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Effect of porosity and Biot's parameters on Rayleigh waves in thermoelastic saturated porous materials

Research Article

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Abstract: This article investigates the propagation of surface waves particularly Rayleigh type waves in the heat conducting porous materials saturated by non-viscous fluid. The frequency equations are obtained separately for thermally insulated and isothermal boundary conditions. We have observed two modes of dispersive Rayleigh type waves, type I and II. The propagation speed, attenuation and specific loss are computed numerically to see the effect of porosity and Biot's parameters. It has been observed that the parameters due to porosity, thermoelasticity and fluid present in the material are effecting the velocity curves. The propagation speed of Rayleigh type-I is just lower than that of transverse waves, while that of Rayleigh type-II is faster than body waves in the material. Some known results are also recovered from the present analysis as particular cases.

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Keywords: Thermodynamics saturated porous materials • Rayleigh wave • phase velocity • attenuation • frequency equation

1. Introduction

The solid and voids in porous materials are connected in a continuous form within the volume of the materials forming looseness between the particles. Nunziato and Cowin [1] developed a non-linear theory for elastic material with voids and showed that an internal dissipation has been caused due to the changes in volume fraction of the materials. This theory was linearized by Cowin and Nunziato [2] taking void volume fraction as independent kinematical variable. Puri and Cowin [3] studied the nature of distribution of plane harmonic waves in a linear elastic material with voids and explained the existence of two dilatational waves and a transverse wave. One of the dilatational waves is due to linear elasticity and the second dilatational wave is due to change in void volume fraction. The study of elastic material with voids is very useful in the exploration of geological materials like rocks and soils and also in the production of ceramics and powders.
Thermoelasticity discusses the heat conduction, thermal stress and strain due to the flow of thermal and the reverse effect of temperature of the materials. Biot [4–7] investigated the interesting features of thermoelastic materials and porosity with the propagation of acoustic waves. Kupradze et al. [8] discussed the three dimensional problems of steady state vibrations using potential methods. Iesan [9] derived the basic field equations for a homogeneous isotropic thermo-elastic material with voids and showed that the presence of thermal does not effect the character of transverse waves. The applications of thermoelasticity can be seen in many fields like Seismology, soil dynamics,

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Nomenclature

σ_{mn}	Stress tensor in solid
τ_{mn}	Stress tensor in thermal conducting porous solid
P_f	Fluid pressure
α	Biot's parameter
δ_{mn}	Kronecker's delta
λ, μ	Lamé parameters
M	Elastic parameter due to bulk coupling
w_m	Average fluid motion
U_m	Displacement component in fluid phase
u_m	Displacement component in solid phase
T_0	Temperature at undisturbed state
f	Porosity
β_f	Coefficient of thermal stress in fluid
β_s	Coefficient of thermal stress in solid
ρ	Density of porous aggregate
ρ_f	Density of pore fluid
K	Thermal parameter
q	Inertial coupling between poro-fluid and solid matrix
ω	Angular frequency
τ_0	Thermal relaxation time
k	Wavenumber

physical sciences, aeronautics, atomic smasher and nuclear reactors. Thermally conducting porous materials saturated by fluids are commonly found in the Earth. A Mathematical model for such materials under non-isothermal conditions was developed by Levy et al. [10]. Sharma [11] considered the problem on two dimensional elastic waves propagation in the saturated material of thermoelastic and obtained their phase velocities. The problem of reflection of those waves was presented by Zorammuana and Singh [12] and they derived amplitude and energy ratios of the reflected waves. Iesan [13] developed the constitutive relations and equations for non-linear and linear theory of thermo-viscoelastic materials with voids discussing the viscous effect in the material. The effect of the heat conduction on the behavior of the elastic materials and the properties of different types of porous materials can be explored through the studies from Yew and Jogi [14], Sorek et al. [15], Boer et al. [16], Singh [17, 18], Sur and Kanoria [19], Bhad et al. [20], Purkait et al. [21], Lianngenga and Singh [22] and Lalawmpuia and Singh [23].

Rayleigh [24] discovered the existence of surface waves which propagated only at the free surface of the medium. The investigation of Rayleigh waves has been utilized in materials science, acoustics, telecommunication industries, geophysics, seismology and acoustics. It was Lockett [25] who examined the effect of thermal on the velocity of Rayleigh waves. Chadwick and Windle [26] studied theoretically the effects of heat conduction on the propagation of Rayleigh waves in isothermal and thermally insulated boundary. The possibility of propagation of more than one type of Rayleigh surface waves in viscoelastic materials was proved and explained by Currie et al. [27]. Hirai [28] analyzed Rayleigh waves in saturated porous media using finite element method. Sharma [29–31] and Goyal et al. [32] discussed the possibility of propagation of the two types of Rayleigh surface waves in porous materials and presented their frequency equations. Vinh et al. [33] derived the secular equation for Rayleigh waves in the fluid-saturated orthotropic porous medium. Khurana and Tomar [34] found two Rayleigh type waves in the half-space of non-local micropolar solid. Kaur et al. [35] explored the existence of Rayleigh surface wave in the half-space of non-local elastic solid containing voids. Tong et al. [36] used Biot's theory to propose a wave model for Rayleigh waves in porous materials saturated by fluids. Several papers based on the different theories of solid media for the propagation of Rayleigh waves have been appeared in the literature. Notable authors among them are Kolsky [37], Achenbach [38], Kumar et al. [39], Sharma [40], Bucur [41], Sudheer et al. [42], Singh et al. [43], Biswas et al. [44], Biswas and Mukhopadhyay [45], Biswas and Abo-Dahab [46], Singh and Lalawmpuia [47], Kundu et al. [48] and Painuly and Arora [49].

The objective of this article is to discuss the effects of porosity and Biot's parameters on the propagation of Rayleigh waves in the thermoelastic saturated porous materials. Section 2 presents the basic equations which include the equations of motion and stress strain relations. It is Section 3 where we discuss solution of the equations of motion and the potential forms of displacement and temperature field. The frequency equations are derived separately for thermally insulated and isothermal conditions in Section 4. It is Section 5 where path of the surface particles has been discussed. The amplitudes of the displacement and temperature fields are also obtained in this section. Particular cases and numerical results are discussed respectively in Section 6 and 7. We have recovered the results of Abouelregal [50], Chadwick [51] and Rayleigh [24]. Finally, results of the present paper are presented in the Section 8.

2. Basic equations

The stress tensors for a thermal conducting porous solid in which the voids are saturated by non-viscous fluids are given by [6]

$$\tau_{mn} = \sigma_{mn} + \alpha(-P_f)\delta_{mn}. \quad (1)$$

The constitutive relations for an isotropic fluid saturated porous heat conducting materials are given as [10, 52]

$$\sigma_{mn} = \lambda u_{l,l}\delta_{mn} + \mu(u_{m,n} + u_{n,m}) - \beta_s(T - T_0)\delta_{mn}, \quad (2)$$

$$-P_f = \alpha M u_{l,l} + M w_{l,l} - \beta_f(T - T_0)\delta_{mn}, \quad (3)$$

where $w_l = f(U_l - u_l)$.

