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Analytical solution for Porous Fin with temperature-dependent heat generation via Homotopy perturbation method

Research Article

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Abstract: In the present work, the solution of a non-linear the problem of porous fin with temperature dependent internal heat generation is obtained by means of the Homotopy Perturbation Method. The heat transfer through porous media is simulated using passage velocity from the DarcyâĂŹs model. The results were also compared with Numerical solution in order to verify the accuracy of the proposed method. The analytic solution was found to be in good agreement with the direct numerical solution. Analytical and Numerical results for the temperature distribution are presented through the graphs and the tables in various values of parameter.

MSC: 34D10 • 76S05

Keywords: Homotopy Perturbation Method • Porous fin • Temperature-dependent heat generation

1. Introduction

Fins are frequently used in many heat transfer applications to improve performance. In the other hand, for many years, High rate of heat transfer with reduced size and cost of fins are main targets for a number of engineering applications such as heat exchangers, economizers, super heaters, conventional furnaces, gas turbines, etc. Some engineering applications such as airplane and motorcycle also require lighter fin with higher rate of heat transfer. Increasing the heat transfer mainly depend on heat transfer coefficient (h), surface area available and the temperature difference between surface and surrounding fluid. However, this requirement is often justified by the high cost of the high-thermal-conductivity metals, that cost of high thermal conductivity metals is also high. fin is porous to allow the flow of infiltrate through it. Extensive research has been done in this area and many references are available especially for heat transfer in porous fins. Described below are a few papers relevant to the study described herein. The theoretical study of MHD has been a subject of great interest due to its widespread applications, such as plasma studies, petroleum industries, MHD power generators, cooling of nuclear reactors, the boundary layer control in aerodynamics, and crystal growth. For instance, MHD induced in rockets can improve heat transfer through porous fins, located on rocket surface. On the effect of MHD flow, although there are many studies regarding the free convection regime, there are only a few regarding the mixed convection regime.Chamkha et al [1] studied the effects of localized heating (cooling), suction (injection), buoyancy forces, and magnetic field for the mixed convection flow on a heated vertical plate. Aldoss et a [2] investigated the effect of MHD on heat transfer from a circular cylinder. Nonlinear problems and phenomena play an important role in applied mathematics, physics, engineering and other branches of science specially some heat transfer equations. Except for a limited number of these problems, most of them do not have precise analytical solutions. Therefore, these nonlinear

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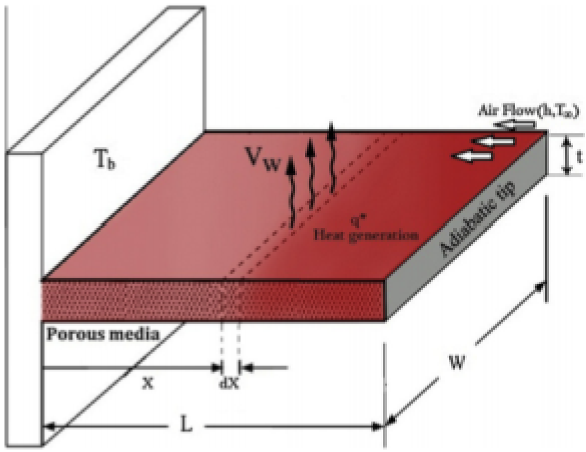

Fig. 1. Schematic diagram for the problem under consideration.

equations should be solved using approximation methods. Perturbation techniques are too strongly dependent upon the so-called "small parameters" [3] . Other many different methods have introduced to solve nonlinear equation such as the Ît'-expansion method [4], Adomian's decomposition method [5] , Homotopy Perturbation Method (HPM) [6–12], Variational Iteration Method (VIM) [13–22] and Homotopy analysis method [23–29].

In this work, we have applied Homotopy perturbation Method to find the approximate solutions of nonlinear differential equations governing on porous fin with temperature dependent internal heat generation. Results demonstrate that HPM is simple and accuracy compared with the BVP as a numerical method. Also, it is found that this method is powerful mathematical tools and that they can be applied to a large class of linear and nonlinear problems arising in different fields of science and engineering.

