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On the Approximate Solutions of the Korteweg-de Vries and Viscid Burgers Equations

Research Article

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Abstract: We solve the Korteweg-de Vries and viscid Burgers equations using Picard's successive approximations method, the Adomian decomposition method and the semi analytic iterative method. Numerical examples comparing these three methods show that though they all converge to the exact solution, the semi analytic method converges much faster and is therefore more accurate and efficient for solving all kinds of linear and nonlinear differential equations.

MSC: 34A34 • 34G20 • 49M27

Keywords: KdV equation • Burgers equation • Adomian decomposition method • Picard's successive approximations method • Semi analytic iterative method

1. Introduction

In this paper, the Adomian decomposition method (ADM) [1], Picard's successive approximations method (SAM) [2] and the novel semi analytic iterative method (SAIM), proposed by Temimi and Ansari [3] and used by Yassein and Aswhad [4], will be applied on the viscid Burgers equation (VBE) and the KdV equation. The results obtained will be compared with those from methods used in the literature. Some of the existing methods for solving these equations include the variational iteration method [5], the linearised implicit numerical method [6], the mixed finite volume element method [7], the mixed finite difference and boundary element method [8], the fractional Sumudu decomposition method [9], the Laplace Adomian decomposition method [10] and the restrictive Taylor approximation method [11].

While the methods used in this paper can be used for solving ordinary differential equations and initial and boundary value problems, the paper will serve as an illustration of methods for solution of nonlinear partial differential equations. The rest of the paper is organized as follows: we give the mathematical models to be solved in Section 2 while Section 3 outlines the proposed methods to be used. Section 4 gives applications in the form of two numerical examples and Section 5 provides some conclusions.

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2. Mathematical Models

One of the most important nonlinear diffusion equations is the generalized Burgers-Huxley (gBH) equation

$$u_t + \alpha u^{\delta} u_x - \nu u_{xx} = \beta u(1-u^{\delta})(u^{\delta}-\eta), \tag{1}$$

where $\alpha, \beta, \delta, \nu$ and η are parameters such that $\alpha, \beta \geq 0$, $\delta, \nu > 0$ and $-1 \leq \eta \leq 1$. In fluid flow ν represents the kinematic viscosity of the fluid. The viscid Burgers equation is a special case of equation (1) when $\beta = 0$ and $\delta = 1$, i.e.,

$$u_t + \alpha u u_x - \nu u_{xx} = 0, \tag{2}$$

with initial conditions $u(x,0) = f(x),\ u_t(x,0) = g(x)$. Burgers equation is a widely studied evolution equation initially derived by Bateman [12] and extensively developed by Burgers [13] as a simplified model for turbulence behaviour in fluid flow. Burgers equation is a fundamental PDE from fluid mechanics that results from a reduction of the Navier-Stokes equation to the one-dimensional case. This equation has several practical applications such as gas dynamics, heat conduction, acoustic waves, shock theory, traffic flow, viscous flow, turbulence and wave propagation in nonlinear dissipative systems.

Remark 2.1.
A negligible kinematic viscosity (i.e., $\nu = 0$) reduces (2) to the inviscid Burgers equation $u_t + \alpha u u_x = 0$, while a negligible fluid velocity u results in the heat equation which corresponds to the linearised Burgers equation $u_t - \nu u_{xx} = 0$.

The Korteweg-de Vries (KdV) equation plays an important role for characterising the diffusion of plasma waves in dispersive media and describing long waves travelling in canals [14]. The generalized KdV equation is given by

$$u_t + \alpha u^{\delta} u_x + \gamma u_{xxx} = 0, \tag{3}$$

where α, δ and γ are arbitrary constants. The parameter α can be scaled to any real number, with commonly used values being $\alpha = \pm 1$ or $\alpha = \pm 6$. Setting $\delta = 1$ reduces equation (3) to the KdV equation

$$u_t + \alpha u u_x + \gamma u_{xxx} = 0, \tag{4}$$

and setting $\delta = 2$ in (3) yields the modified KdV equation

$$u_t + \alpha u^2 u_x + \gamma u_{xxx} = 0. \tag{5}$$

Equation (4) is the pioneering equation that gives rise to solitary wave solitons, i.e., waves with infinite support generated as a result of the balance between the nonlinear convection $u u_x$ and the linear dispersion u_{xxx} [5]. This paper is concerned with solving equations (2) and (4).