The equation of motion in the absence of body and internal forces for such materials are

$$\tau_{mn,m} = \rho\ddot{u}_m + \rho_f\ddot{w}_m, \quad (4)$$

$$(-P_f)_{,m} = \rho_f\ddot{u}_m + q\ddot{w}_m, \quad (5)$$

$$KT_{,nn} - \rho C_e(\dot{T} + \tau_0\ddot{T}) = T_0\beta\{\tau_0(\ddot{u}_{n,n} + \ddot{w}_{n,n}) + \dot{u}_{n,n} + \dot{w}_{n,n}\}, \quad (6)$$

where $\beta = \beta_s + \alpha\beta_f$.

Equation (6) is the energy balance equation for the heat conducting porous materials. This equation was presented by Sharma (2008) using the theory of Lord and Shulman [53].

Using Eqs. (1)-(3) into Eqs. (4)-(6), we get

$$(\lambda + \mu + \alpha^2 M)\nabla(\nabla\cdot\mathbf{u}) + \mu\nabla^2\mathbf{u} - \rho\ddot{\mathbf{u}} + \alpha M\nabla(\nabla\cdot\mathbf{w}) - \rho_f\ddot{\mathbf{w}} - \beta\nabla T = 0, \quad (7)$$

$$\alpha M\nabla(\nabla\cdot\mathbf{u}) - \rho_f\ddot{\mathbf{u}} + M\nabla(\nabla\cdot\mathbf{w}) - q\ddot{\mathbf{w}} - \beta_f\nabla T = 0, \quad (8)$$

$$\beta T_0\{(\nabla\cdot\dot{\mathbf{u}} + \tau_0\nabla\cdot\ddot{\mathbf{u}}) + (\nabla\cdot\dot{\mathbf{w}} + \tau_0\nabla\cdot\ddot{\mathbf{w}})\} - \{K\nabla^2 T - \rho C_e(\dot{T} + \tau_0\ddot{T}) = 0. \quad (9)$$

Using Helmholtz's theorem, **u** and **w** can be decomposed as

$$\mathbf{u} = \nabla\phi_s + \nabla\times\boldsymbol{\psi}_s, \quad \nabla\cdot\boldsymbol{\psi}_s = 0, \quad (10)$$

$$\mathbf{w} = \nabla\phi_f + \nabla\times\boldsymbol{\psi}_f, \quad \nabla\cdot\boldsymbol{\psi}_f = 0, \quad (11)$$

where ϕ_s and $\boldsymbol{\psi}_s$ are potentials representing solid phase and ϕ_f and $\boldsymbol{\psi}_f$ are representing fluid phase.

3. Surface wave

We consider Cartesian coordinates as x and z-axis lying horizontal with vertical y-axis. Using harmonic nature of traveling waves that potentials vary with $\exp(-\imath\omega t)$ and inserting Eqs. (10) & (11) into Eqs. (7)-(9), we get two sets of equations

$$\{\Omega_3\nabla^6 - \omega^2\Omega_2\nabla^4 + \omega^4\Omega_1\nabla^2 - \omega^6\Omega_0\}\{\phi_s, \phi_f, T\}(x, y, z) = 0, \quad (12)$$

$$\{\nabla^2 - \frac{\omega^2(\rho_f^2 - \rho q)}{\mu q}\}\{\boldsymbol{\psi}_s, \boldsymbol{\psi}_f\}(x, y, z) = 0, \quad (13)$$

where $\Omega_0 = \rho C_e\tau(\rho_f^2 - \rho q)$, $\Omega_3 = MK(\lambda + 2\mu)$, $\tau = \tau_0 + \frac{\imath}{\omega}$, $D_1 = \lambda + 2\mu + \alpha^2 M$, $\Omega_1 = \rho C_e\tau(D_1 q + M\rho - 2\alpha M\rho_f) + K(\rho q - \rho_f^2) + T_0\tau\beta\beta_f(\rho - \rho_f) + \beta^2 T_0\tau(q - \rho_f)$ and $\Omega_2 = -K(D_1 q + M\rho - 2\alpha M\rho_f) - \rho C_e\tau(D_1 M - \alpha^2 M^2) - T_0\tau\beta\beta_f(D_1 - \alpha M) + \beta^2 T_0\tau(\alpha M - M)$.

We consider the two dimensional problem of wave propagation in xy-plane $\{(x, y) : -\infty < x < \infty, y \geq 0\}$ in the half-space of thermoelastic saturated porous materials. The full potential structures for surface waves can be represented as

$$\langle\phi_s, \phi_f, T - T_0\rangle(x, y, t) = \sum_{n=1}^{3}\langle A_n, a_n A_n,\ b_n A_n\rangle e^{(\imath kx - m_n y - \imath\omega t)},$$
$$\langle\psi_s, \psi_f\rangle(x, y, t) = \langle A_4, dA_4\rangle e^{(\imath kx - m_4 y - \imath\omega t)}, \quad (14)$$

where d, a_n and b_n are the coupling constants given by

$$a_n = \frac{\beta_f(D_1 - \rho c_n^2) - \beta(\alpha M - \rho_f c_n^2)}{\beta(M - qc_n^2) - \beta_f(\alpha M - \rho_f c_n^2)}, \quad d = \frac{\mu - \rho c_4^2}{\rho_f c_4^2},$$

$$b_n = \frac{k_n^2\{(D_1 - \rho c_n^2)(M - qc_n^2) - (\alpha M - \rho_f c_n^2)^2\}}{\{\beta_f(\alpha M - \rho_f c_n^2) - \beta(M - qc_n^2)\}}, \quad (n = 1, 2, 3).$$

where ψ_s and ψ_f are z-components of $\boldsymbol{\psi}_s$ and $\boldsymbol{\psi}_f$ respectively, A_n are amplitude constants, $c_n = \frac{\omega}{k_n}$ is the phase speed of the body wave with wavenumber, k_n and $m_n^2 = k^2 - k_n^2$. Here, m_n denotes the penetration depth of the surface waves which decay exponentially as they move away from the surface. All the real part of m_n need to be positive, i.e., $\Re(m_n) > 0$ due to the choice of the solution. This solution (14) depends on the parameter k, that has to be determined from the boundary conditions.

4. Frequency Equations

At the free surface of the thermo-elastic saturated porous medium, the stress tensors, gradient of temperature and fluid flux vanished. These conditions are (at $y=0$)

$$(\lambda+\alpha^2 M)\nabla^2\phi_s+2\mu\frac{\partial^2\phi_s}{\partial y^2}+\alpha M\nabla^2\phi_f - \beta(T-T_0)-2\mu\frac{\partial^2\psi_s}{\partial x\partial y}=0, \tag{15}$$

$$2\frac{\partial^2\phi_s}{\partial x\partial y}+\frac{\partial^2\psi_s}{\partial y^2}-\frac{\partial^2\psi_s}{\partial x^2} = 0, \tag{16}$$

$$\frac{\partial T}{\partial y}+hT = 0, \tag{17}$$

$$\frac{\partial\dot{\phi}_f}{\partial y}-\frac{\partial\dot{\psi}_f}{\partial x} = 0, \tag{18}$$

where $h\to 0$ for thermally insulated surface and $h\to\infty$ for isothermal surface.
Inserting Eq. (14) into (15)-(18), we have

$$a_{11}A_1+a_{12}A_2+a_{13}A_3+\imath a_{14}m_4A_4=0, \tag{19}$$

$$m_1a_{21}A_1+m_2a_{22}A_2+m_3a_{23}A_3+\imath a_{24}A_4=0, \tag{20}$$

$$m_1a_{31}A_1+m_2a_{32}A_2+m_3a_{33}A_3=0, \tag{21}$$

$$a_{31}A_1+a_{32}A_2+a_{33}A_3=0, \tag{22}$$

$$m_1a_{41}A_1+m_2a_{42}A_2+m_3a_{43}A_3+\imath a_{44}A_4=0, \tag{23}$$

where

$$a_{1n}=(\lambda+\alpha^2M+\alpha Ma_n)(m_n^2-k^2)+2\mu m_n^2-\beta b_n,\ \ a_{14}=2\mu k,\ \ a_{2n}=2k,$$
$$a_{24}=(m_4^2+k^2), a_{3n}=b_n,\ \ a_{4n}=a_n,\ \ a_{44}=kd,\quad (n=1,2,3).$$