2. Analysis

As shown in Fig. 1, a rectangular porous fin profile is considered. The dimensions of this fin are length L, width wand thickness t. The cross section area of the fin is constant and the fin has temperature-dependent internal heat generation. Also, the heat loss from the tip of the fin compared with the top and bottom surfaces of the fin is assumed to be negligible. Since the transverse Biot number should be small for the fin to be effective[30] , the temperature variation in the transverse direction are neglected. Thus heat conduction is assumed to occur solely in the longitudinal direction. Energy balance can be written as:

$$q(x)-q(x+\Delta x)+q^{*}.A\Delta x=m c_p(T(x)-T_\infty)+h(p\Delta x)(T(x)-T_\infty) \quad (1)$$

The mass flow rate of the fluid passing through the porous material can be written as:

$$dotm=\rho\,\overline{\vartheta_w}\,\Delta x\,w \quad (2)$$

The value of $\bar{\vartheta}_w$ should be estimated from the consideration of the flow in the porous medium[31]. From the DarcyâĂŹs model we have:

$$\overline{\vartheta_w}=\frac{gk\beta}{\upsilon}\left[T_{(x)}-T_\infty\right] \quad (3)$$

Substitutions of Eqs. (2) and (3) into Eq. (1) yields:

$$\frac{q(x)-q(x+\Delta x)}{\Delta x}+q*.A=\frac{\rho\, c_p g\, k\,\beta\, w}{\Delta x}[T(x)-T_\infty]^2+hp[T(x)-T_\infty] \quad (4)$$

As,$\Delta x\rightarrow 0$ Eq. (4) becomes

$$\frac{dq}{dx}+q*.A=\frac{\rho\, c_p g\, k\,\beta\, w}{\Delta x}[T(x)-T_\infty]^2+hp[T(x)-T_\infty] \quad (5)$$

Also from FourierâĂŸs Law of conduction:

$$q = -k_{eff} A \frac{dT}{dx} \tag{6}$$

Where A is the cross-sectional area of the fin $A = w.$, t and k_{eff} is the effective thermal conductivity of the porous fin that can be obtained from following equation[33]:

$$k_{eff} = \varphi . k_f + (1-\varphi) k_s \tag{7}$$

Where $varphi$ is the porosity of the porous fin. Substitution Eq. (6) into Eq. (5) leads to:

$$fracd^2 T dx^2 - \frac{\rho c_p g k \beta w}{t k_{eff} \upsilon} [T(x) - T_\infty]^2 + \frac{hp}{k_{eff} A} [T(x) - T_\infty] + \frac{q*}{k_{eff}} = 0 \tag{8}$$

It is assumed that heat generation in the fin varies with temperature as Eq. (9) [16]:

$$q* = q^*_\infty [1 + \varepsilon (T - T_\infty)] \tag{9}$$

Where q^*_∞ is the internal heat generation at temperature T_∞ . For simplifying the above equations some dimensionless parameters are introduced as follows:

$$\begin{array}{l} \theta = \frac{(T-T_\infty)}{(T_b - T_\infty)}, \quad X = \frac{x}{L}, \quad M^2 = \frac{hpL^2}{k_0 A}, \quad Sh = \frac{Da\,x\,Ra}{kr}(\frac{L}{t})^2 \\ G = \frac{q^*_\infty}{hp(T_b - T_\infty)}, \quad \varepsilon g = \varepsilon (T_b - T_\infty) \end{array} \tag{10}$$

Where sh is a porous parameter that indicates the effect of the permeability of the porous medium as well as buoyancy effect so higher value of sh indicates higher permeability of the porous medium or higher buoyancy forces. M is a convection parameter that indicates the effect of surface convecting of the fin. Finally, Eq. (8) can be rewritten as:

$$\frac{d^2\theta}{dX^2} - M^2\theta + M^2 G (1 + \varepsilon g\, \theta) - Sh\, \theta^2 = 0 \tag{11}$$

In this research we study finite-length fin with insulated tip. For this case, the fin tip is insulated so that there will not be any heat transfer at the insulated tip and boundary condition will be,

$$\theta(1) = 1 \quad , \quad \theta'(0) = 0 \tag{12}$$

3. Implementation of Homotopy perturbation method (HPM)

In this section, we will apply HPM to nonlinear ordinary differential Eq. (11) with a boundary condition (12). According to HPM, we can construct Homotopy of Eq. (11) as described in the following [6]:

$$H(\theta, P) = (1-P)\left[\frac{d^2\theta}{dX^2}\right] + (1-P)\left[\frac{d^2\theta}{dX^2} - M^2\theta + M^2 G(1+\varepsilon g\,\theta) - Sh\,\theta^2\right] \tag{13}$$