3. Methods and Materials

In this section, we outline the proposed methods for solving the VBE and KdV equation, viz., the successive approximations method (SAM), Adomian decomposition method (ADM) and the semi analytic iterative method (SAIM).

3.1. Successive Approximations Method

Picard's successive approximations method provides a scheme for solving initial value problems or integral equations [2, 15]. The method solves any problem through successive approximations by starting with an initial guess or zeroth approximation $u_0(x)$. The initial guess can be any real-valued function but it commonly takes the forms 0, 1 or x and can be used in a recurrence relation to determine subsequent approximations. Let $u' = f(t,u),\ u(t_0) = u_0$. Then any solution of this DE should also be a solution of the integral equation

$$u(t) = u_0 + \int_0^t f(s, u(s))\,ds. \tag{6}$$

Picard's SAM is based on the initial approximation $u_0(t) = u_0$ and the rest of the approximations are

$$\begin{aligned}
u_1(t) &= u_0 + \int_0^t f(s, u_0(s))\,ds \\
u_2(t) &= u_0 + \int_0^t f(s, u_1(s))\,ds \\
u_3(t) &= u_0 + \int_0^t f(s, u_2(s))\,ds \\
&\vdots \\
u_{n+1}(t) &= u_0 + \int_0^t f(s, u_n(s))\,ds,\ n \geq 0.
\end{aligned}$$

Thus, for the VBE, SAM gives the successive approximations

$$u_{n+1}(x,t) = u_0 + \int_0^t \left[\nu u_{n_{xx}} - \alpha u_n u_{n_x}\right] ds, \; n \geq 0, \tag{7}$$

and for the KdV equation we have

$$u_{n+1}(x,t) = u_0 - \int_0^t \left[u_{n_{xxx}} + \alpha u_n u_{n_x}\right] ds, \; n \geq 0. \tag{8}$$

3.2. Adomian Decomposition Method

The ADM is well-known and has been widely applied in the literature for solving ordinary and partial differential equations [16, 17]. In this section we give a brief presentation of the ADM. We start by writing equations (2) and (4) in Adomian's operator-theoretic notation. Thus, the VBE can be expressed as

$$Lu = [R-N]u, \tag{9}$$

where $L = \frac{\partial}{\partial t}$ is a linear differential operator, $R = \frac{\partial^2}{\partial x^2}$ is a linear remainder operator and N is a nonlinear operator corresponding to the nonlinear term $\alpha u u_x$. Assuming that the inverse operator L^{-1} exists, we can define it as the one-fold integral operator $L^{-1}(\cdot) = \int_0^t (\cdot) ds$ and apply it to both sides of (9), i.e.,

$$L^{-1}Lu = L^{-1}[R-N]u. \tag{10}$$

With initial condition $u(x,0) = u_0 = f(x)$, we have the solution

$$u(x,t) = f(x) + L^{-1}[R-N]u. \tag{11}$$

Since the nonlinear term is represented by Adomian polynomials, the ADM solution is given by the recursive scheme

$$\begin{aligned} u_0 &= f(x), \\ u_{n+1} &= L^{-1}[Ru_n] - L^{-1}[A_n], \; n \geq 0. \end{aligned} \tag{12}$$

The solution $u(x,t)$ is given as an infinite series

$$u(x,t) = \sum_{n=0}^{\infty} u_n(x,t). \tag{13}$$

The nonlinear term $Nu = g(u)$ is usually represented by the series

$$Nu = \sum_{n=0}^{\infty} A_n, \tag{14}$$

where the A_n, which depend on $u_0, \; u_1, \; u_2, \ldots, \; u_n$, are the Adomian polynomials defined by

$$A_n = \frac{1}{n!}\left[\frac{d^n}{d\lambda^n} g\left(\sum_{k=0}^{\infty} \lambda^k u_k\right)\right]_{\lambda=0}, \; n = 0, \; 1, \; 2, \ldots \tag{15}$$

or, equivalently,

$$\begin{aligned} A_0 &= g(u_0), \\ A_1 &= g'(u_0)u_1, \\ A_2 &= g'(u_0)u_2 + \frac{u_1^2}{2!}g''(u_0), \\ A_3 &= g'(u_0)u_3 + u_1u_2g''(u_0) + \frac{u_1^3}{3!}g'''(u_0), \\ A_4 &= g'(u_0)u_4 + \left(u_1u_3 + \frac{u_2^2}{2!}\right)g''(u_0) + \frac{u_1^2u_2}{2!}g'''(u_0) + \frac{u_1^4}{4!}g^{(4)}(u_0), \\ &\vdots \end{aligned} \tag{16}$$