These equations help to derive the frequency equations of the Rayleigh waves corresponding to thermally insulated and isothermal surfaces respectively as

$$a_{11}m_2m_3D_{11}-a_{12}m_1m_3D_{12}+a_{13}m_1m_2D_{13}-a_{14}m_1m_2m_3D_{14}=0, \tag{24}$$

$$\text{and}\ \ D_{21}m_1m_2+D_{22}m_1m_3+D_{23}m_1m_4-D_{24}m_2m_3-D_{25}m_2m_4-D_{26}m_3m_4=0, \tag{25}$$

where

$$D_{11}=(a_{22}a_{33}-a_{23}a_{32})a_{44}+(a_{32}a_{43}-a_{33}a_{42})a_{24},\ \ D_{12}=(a_{21}a_{33}-a_{23}a_{31})a_{44}+$$
$$(a_{31}a_{43}-a_{33}a_{41})a_{24},\ \ D_{13}=(a_{21}a_{32}-a_{22}a_{31})a_{44}+(a_{31}a_{42}-a_{32}a_{41})a_{24},$$
$$D_{14}=(a_{32}a_{43}-a_{33}a_{42})a_{21}+(a_{33}a_{41}-a_{31}a_{43})a_{22}+(a_{31}a_{42}-a_{32}a_{41})a_{23},$$
$$D_{21}=a_{33}a_{14}m_4^2(a_{41}a_{22}-a_{42}a_{21}),\ \ D_{22}=a_{32}a_{14}m_4^2(a_{43}a_{21}-a_{41}a_{23}),$$
$$D_{23}=(a_{32}a_{13}-a_{33}a_{12})(a_{41}a_{24}-a_{44}a_{21}),\ \ D_{24}=a_{31}a_{14}m_4^2(a_{43}a_{22}-a_{42}a_{23}),$$
$$D_{25}=(a_{31}a_{13}-a_{33}a_{11})(a_{42}a_{24}-a_{44}a_{22}),\ \ D_{26}=(a_{31}a_{12}-a_{32}a_{11})(a_{44}a_{23}-a_{43}a_{24}).$$

Equations (24) & (25) contain radical powers in the expressions of m_n and difficult to solve directly for k. The radical powers are removed by squaring and obtained the frequency equations as

$$(\alpha_{11}^2-\alpha_{12}^2m_1^2m_3^2-\alpha_{13}^2m_2^2m_4^2)^2-4\alpha_{12}^2\alpha_{13}^2m_1^2m_2^2m_3^2m_4^2=0, \tag{26}$$

$$\alpha_{21}^2-\alpha_{22}^2m_2^2m_3^2=0, \tag{27}$$

where

$$\alpha_{11}=D_{11}^2a_{11}^2m_2^2m_3^2+D_{13}^2a_{13}^2m_1^2m_2^2-D_{12}^2a_{12}^2m_1^2m_3^2-D_{14}^2a_{14}^2m_1^2m_2^2m_3^2m_4^2,$$
$$\alpha_{12}=2D_{11}D_{13}a_{11}a_{13}m_2^2,\ \ \alpha_{13}=2D_{12}D_{14}a_{12}a_{14}m_1^2m_3^2,\ \ \alpha_{22}=2(D_{33}D_{34}m_4^2-D_{31}D_{32}),$$
$$\alpha_{21}=D_{31}^2+D_{32}^2m_2^2m_3^2-D_{33}^2m_2^2m_4^2-D_{34}^2m_3^2m_4^2,\ \ D_{32}=2(D_{21}D_{22}m_1^2-D_{25}D_{26}m_4^2),$$
$$D_{31}=D_{21}^2m_1^2m_2^2+D_{22}^2m_1^2m_3^2+D_{23}^2m_1^2m_4^2-D_{24}^2m_2^2m_3^2-D_{25}^2m_2^2m_4^2-D_{26}^2m_3^2m_4^2,$$
$$D_{33}=2(D_{24}D_{26}m_3^2-D_{21}D_{23}m_1^2),\ \ D_{34}=2(D_{24}D_{25}m_2^2-D_{22}D_{23}m_1^2).$$

Eqs. (26) and (27) are equations of 48 powers and all roots do not satisfy the boundary conditions. The solutions of these equations are complex and if $k=R+\imath Q$ be a solution, then the phase speed, $c_r=\frac{\omega}{R}$ and attenuation, $A_r=Q$ satisfy

$$c^{-1}=c_r^{-1}+\imath\omega^{-1}Q. \tag{28}$$

It may note that the exponent in Eq. (14) becomes $\imath R(x-c_rt)-Qx-m_ny$.

4.1. Specific Loss

The direct method to find internal friction for a material is finding the specific loss. It may be defined as the ratio of energy dissipated (ΔW) in a specimen through a stress cycle to the elastic energy (W) stored in the specimen at the maximum strain. Numerical values of this factor is calculated as

$$\text{Specific Loss} = 4\pi\left|\frac{c_r Q}{\omega}\right|. \tag{29}$$

5. Path of surface particles

The amplitude of displacement and temperature functions due to the propagation of Rayleigh type waves at the surface, $y=0$ are obtained as

$$\{u_k, w_k, T\} = \left\{|U_k|\,e^{\imath\theta_k}, |W_k|\,e^{\imath\theta_{(k+2)}}, T_\theta e^{\imath\theta_5}\right\} A_1 e^{(\imath q - Qx)}, \quad k=1,2 \tag{30}$$

