^T T(x) \tag{6}$$

where C and T are two vectors given by:

$$C = (c_0, c_1, c_2, ..., c_N), \quad T(x) = (T_0(x), T_1(x), ... T_N(x))^T \tag{7}$$

Then by substituting $u_N(t)$ into eq. (4), we get

$$C^T T(x) \pm \frac{1}{\pi} \int_{-1}^{1} K(x,t) \sum_{n=0}^{\infty} c_n T_n(x)\, dt = 1 \tag{8}$$

Now we use the Chebyshev collocation method which is a matrix method based on the Chebyshev collocation points depended by

$$x_j = -1 + \frac{2}{n}, \quad j = 0, 1, 2, \ldots . N \tag{9}$$

We collocate eq. (8) with the points (9) to obtain

$$C^T T(x_j) \pm \frac{1}{\pi} \int_{-1}^{1} K(x_j, t, C^T T(x)) dx \tag{10}$$

The integral terms in eq. (10) can be found using composite Trapezoidal integration technique as:

$$\int_{-1}^{1} K\left(x_j, t, C^T T(x)\right) dx \approx \frac{h}{2} \left(g(x_0) + g(x_m) + 2 \sum_{k=1}^{m-1} g(x_k) \right) \tag{11}$$

where $g(x) = \int_{-1}^{1} K\left(x_j, t, C^T T(x)\right) dt$ and $h = \frac{1}{m}$ for an arbitrary $x_i = ih, i = 0, 1, ...m$. Therefore eq. (9) together with eq. (10) gives an $(N+1)x(N+1)$ system of linear algebraic equations, which can be solved for $c_k, k = 0, 1, \ldots, N$. Hence the unknown function $x_N(t)$ can be found.

4. Numerical examples

In this section, the method presented in this paper is used to find numerical solution of two illustrative examples. The solution of the equations obtained here. All calculations in the following tables are performed using Matlab.

Example 4.1.
Consider the following Love's integral equation [15]

$$u(x) - \frac{1}{\pi} \int_{-1}^{1} \frac{u(t)}{1 + (x-t)^2} dt = 1 \quad -1 \leqslant x \leqslant 1 \tag{12}$$

Where $u(x)$ is a real and continuous. The numerical result for $u(x)$ where $0 \leqslant x \leqslant 1$ is shown in Table 1. Our approximate solutions agree well with Chebyshev series expansion method [22]. We observe that using $n = 4$ obtain a good approximate solution as well as the absolute error.

Example 4.2.
Consider the following Love's integral equation

$$u(x) + \frac{1}{\pi} \int_{-1}^{1} \frac{u(t)}{1 + (x-t)^2} dt = 1 \quad -1 \leqslant x \leqslant 1 \tag{13}$$

The difference between this equation and the previous equation is the sign in front of the integral term. The numerical result for $u(x)$ and its first derivative are shown in table 2. Our approximate solution agrees well with Chebyshev series expansion method. We observe that using $n = 4$ obtain a good approximate solution as well as the absolute error.

Table 1. The approximate solution for $u(x)$ in Example 4.1 with $n = 4$

x	Chebyshev Approx.	Our Approx.	Error
0.0	1.91903	1.91924	2.10E-04
0.1	1.91592	1.91557	3.50E-04
0.2	1.90659	1.90676	1.70E-04
0.3	1.89112	1.89113	1.00E-05
0.4	1.86964	1.86990	2.60E-04
0.5	1.84238	1.84733	4.95E-03
0.6	1.80974	1.80946	2.80E-04
0.7	1.77227	1.77161	6.60E-04
0.8	1.73075	1.73247	1.72E-03
0.9	1.68617	1.68674	5.70E-04
1.0	1.63969	1.54340	9.63E-02