For the KdV equation, the operator form is

$$Lu = -[R+N]u, \tag{17}$$

where, in this case, $R = \frac{\partial^3}{\partial x^3}$. Thus, for the initial condition $u(x,0) = f(x)$ the solution is

$$u(x,t) = f(x) - L^{-1}[R+N]u, \tag{18}$$

or

$$\begin{aligned} u_0 &= f(x), \\ u_{n+1} &= -L^{-1}[Ru_n] - L^{-1}[A_n]. \end{aligned} \tag{19}$$

The solution is given by the decomposition series (13) and the Adomian polynomials by (16).

3.3. Semi Analytic Iterative Method

The SAIM was used by [4] to solve KdV equations. This method offers several advantages over existing methods such as SAM and ADM. The SAIM is very easy to implement in that it avoids the calculation of Adomian polynomials for the nonlinear term in the ADM or Lagrange multipliers in the VIM, thus demanding less computational work [18]. Consider the homogeneous VBE

$$u_t + \alpha u u_x - \nu u_{xx} = 0, \tag{20}$$

subject to the initial condition

$$u(x,0) = f(x),\ 0 \le x \le 1,\ t > 0. \tag{21}$$

Equation (20) can be written as

$$L[u(x,t)] + N[u(x,t)] + h(x,t) = 0, \tag{22}$$

with the condition $C\left(u, \frac{\partial u}{\partial t}\right) = 0$, where $L[u(x,t)] = u_t$, $N[u(x,t)] = uu_x - \nu u_{xx}$ and $h(x,t)$ is the source term. The first step of the SAIM is to find the initial approximation by solving

$$L[u_0(x,t)] + h(x,t) = 0 \text{ with } C\left(u_0, \frac{\partial u_0}{\partial t}\right) = 0. \tag{23}$$

The next iteration to the solution can be obtained by solving

$$L[u_1(x,t)] + N[u_0(x,t)] + h(x,t) = 0 \text{ with } C\left(u_1, \frac{\partial u_1}{\partial t}\right) = 0. \tag{24}$$

After several iterations we obtain the general form of the SAIM solution which is

$$L[u_{n+1}(x,t)] + N[u_n(x,t)] + h(x,t) = 0 \text{ with } C\left(u_{n+1}, \frac{\partial u_{n+1}}{\partial t}\right) = 0. \tag{25}$$

Each iteration of the function $u_n(x,t)$ effectively represents a complete solution for equation (22). For the homogeneous VBE, $h(x,t) = 0$. For the KdV equation, the solution is found in a similar manner.

4. Applications

In this section we give a couple of numerical examples that illustrate the use and effectiveness of the proposed methods. The examples show that the SAM, ADM and SAIM give solutions that compare favourably with the exact solution. However, the SAIM appears to perform better than the other two. All the computations associated with these examples were performed using a Samsung Series 3 PC with an Intel Celeron CPU 847 at 1.10GHz and 6.0GB internal memory. The figures were constructed using MATLAB R2016a.

Example 4.1.
Consider the homogeneous one-dimensional viscid Burgers equation

$$u_t + uu_x - u_{xx} = 0,\ u(x,0) = x,\ 0 \le x \le 1,\ t > 0, \tag{26}$$

with exact solution $u(x,t) = \frac{x}{1+t}$, $t \ne -1$ (Wazwaz [5]). Here, $\alpha = \nu = 1$ in equation (2).

Solution of the viscid Burgers equation by SAM

Rearranging (26) gives

$$u_t = u_{xx} - uu_x.$$

Integrating both sides over $(0,t)$ and applying the initial condition yields

$$u_{n+1}(x,t) = x + \int_0^t \left(u_{n_{xx}} - u_n u_{n_x}\right) ds.$$

If the zeroth approximation is $u_0(x,t) = 0$, we have

$$u_1(x,t) = x,$$

$$u_2(x,t) = x + \int_0^t \left(u_{1_{xx}} - u_1 u_{1_x}\right) ds = x - xt,$$

$$u_3(x,t) = x + \int_0^t \left(u_{2_{xx}} - u_2 u_{2_x}\right) ds = x - xt + xt^2 - \frac{1}{3}xt^3,$$

$$u_4(x,t) = x + \int_0^t \left(u_{3_{xx}} - u_3 u_{3_x}\right) ds = x - xt + xt^2 - xt^3 + \frac{2}{3}xt^4 - \frac{1}{3}xt^5 + \frac{1}{9}xt^6 - \frac{1}{63}xt^7,$$

$$\vdots$$

The results are given in Table 1 and are compared with those obtained from He's variational iteration method [5].