where

$$U_1 = \imath k\{1+\frac{L_4}{L_5}-\frac{L_6}{L_3L_5}\} - \imath m_4\frac{L_7}{L_3L_5},\; U_2 = -m_1 - m_2\frac{L_4}{L_5} + m_3\frac{L_6}{L_3L_5} + \imath k\frac{L_7}{L_3L_5},$$
$$W_1 = \imath k\{a_1 + a_2\frac{L_4}{L_5} - a_3\frac{L_6}{L_3L_5}\} - \imath d m_4\frac{L_7}{L_3L_5},\quad q = R(x - c_r t),$$
$$W_2 = -a_1 m_1 - a_2 m_2\frac{L_4}{L_5} + a_3 m_3\frac{L_6}{L_3L_5} + \imath d k\frac{L_7}{L_3L_5},\quad T_\theta = b_1 m_1 + b_2 m_2\frac{L_4}{L_5} - b_3 m_3\frac{L_6}{L_3L_5},$$
$$L_1 = a_{14}a_{21}m_1m_4 - a_{11}a_{24},\; L_2 = a_{14}a_{22}m_2m_4 - a_{12}a_{24},\; L_3 = a_{14}a_{23}m_3m_4 - a_{13}a_{24},$$
$$L_6 = L_1L_5 + L_2L_4,\; L_7 = a_{11}L_3L_5 + a_{12}L_3L_4 - a_{13}\left(L_1L_5 + L_2L_4\right)$$
$$L_4 = \begin{cases} L_1a_{33}m_3 - L_3a_{31}m_1, & \text{for thermally insulated} \\ L_1a_{33} - L_3a_{31}, & \text{for isothermal}\end{cases}$$
$$L_5 = \begin{cases} L_2a_{33}m_3 - L_3a_{32}m_1, & \text{for thermally insulated} \\ L_2a_{33} - L_3a_{32}, & \text{for isothermal}\end{cases}$$
$$(\theta_1,\theta_2,\theta_3,\theta_4,\theta_5) = \left(\arg(U_1),\arg(U_2),\arg(W_1),\arg(W_2),\arg(T_\theta)\right).$$

We know that the surface of thermoelastic saturated porous material damped out the vibration of Rayleigh type wave. As a result of this, the phase differences are developed between u_1 and u_2 in the solid and w_1 and w_2 in the fluid phase. Eq. (30) retains the real part on the surface $y=0$ as

$$\{u_k, w_k\} = \left\{|U_k|\cos(q+\theta_k), |W_k|\cos(q+\theta_{(k+2)})\right\} N,\quad k=1,2 \tag{31}$$

where $N = A_1 e^{-Qx}$.
Eliminating q from Eq. (31), we get

$$\left(\frac{u_1}{|U_1|}\right)^2 + \left(\frac{u_2}{|U_2|}\right)^2 - 2\left(\frac{u_1}{|U_1|}\right)\left(\frac{u_2}{|U_2|}\right)\cos(\theta_1-\theta_2) = N^2\sin^2(\theta_1-\theta_2), \tag{32}$$
$$\left(\frac{w_1}{|W_1|}\right)^2 + \left(\frac{w_2}{|W_2|}\right)^2 - 2\left(\frac{w_1}{|W_1|}\right)\left(\frac{w_2}{|W_2|}\right)\cos(\theta_3-\theta_4) = N^2\sin^2(\theta_3-\theta_4). \tag{33}$$

These Eqs. (32) and (33) represent ellipse in (u_1-u_2) plane and (w_1-w_2) plane respectively due to the fact that

$$\frac{4}{|U_1|^2|U_2|^2}\cos^2(\theta_1-\theta_2) - \frac{4}{|U_1|^2|U_2|^2} = -\frac{4}{|U_1|^2|U_2|^2}\sin^2(\theta_1-\theta_2) < 0 \text{ and}$$
$$\frac{4}{|W_1|^2|W_2|^2}\cos^2(\theta_3-\theta_4) - \frac{4}{|W_1|^2|W_2|^2} = -\frac{4}{|W_1|^2|W_2|^2}\sin^2(\theta_3-\theta_4) < 0.$$

The semi major (X_s, X_f), minor axes (Y_s, Y_f) and eccentricities (e_s, e_f) of the elliptical path in Eqs. (32) and (33) are given by

$$\begin{aligned} X_s^2 &= \frac{N^2}{2}\left[|U_1|^2 + |U_2|^2 + \sqrt{\left(|U_1|^2-|U_2|^2\right)^2 + 4|U_1|^2|U_2|^2\cos^2(\theta_1-\theta_2)}\right], \\ Y_s^2 &= \frac{N^2}{2}\left[|U_1|^2 + |U_2|^2 - \sqrt{\left(|U_1|^2-|U_2|^2\right)^2 + 4|U_1|^2|U_2|^2\cos^2(\theta_1-\theta_2)}\right], \\ e_s^2 &= \frac{2\sqrt{\left(|U_1|^2-|U_2|^2\right)^2 + 4|U_1|^2|U_2|^2\cos^2(\theta_1-\theta_2)}}{|U_1|^2 + |U_2|^2 + \sqrt{\left(|U_1|^2-|U_2|^2\right)^2 + 4|U_1|^2|U_2|^2\cos^2(\theta_1-\theta_2)}} \end{aligned} \tag{34}$$

and

$$
\begin{aligned}
X_f^2 &= \frac{N^2}{2}\left[|W_1|^2+|W_2|^2+\sqrt{\left(|W_1|^2-|W_2|^2\right)^2+4|W_1|^2|W_2|^2\cos^2(\theta_3-\theta_4)}\right],\\
Y_f^2 &= \frac{N^2}{2}\left[|W_1|^2+|W_2|^2-\sqrt{\left(|W_1|^2-|W_2|^2\right)^2+4|W_1|^2|W_2|^2\cos^2(\theta_3-\theta_4)}\right],\\
e_f^2 &= \frac{2\sqrt{\left(|W_1|^2-|W_2|^2\right)^2+4|W_1|^2|W_2|^2\cos^2(\theta_3-\theta_4)}}{|W_1|^2+|W_2|^2+\sqrt{\left(|W_1|^2-|W_2|^2\right)^2+4|W_1|^2|W_2|^2\cos^2(\theta_3-\theta_4)}}.
\end{aligned}
\tag{35}
$$

If α_s and α_f are the inclination of major-axes to the wave normal, then

$$
\begin{aligned}
\tan(2\alpha_s) &= \frac{2\left\{\left(\tan^2\delta-1\right)|U_1||U_2|\cos(\theta_1-\theta_2)-\left(|U_1|^2-|U_2|^2\right)\tan\delta\right\}}{\left(\tan^2\delta-1\right)\left(|U_1|^2-|U_2|^2\right)+4|U_1||U_2|\cos(\theta_1-\theta_2)\tan\delta},\\
\tan(2\alpha_f) &= \frac{2\left\{\left(\tan^2\delta-1\right)|W_1||W_2|\cos(\theta_3-\theta_4)-\left(|W_1|^2-|W_2|^2\right)\tan\delta\right\}}{\left(\tan^2\delta-1\right)\left(|W_1|^2-|W_2|^2\right)+4|W_1||W_2|\cos(\theta_3-\theta_4)\tan\delta},
\end{aligned}
\tag{36}
$$

where δ is angle of propagation.
For Rayleigh waves propagating along x-axis, $\delta=\frac{\pi}{2}$, then Eq. (36) becomes

$$
\begin{aligned}
\tan(2\alpha_s) &= \frac{2|U_1||U_2|\cos(\theta_1-\theta_2)}{|U_1|^2-|U_2|^2},\\
\tan(2\alpha_f) &= \frac{2|W_1||W_2|\cos(\theta_3-\theta_4)}{|W_1|^2-|W_2|^2}.
\end{aligned}
\tag{37}
$$

The horizontal and vertical components of displacement in solid and fluid phase of the thermoelastic saturated porous material have equal magnitude, i.e., $|U_1|=|U_2|$ and $|W_1|=|W_2|$ when $\alpha_s=\alpha_f=\frac{\pi}{4}$. This means that the surface particles moving in the elliptical path of Eqs. (32) and (33) are parallel to the direction of propagation of Rayleigh waves. Since the semi-axes depend upon $N=A_1e^{-Qx}$, they are increasing or decreasing exponentially. Thus, the decay of surface particles in thermoelastic saturated porous material are retrograde and prograde when X_s and Y_s in solid as well as X_f and Y_f in fluid have same sign and opposite sign respectively.