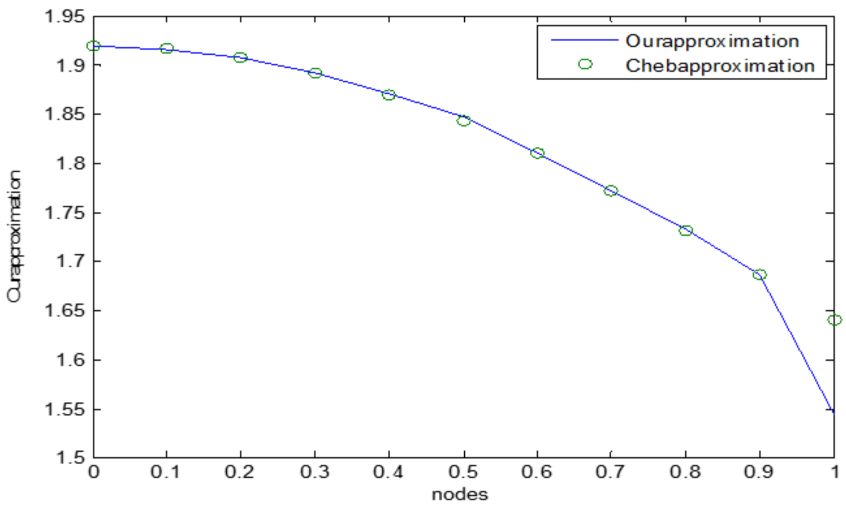

Fig. 2. The approximate solution for $u(x)$ in Example 4.1.

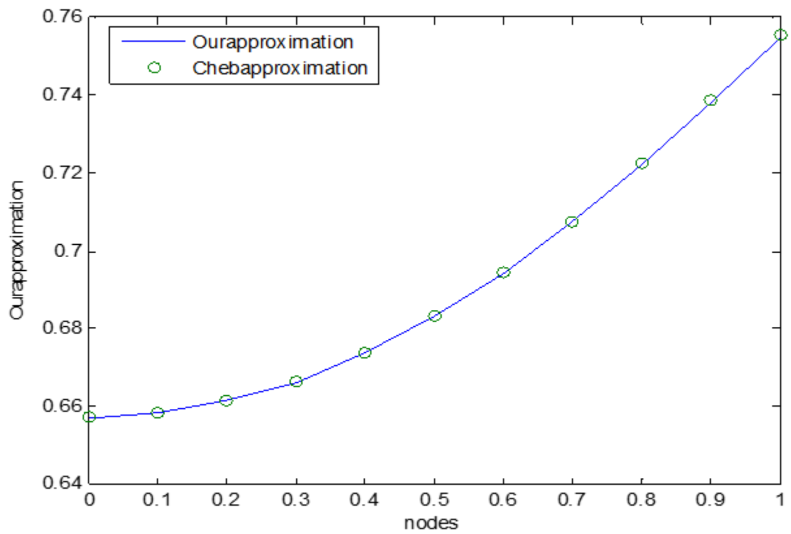

Fig. 3. The approximate solution for $u(x)$ in Example 4.2.

Table 2. The approximate solution for $u(x)$ in Example 4.2 with $n = 4$.

x	Chebyshev Approx.	Our Approx.	Error
0.0	0.65741	0.65708	3.30E-04
0.1	0.65844	0.65848	4.00E-05
0.2	0.66151	0.66151	0.00E+00
0.3	0.66666	0.66611	5.50E-04
0.4	0.67389	0.67382	7.00E-05
0.5	0.68318	0.68319	1.00E-05
0.6	0.69448	0.69429	1.90E-04
0.7	0.70766	0.70765	1.00E-05
0.8	0.72249	0.72224	2.50E-04
0.9	0.73865	0.73813	5.20E-04
1.0	0.75572	0.75506	6.60E-04

5. Conclusion

In this paper, we presented a useful numerical method that originated mainly from the Chebyshev polynomials for solving Love's integral equation. As we explained above, this method converts the present problem to a system of linear algebraic equations which may be solvable easily. Having determined the unknown Chebyshev coefficients of the solution function, the series solution is produced for numerical purposes immediately. It is important to be noted that, the more terms must be evaluated to the higher accuracy level. The obtained numerical results from analyzed examples illustrated that in applications involving computations with polynomials, the Chebyshev form offers an efficient algorithm for many basic functions.

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