Table 1. Comparison of approximate and exact solutions from Picard's SAM and He's variational iteration method (VIM) for Example 4.1 ($t = 0.1$)

x	$u(x)$	$u_{SAM}(x)$	$u_{VIM}(x)$	e_{SAM}	e_{VIM}
0	0	0	0	0	0
0.1	0.090909	0.090906	0.090909	0.000003	0
0.2	0.181818	0.181813	0.181818	0.000005	0
0.3	0.272727	0.272719	0.272727	0.000008	0
0.4	0.363636	0.363625	0.363636	0.000011	0
0.5	0.454545	0.454532	0.454545	0.000013	0
0.6	0.545455	0.545438	0.545454	0.000017	0.000001
0.7	0.636364	0.636344	0.636364	0.000020	0
0.8	0.727273	0.727251	0.727272	0.000022	0.000001
0.9	0.818182	0.818157	0.818181	0.000025	0.000001
1.0	0.909091	0.909063	0.909090	0.000028	0.000001

Solution of the viscid Burgers equation by ADM

The ADM solution of (26) is given by the recursive scheme

$$\begin{aligned} u_0(x,t) &= x, \\ u_{n+1}(x,t) &= L^{-1}[Ru_n] - L^{-1}[A_n], \end{aligned} \tag{27}$$

where A_n are Adomian polynomials obtained as

$$\begin{aligned} A_0 &= u_0 u_{0_x} = x, \\ A_1 &= (u_0 u_{0_x})' u_1 = -xt, \\ A_2 &= (u_0 u_{0_x})' u_2 = \frac{1}{2} x t^2, \\ A_3 &= (u_0 u_{0_x})' u_3 + u_1 u_2 (u_0 u_{0_x})'' + \frac{u_1^3}{3!} (u_0 u_{0_x})''' = -\frac{1}{6} x t^3, \end{aligned}$$

and so on, and the first few approximants are

$$\begin{aligned} u_0 &= x, \\ u_1 &= L^{-1}[u_{0_{xx}}] - L^{-1}[A_0] = -xt, \\ u_2 &= L^{-1}[u_{1_{xx}}] - L^{-1}[A_1] = \frac{1}{2} x t^2, \\ u_3 &= L^{-1}[u_{3_{xx}}] - L^{-1}[A_2] = -\frac{1}{6} x t^3, \\ &\vdots \end{aligned}$$

Hence, the ADM solution is

$$u(x,t) = x\left(1 - t + \frac{t^2}{2!} - \frac{t^3}{3!} + \cdots\right) = x\mathrm{e}^{-t}.$$

The ADM results are compared with He's VIM results obtained by [5] in Table 2 which also shows the absolute errors from the two methods.

Table 2. Comparison of approximate and exact solutions from ADM and He's variational iteration method (VIM) for Example 4.1 ($t = 0.1$)

x	$u(x)$	$u_{ADM}(x)$	$u_{VIM}(x)$	e_{ADM}	e_{VIM}
0	0	0	0	0	0
0.1	0.090909	0.090484	0.090909	0.000425	0
0.2	0.181818	0.180967	0.181818	0.000851	0
0.3	0.272727	0.271451	0.272727	0.001276	0
0.4	0.363636	0.361935	0.363636	0.001701	0
0.5	0.454545	0.452419	0.454545	0.002126	0
0.6	0.545455	0.542902	0.545454	0.002553	0.000001
0.7	0.636364	0.633386	0.636364	0.002974	0
0.8	0.727273	0.723870	0.727272	0.003403	0.000001
0.9	0.818182	0.814354	0.818181	0.003828	0.000001
1.0	0.909091	0.904837	0.909090	0.004254	0.000001