6. Particular Cases

Case I: If the porosity of the material is neglected, then the fluid flow in it will no more and the problem becomes Rayleigh waves propagation in the thermoelastic solid. Thus, $\alpha=f=q=\rho_f=M=\beta_f=a_n=0$ and Eqs.(12) & (13) reduce to

$$
\begin{aligned}
\{\Omega_2\nabla^4+\omega^2\Omega_1\nabla^2+\omega^4\Omega_0\}\{\phi_s,T\}(x,y,z) &= 0,\\
\{\mu\nabla^2+\rho\omega^2\}\{\boldsymbol{\psi}_s\}(x,y,z) &= 0,
\end{aligned}
\tag{38}
$$

where

$$\Omega_0=\rho^2C_e\tau,\ \ \Omega_1=\rho C_e\tau(\lambda+2\mu)+K\rho+\beta_s^2T_0\tau,\ \ \Omega_2=K(\lambda+2\mu).$$

The coupling parameter b_n is given by

$$b_n=\frac{1}{\beta_s}\{(\lambda+2\mu)(m_n^2-k^2)+\rho\omega^2\},\quad (n=1,2).$$

The frequency equations are also reduced as

$$(\alpha_{11}^2m_1^2+\alpha_{13}^2m_1^2m_2^2m_4^2-\alpha_{12}^2m_2^2)^2-4\alpha_{11}^2\alpha_{13}^2m_1^4m_2^2m_4^2=0,\qquad \text{(for thermally insulated)} \tag{39}$$

$$\alpha_{21}^2-\alpha_{22}^2m_1^2m_2^2=0,\qquad \text{(for isothermal)} \tag{40}$$

with the following modified values

$$
\begin{aligned}
&\alpha_{11}=a_{12}a_{24}a_{31},\ \ \alpha_{12}=a_{11}a_{24}a_{32},\ \ \alpha_{13}=a_{14}(a_{21}a_{32}-a_{22}a_{31}),\ \ \alpha_{22}=a_{14}^2m_4^2a_{21}a_{22}a_{31}a_{32},\\
&\alpha_{21}=a_{24}^2(a_{31}a_{12}-a_{32}a_{11})^2-a_{14}^2m_4^2(a_{31}^2a_{22}^2m_2^2+a_{32}^2a_{21}^2m_1^2),\\
&a_{11}=(\lambda+\alpha^2M+\alpha M)(m_1^2-k^2)+2\mu m_1^2,\ \ a_{12}=(\lambda+\alpha^2M+\alpha M)(m_2^2-k^2)+2\mu m_2^2.
\end{aligned}
$$

Table 1. Parametric values

Parameters	Value	Parameters	Value
λ	3.7 Gpa	q	$1.05\rho_f/f\ kgm^{-3}$
μ	7.9 Gpa	C_e	1040 JKg^{-1}/K
M	6 Gpa	K	170 Wm^{-1}/K
ρ	2216 kgm^{-3}	β_f	$2.37\times10^{-3}\ Gpa/K$
ρ_f	950 kgm^{-3}	β_s	$2\beta_f\ Gpa/K$
α	0.4	T_0	300 K
f	0.16	τ_0	10^{-10}

Eq. (39) is exactly match with the result of Abouelregal [50] for the Lord and Shulman theory. If $\tau_0 = 0$, then Eqs. (39) and (40) reduced to the result of Chadwick [51] for thermally insulated and isothermal condition respectively.
Case II: When we neglect thermal effect, $\beta_s = \beta_f = \tau_0 = K = C_e = b_n = 0$ and Eq.(12) becomes

$$\{\Omega_2\nabla^4 + \omega^2\Omega_1\nabla^2 + \omega^4\Omega_0\}\{\phi_s, \phi_f\}(x, y, z) = 0, \tag{41}$$

where $\Omega_0 = \rho q - \rho_f^2$, $\Omega_1 = D_1 q + M\rho - 2\alpha M\rho_f$, $\Omega_2 = D_1 M - \alpha M^2$ and the coupling parameter a_n reduces to

$$a_n = \frac{D_1(k^2 - m_n^2) - \rho\omega^2}{\alpha M(m_n^2 - k^2) + \rho_f\omega^2}, \quad (n = 1, 2).$$

The frequency equation, in this case, becomes

$$(\alpha_{11}^2 m_2^2 + \alpha_{13}^2 m_1^2 m_2^2 m_4^2 - \alpha_{12}^2 m_1^2)^2 - 4\alpha_{11}^2\alpha_{13}^2 m_1^2 m_2^4 m_4^2 = 0, \tag{42}$$

with the modified values of

$$\alpha_{11} = a_{11}(a_{22}a_{44} - a_{24}a_{42}),\ \alpha_{12} = a_{12}(a_{21}a_{44} - a_{24}a_{41}),\ \alpha_{13} = a_{14}(a_{21}a_{42} - a_{22}a_{41}),$$
$$a_{11} = \lambda(m_1^2 - k^2) + 2\mu m_1^2 - b_1\beta,\ \ a_{12} = \lambda(m_2^2 - k^2) + 2\mu m_2^2 - b_2\beta.$$

Case III: If we neglect the effect of thermal and porosity of the material, all the parameters except ρ, λ and μ are zero and Eqs.(12) & (13) reduce to

$$\begin{aligned} \{(\lambda + 2\mu)\nabla^4 + \rho\omega^2\}\{\phi_s\}(x, y, z) &= 0, \\ \{\mu\nabla^2 + \rho\omega^2\}\{\boldsymbol{\psi}_s\}(x, y, z) &= 0. \end{aligned} \tag{43}$$

Eq. (24) also reduces to

$$\{\lambda(m_1^2 - k^2) + 2\mu m_1^2\}(m_4^2 + k^2) - 4\mu k^2 m_1 m_4 = 0$$

which may be expressed as

$$\left(2 - \frac{c^2}{c_4^2}\right)^2 - 4\sqrt{1 - \frac{c^2}{c_1^2}}\sqrt{1 - \frac{c^2}{c_4^2}} = 0, \tag{44}$$

where $c_1^2 = \dfrac{\lambda + 2\mu}{\rho}$ and $c_4^2 = \dfrac{\mu}{\rho}$.
Eq. (44) is the frequency equation of Rayleigh wave in classical elasticity [24].

7. Numerical Results

For investigating the effect of porosity and Biot's parameter on Rayleigh waves, we consider the numerical values of liquid saturated reservoir rock like North-sea Sandstone [11] given in Table 1
We have solved the polynomial Eqs.(26) and (27) numerically for the wavenumber of Rayleigh waves at the thermally insulated and isothermal surface respectively. There are 48 roots for each cases and we drop those roots that arise due to squaring. We get only two roots each for both the cases satisfying Eqs. (24) and (26) as well as Eqs. (25) and (27). These two roots correspond to Rayleigh type I and II which are of dispersive nature. Note that $(c_{r1}, A_{r1}, SL1)$ and $(c_{r2}, A_{r2}, SL2)$ correspond to Rayleigh type I and II respectively.
We have observed that Rayleigh type I is found to propagate with the speed just less than that of the transverse wave and the second type is faster than all the body waves in the thermoelastic saturated porous material. The velocity curves are depicted with angular frequency(ω) with different α and f in Fig. 1-Fig. 6 and Fig. 7-Fig. 12 respectively. In each figures, (a) corresponds for thermally insulated and (b) represents for isothermal surface boundary.