Solution of the viscid Burgers equation by SAIM

We rewrite (26) in the form

$$Lu + Nu = 0,$$

i.e., $Lu = u_t$, $Nu = uu_x - u_{xx}$ and $h(x,t) = 0$ (since there is no source term). So the primary problem is to find the initial approximation by solving

$$L[u_0(x,t)] = 0 \text{ with } u_0(x,0) = x. \tag{28}$$

The general recursive relation is

$$L[u_{n+1}(x,t)] = -N[u_n(x,t)] \text{ with } u_{n+1}(x,0) = x. \tag{29}$$

By solving (28) we obtain

$$u_0(x,t) = x.$$

The first iteration is

$$L[u_1(x,t)] = u_{0_{xx}} - u_0 u_{0_x} \text{ with } u_1(x,0) = x. \tag{30}$$

The solution of (30) is

$$u_1(x,t) = x - xt.$$

The second iteration is

$$L[u_2(x,t)] = u_{1_{xx}} - u_1 u_{1_x} \text{ with } u_2(x,0) = x. \tag{31}$$

The solution of (31) is

$$u_2(x,t) = x - xt + xt^2 - \frac{1}{3}xt^3.$$

The third iteration is

$$L[u_3(x,t)] = u_{2_{xx}} - u_2 u_{2_x} \text{ with } u_3(x,0) = x. \tag{32}$$

The solution of (32) is

$$u_3(x,t) = x - xt + xt^2 - xt^3 + \frac{2}{3}xt^4 - \frac{1}{3}xt^5 + \frac{1}{9}xt^6 - \frac{1}{63}xt^7.$$

The solution of the given VBE is the limit as $n \to \infty$ of

$$u_n(x,t) = x(1 - t + t^2 - t^3 + \cdots),$$

which is the exact solution $u(x,t) = \frac{x}{1+t}$. Since SAIM leads to the exact solution, it performs far better than VIM which leads to bigger errors as shown in Table 3. Fig. 1 compares the approximate solutions of the VBE (26) with the exact solution. Table 1, Table 2 and Table 3 highlight the differences more clearly than Fig. 1.

Example 4.2.
Consider the homogeneous one-dimensional KdV equation

$$u_t - 6uu_x + u_{xxx} = 0,\ u(x,0) = \frac{1}{6}(x-1),\ 0 \le x \le 1,\ t > 0, \tag{33}$$

with exact solution $u(x,t) = \frac{x-1}{6(1-t)}$, $|t| < 1$ (Yassein and Aswhad [4]). Here, $\alpha = -6$ and $\gamma = 1$ in equation (4).

Table 3. Comparison of approximate and exact solutions from SAIM and He's variational iteration method (VIM) for Example 4.1 ($t = 0.1$)

x	$u(x)$	$u_{\text{SAIM}}(x)$	$u_{\text{VIM}}(x)$	e_{SAIM}	e_{VIM}
0	0	0	0	0	0
0.1	0.090909	0.090909	0.090909	0	0
0.2	0.181818	0.181818	0.181818	0	0
0.3	0.272727	0.272727	0.272727	0	0
0.4	0.363636	0.363636	0.363636	0	0
0.5	0.454545	0.454545	0.454545	0	0
0.6	0.545455	0.545455	0.545454	0	0.000001
0.7	0.636364	0.636364	0.636364	0	0
0.8	0.727273	0.727273	0.727272	0	0.000001
0.9	0.818182	0.818182	0.818181	0	0.000001
1.0	0.909091	0.909091	0.909090	0	0.000001