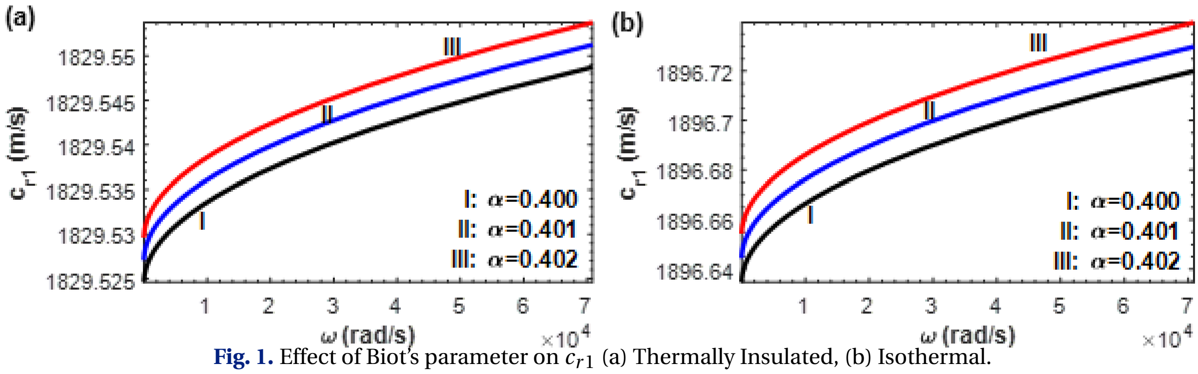

Fig. 1. Effect of Biot's parameter on c_{r1} (a) Thermally Insulated, (b) Isothermal.

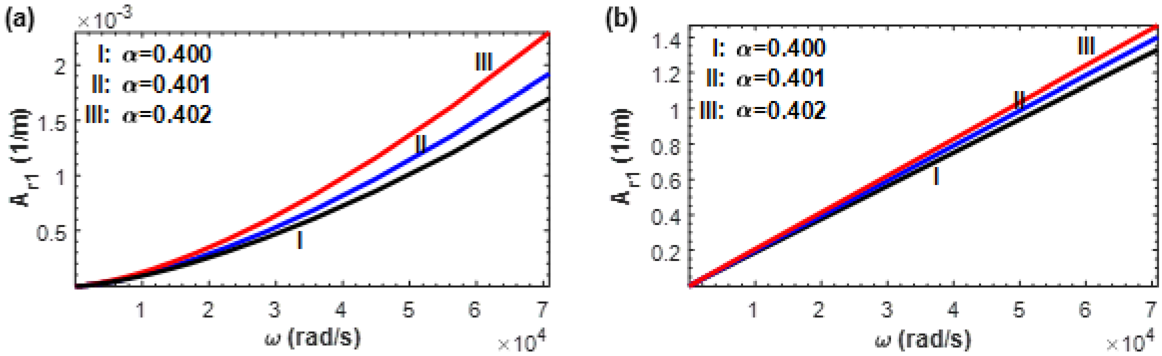

Fig. 2. Effect of Biot's parameter on A_{r1} (a) Thermally Insulated, (b) Isothermal.

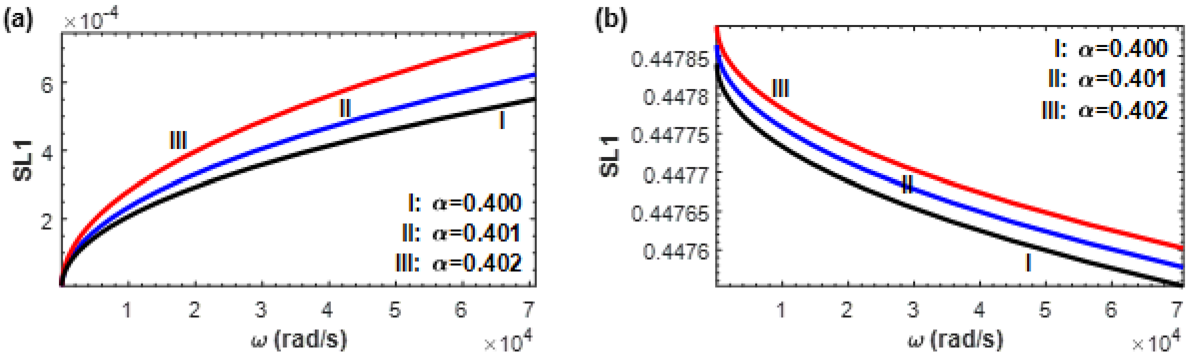

Fig. 3. Effect of Biot's parameter on $SL1$ (a) Thermally Insulated, (b) Isothermal.

7.1. Effect of Biot's parameter

In Fig. 1, the phase speed (c_{r1}) corresponding to Rayleigh type I increases with the increase of angular frequency (ω) for both the thermally insulated and isothermal surface boundary. We have seen that the Rayleigh type I is faster in isothermal than thermally insulated surface. Their values also increase with the increase of α. Similar nature of variations in attenuation A_{r1} are observed in Fig. 2. The specific loss ($SL1$) in Fig. 3 for thermally insulated surface increases with the increase of ω, while it is decreased for isothermal surface. The values of $SL1$ increase with the increase of α. For the thermally insulated and isothermal surface, the values of c_{r2}, A_{r2} and $SL2$ corresponding to Rayleigh type II in Fig. 4, Fig. 5 and Fig. 6 are all increase with the increase of ω. Rayleigh type II is found to be faster in thermally insulated than isothermal surface. In Fig. 4, we have observed that with the increase in the value of α, the values of c_{r2} for both the thermally insulated and isothermal surface decrease. The values of A_{r2} and $SL2$ for the thermally insulated surface in Fig. 5(a) and Fig. 6(a) decrease, while those in Fig. 5(b) and Fig. 6(b) for isothermal surface increase with the increase of α.

7.2. Effect of porosity

In Fig. 7 and Fig. 10, the values of c_{r1} and c_{r2} for both thermally insulated and isothermal surface increase with the increase of porosity (f). We have observed that the values of A_{r1} and $SL1$ in Fig. 8 and Fig. 9 decrease with the increase of f. In Fig. 11 and Fig. 12, the values of A_{r2} and $SL2$ for thermally insulated surface decrease with the increase of f,

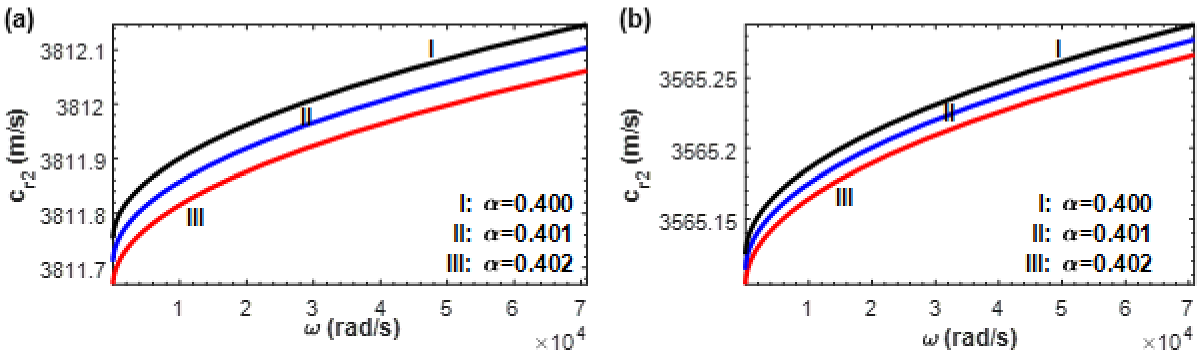

Fig. 4. Effect of Biot's parameter on c_{r2} (a) Thermally Insulated, (b) Isothermal.

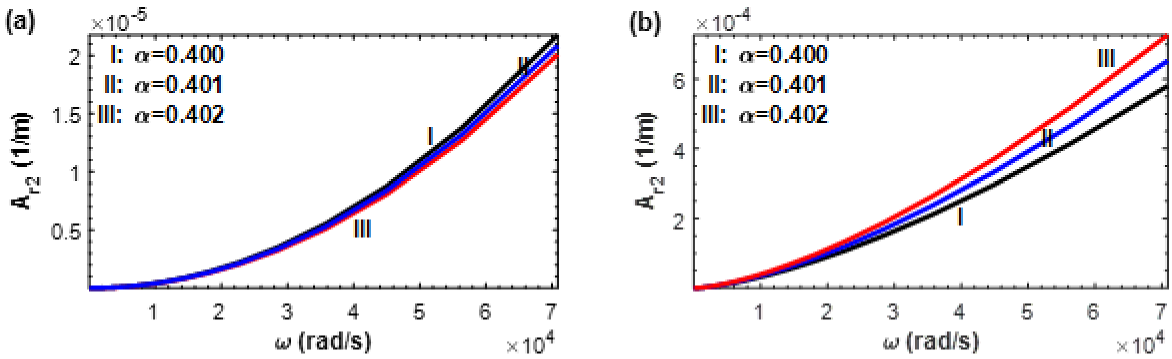

Fig. 5. Effect of Biot's parameter on A_{r2} (a) Thermally Insulated, (b) Isothermal.

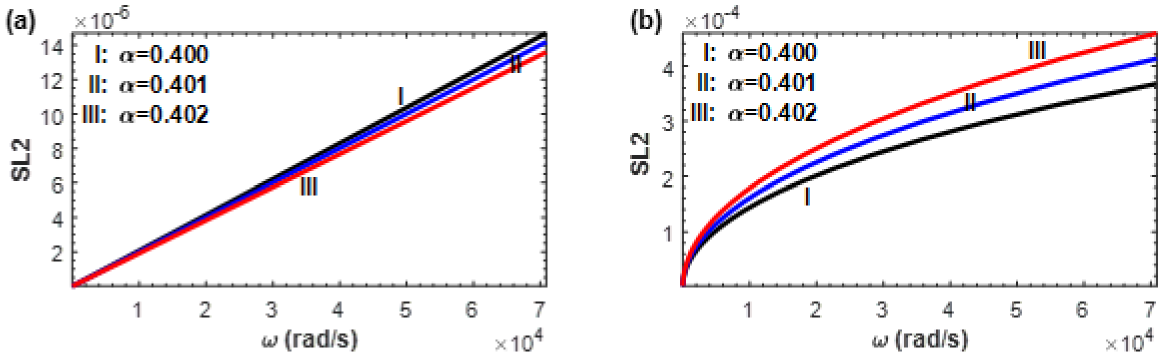

Fig. 6. Effect of Biot's parameter on $SL2$ (a) Thermally Insulated, (b) Isothermal.

while those of isothermal surface increase with the increase of f. The values of A_{r2} and A_{r1} are found to be smaller in the case of thermally insulated surface. In thermally insulated boundary, there is a linear relationship of $SL2$ with ω. Thus, the propagation speed, attenuation and specific loss of Rayleigh type waves in the thermally insulated and isothermal surface of thermoelastic saturated porous material depend on Biot's parameter and porosity.

8. Conclusions

We have analyzed the propagation of Rayleigh type waves at the thermally insulated and isothermal surface of thermo-elastic saturated porous medium. The frequency equations for the Rayleigh type waves have been derived separately using boundary conditions. The velocity curves have been depicted and the results are presented graphically. We conclude with the following points

- Two Rayleigh type waves - I, II exist in both the thermally insulated and isothermal boundary of thermo-elastic saturated porous medium. Notice that the propagation speed of first type is just lower than that of transverse waves and the second type is faster than those of body waves.
- We have observed that Rayleigh type-I is faster in the case of isothermal surface, while Rayleigh type-II is faster in the case of thermally insulated boundary.
- The propagation speeds, attenuation and specific loss except $SL1$ for isothermal surface increase with the in-

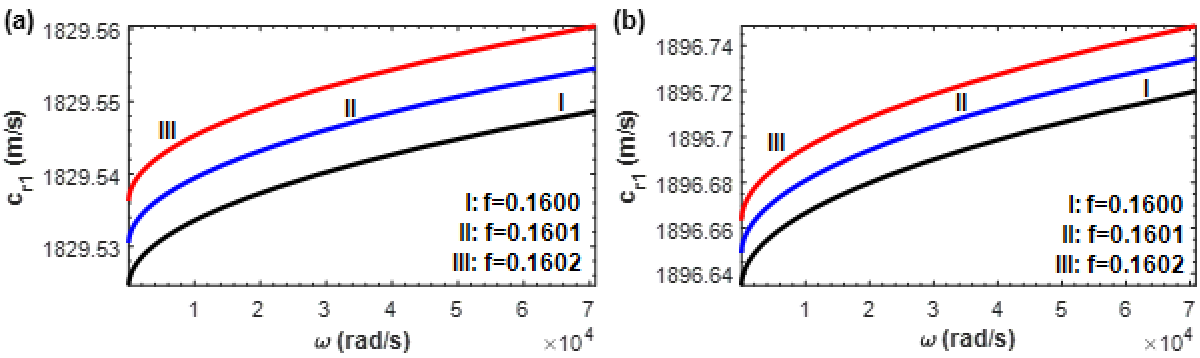

Fig. 7. Effect of porosity on c_{r1} (a) Thermally Insulated, (b) Isothermal.

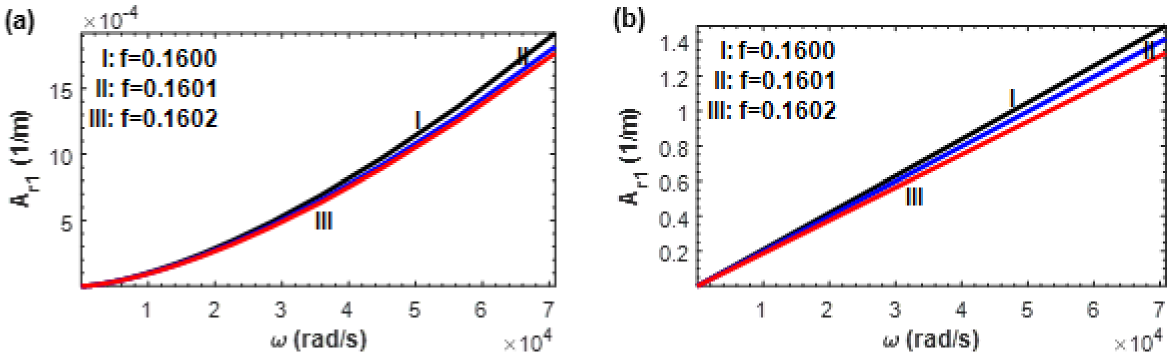

Fig. 8. Effect of porosity on A_{r1} (a) Thermally Insulated, (b) Isothermal.