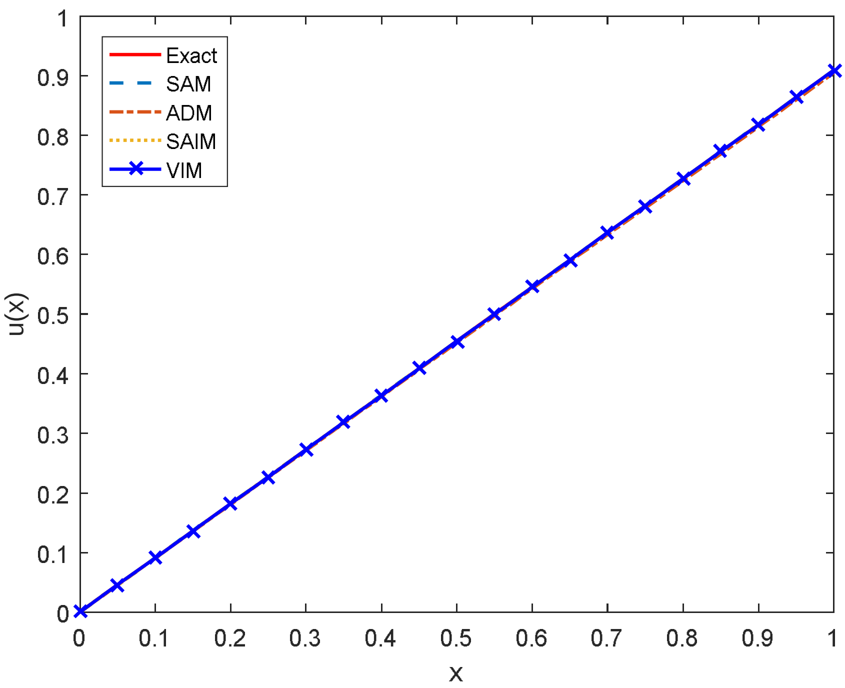

Fig. 1. Comparison of approximate and exact solutions for the viscid Burgers equation in Example 4.1 for fixed $t = 0.1$

Solution of the KdV equation by SAM

Rearranging equation (33) gives

$$u_t = 6uu_x - u_{xxx} = -(u_{xxx} - 6uu_x).$$

Taking initial guess $u_0(x, t) = 0$ and integrating successively over $(0, t)$ gives the successive approximations to the solution $u(x, t)$ as

$$u_{n+1}(x, t) = \frac{1}{6}(x-1) - \int_0^t (u_{n_{xxx}} - 6u_n u_{n_x})\,ds. \tag{34}$$

The rest of the successive approximations are:

$$
\begin{aligned}
u_1(x,t) &= \frac{1}{6}(x-1),\\
u_2(x,t) &= \frac{1}{6}(x-1) - \int_0^t \left(u_{1_{xxx}} - 6u_1 u_{1_x}\right) ds = \frac{1}{6}(x-1)(1+t),\\
u_3(x,t) &= \frac{1}{6}(x-1) - \int_0^t \left(u_{2_{xxx}} - 6u_2 u_{2_x}\right) ds = \frac{1}{6}(x-1)\left(1+t+t^2+\frac{1}{18}t^3\right),\\
u_4(x,t) &= \frac{1}{6}(x-1) - \int_0^t \left(u_{3_{xxx}} - 6u_2 u_{3_x}\right) ds,\\
&= \frac{1}{6}(x-1)\left(1+t+t^2+t^3+\frac{2}{3}t^4+\frac{1}{3}t^5+\frac{1}{9}t^6+\frac{1}{63}t^7\right),
\end{aligned}
$$

and so on. That fourth term approximates the solution to the given KdV equation. The results are summarised in Table 4 and are compared with those obtained by Yassein and Aswhad using SAIM [4].

Table 4. Comparison of approximate and exact solutions from SAM and SAIM for Example 4.2 ($t = 0.1$)

x	$u(x)$	$u_{\text{SAM}}(x)$	$u_{\text{SAIM}}(x)$	e_{SAM}	e_{SAIM}
0	−0.185185	−0.185178	−0.185185	0.000007	0
0.1	−0.166667	−0.166661	−0.166667	0.000006	0
0.2	−0.148148	−0.148143	−0.148148	0.000005	0
0.3	−0.129630	−0.129625	−0.129630	0.000005	0
0.4	−0.111111	−0.111107	−0.111111	0.000004	0
0.5	−0.092593	−0.092589	−0.092593	0.000004	0
0.6	−0.074074	−0.074071	−0.074071	0.000003	0
0.7	−0.055556	−0.055554	−0.055556	0.000002	0
0.8	−0.037037	−0.037036	−0.037037	0.000001	0
0.9	−0.018519	−0.018518	−0.018519	0.000001	0
1.0	0	0	0	0	0

Solution of the KdV equation by ADM

The ADM solution of the KdV equation (33) is given by the recursive scheme

$$
\begin{aligned}
u_0(x,t) &= \frac{1}{6}(x-1),\\
u_{n+1}(x,t) &= -L^{-1}[Ru_n] + 6L^{-1}[A_n],
\end{aligned}
\tag{35}
$$

where $Ru_n = u_{n_{xxx}}$ and the Adomian polynomials A_n are

$$
\begin{aligned}
A_0 &= \frac{1}{36}(x-1),\\
A_1 &= \frac{1}{216}(x-1)t,\\
A_2 &= \frac{1}{2592}(x-1)t^2,\\
A_3 &= \frac{1}{46656}(x-1)t^3,
\end{aligned}
$$