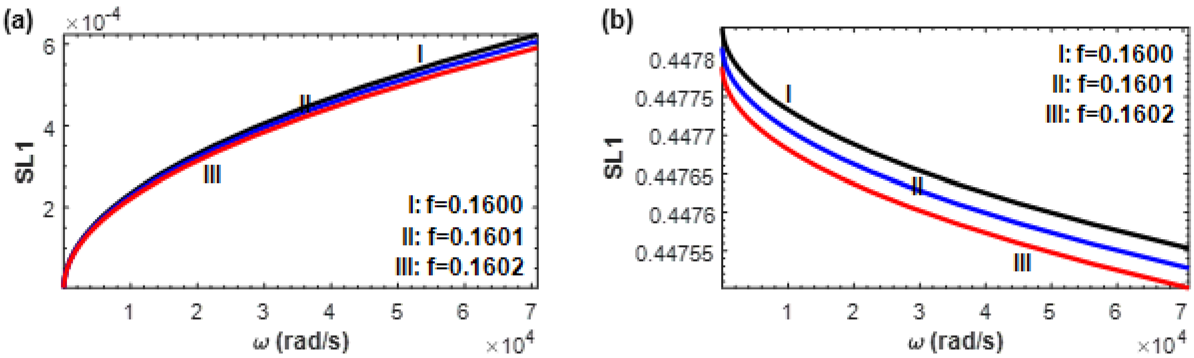

Fig. 9. Effect of porosity on $SL1$ (a) Thermally Insulated, (b) Isothermal.

crease of ω.

- It is observed that the values of c_{r1}, A_{r1} and $SL1$ increase with the increase of α, while c_{r2} decreases.
- The attenuation and specific loss of Rayleigh type II decrease with the increase of α and f for thermally insulated boundary which increase in the case of isothermal boundary.
- The values of c_{r1} and c_{r2} increase, while A_{r1} and $SL1$ decrease with the increase of f for both the boundary conditions.
- With a small change in f and α, there are drastic changed in velocity curves. Thus, the effect of these parameters on the velocity curves of the Rayleigh type waves are very high.

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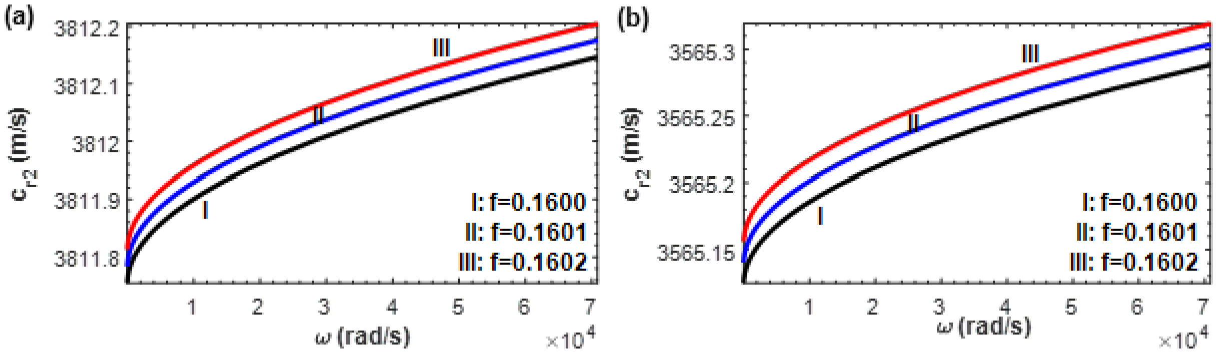

Fig. 10. Effect of porosity on c_{r2} (a) Thermally Insulated, (b) Isothermal.

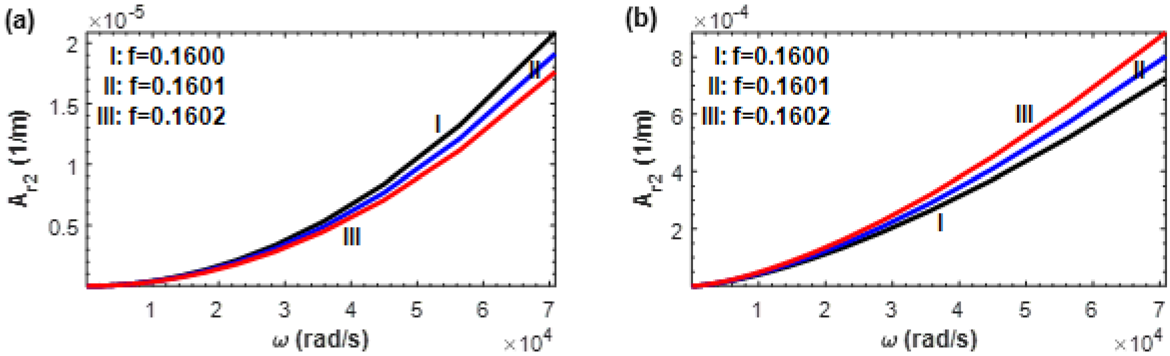

Fig. 11. Effect of porosity on A_{r2} (a) Thermally Insulated, (b) Isothermal.

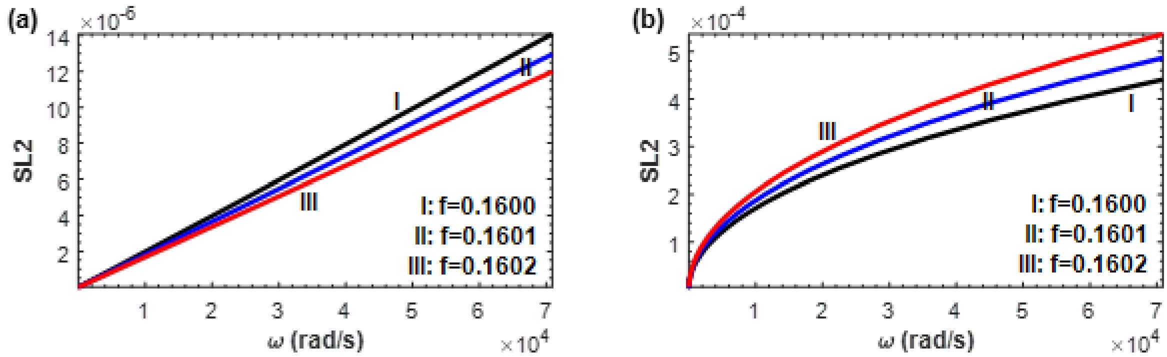

Fig. 12. Effect of porosity on $SL2$ (a) Thermally Insulated, (b) Isothermal.

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