and so on. The first few approximants are

$$
\begin{aligned}
u_0 &= \frac{1}{6}(x-1),\\
u_1 &= -L^{-1}[u_{0_{xxx}}] + 6L^{-1}[A_0] = \frac{1}{6}(x-1),\\
u_2 &= -L^{-1}[u_{1_{xxx}}] + 6L^{-1}[A_1] = \frac{1}{72}(x-1)t^2,\\
u_3 &= -L^{-1}[u_{2_{xxx}}] + 6L^{-1}[A_2] = \frac{1}{1296}(x-1)t^3,\\
u_4 &= -L^{-1}[u_{3_{xxx}}] + 6L^{-1}[A_3] = \frac{1}{31104}(x-1)t^4,\\
&\vdots
\end{aligned}
$$

and the ADM solution is the sum of these approximants. The results are shown in Table 5 which also compares the computed ADM results with those from the SAIM used by Yassein and Aswhad [4]. The semi analytic method is shown to perform far better than the ADM as it produces the exact solution. The approximate solutions of the KdV equation (33) from the SAM, ADM and SAIM are compared with the exact solution in Fig. 2.

Table 5. Comparison of approximate and exact solutions from ADM and SAIM for Example 4.2 ($t = 0.1$)

x	$u(x)$	$u_{ADM}(x)$	$u_{SAIM}(x)$	e_{ADM}	e_{SAIM}
0	−0.185185	−0.183473	−0.185185	0.001712	0
0.1	−0.166667	−0.165126	−0.166667	0.001541	0
0.2	−0.148148	−0.146778	−0.148148	0.001370	0
0.3	−0.129630	−0.128431	−0.129630	0.001199	0
0.4	−0.111111	−0.110084	−0.111111	0.001027	0
0.5	−0.092593	−0.091736	−0.092593	0.000857	0
0.6	−0.074074	−0.073389	−0.074071	0.000685	0
0.7	−0.055556	−0.055042	−0.055556	0.000514	0
0.8	−0.037037	−0.036695	−0.037037	0.000342	0
0.9	−0.018519	−0.018347	−0.018519	0.000172	0
1.0	0	0	0	0	0

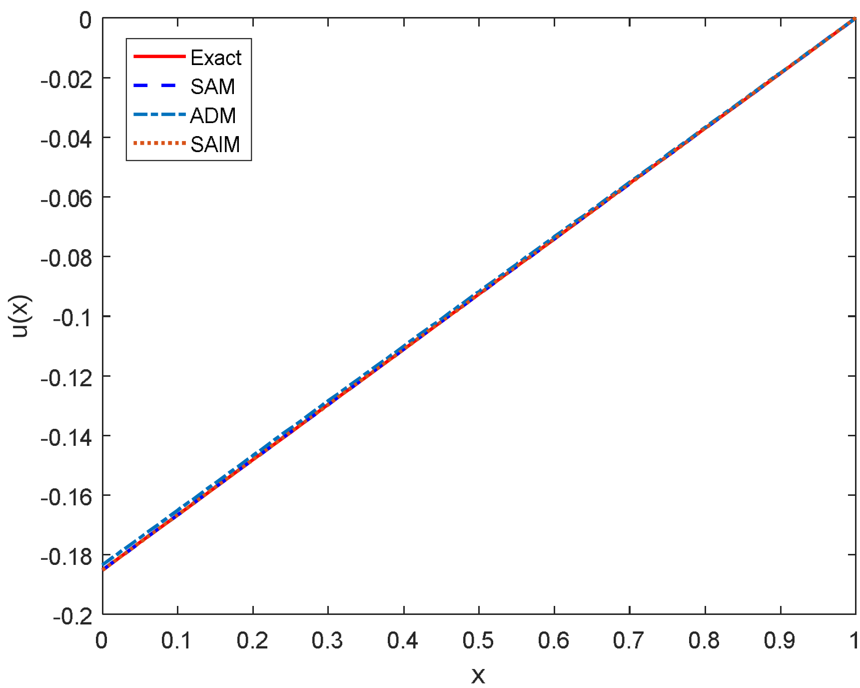

Fig. 2. Comparison of approximate and exact solutions for the KdV equation in Example 4.2 for fixed $t = 0.1$

5. Conclusion

This paper has shown by numerical examples that the SAIM is an excellent method for solution of linear and nonlinear differential equations in that it performs far better than existing methods such as ADM and SAM. These examples validate the accuracy of the method and show that it gives results which rapidly converge to the exact solutions.

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