Where $p \in [0, 1]$ is an embedding parameter. For $p = 0$ and $p = 1$ we have

$$\theta(X, 0) = \theta_0(X) \quad , \quad \theta(X, 1) = \theta(X) \tag{14}$$

Note that when p increases from 0 to 1, $\theta\left(X , p\right)$ varies from $\theta_0(X)$ to$\theta(X)$. By substituting

$$\theta(X) = \theta_0(X) + P\,\theta_1(X) + P^3\,\theta_2(X) + P^3\,\theta_3(X) + \cdots = \sum_{i=0}^{n} P^i\,\theta_i(X) \quad g_0 = 0 \tag{15}$$

Into Eq. (8) and rearranging the result based on powers of p-terms, we have

$$P^0 : \frac{\hat{a}^2}{\hat{a}X^2}\theta_0(X) = 0 \qquad \theta_0(0) = 1 \quad \theta_0(1) = 0 \tag{16}$$

$$P^1 : \frac{\hat{a}^2}{\hat{a}X^2}\theta_1(X) + M^2 G\,\varepsilon g\,\theta_0(X) - Sh\,\theta_0(X)^2 - M^2\theta_0(X) + M^2 G = 0 \qquad \theta_1(0) = 0 \quad \theta_1(1) = 0 \tag{17}$$

$$P^2 : \frac{\hat{a}^2}{\hat{a}X^2}\theta_2(X) + M^2 G\,\varepsilon g\,\theta_1(X) - 2Sh\,\theta_0(X)\theta_1(X) - M^2\theta_1(X) = 0 \qquad \theta_2(0) = 0 \quad \theta_2(1) = 0 \tag{18}$$

$$P^1 : \frac{\hat{a}^2}{\hat{a}X^2}\theta_1(X) + M^2 G\,\varepsilon g\,\theta_0(X) - Sh\,\theta_0(X)^2 - M^2\theta_0(X) + M^2 G = 0 \qquad \theta_1(0) = 0 \quad \theta_1(1) = 0 \tag{19}$$

$$P^4 : \frac{\hat{a}^2}{\hat{a}X^2}\theta_4(X) - M^2\theta_3(X) - 2Sh\,\theta_1(X)\theta_2(X) - 2\,Sh\,\theta_0(X)\theta_3(X) + M^2 G\,E\,\theta_3(X) = 0 \qquad \theta_4(0) = 0 \quad \theta_4(1) = 0 \tag{20}$$

$$P^5 : \frac{\hat{a}^2}{\hat{a}X^2}\theta_5(X) - 2Sh\,\theta_1(X)\theta_3(X) + M^2 G\,\varepsilon g\,\theta_4(X) - M^2\theta_4(X) - Sh\,\theta_2(X)^2 - 2Sh\,\theta_0(X)\theta_4(X) = 0 \qquad \theta_5(0) = 0 \quad \theta_5(1) = 0 \tag{21}$$

$$P^6 : \frac{\hat{a}^2}{\hat{a}X^2}\theta_6(X) + M^2 G\,\varepsilon g\,\theta_5(X) - 2Sh\,\theta_0(X)\theta_5(X) - 2Sh\,\theta_1(X)\theta_4(X) - M^2\theta_5(X) - 2\,Sh\,\theta_2(X)\theta_3(X) = 0 \qquad \theta_6(0) = 0 \quad \theta_6(1) = 0 \tag{22}$$

$$P^7 : \frac{\hat{a}^2}{\hat{a}X^2}\theta_7(X) + M^2 G\,\varepsilon g\,\theta_6(X) - 2Sh\,\theta_1(X)\theta_5(X) - 2\,Sh\,\theta_0(X)\theta_6(X) - Sh\theta_3(X)^2 - M^2\theta_6(X) - 2Sh\theta_2(X)\theta_4(X) = 0 \qquad \theta_7(0) = 0 \quad \theta_7(1) = 0 \tag{23}$$

$$P^8 : \frac{\hat{a}^2}{\hat{a}X^2}\theta_8(X) + M^2 A\,\varepsilon g\,\theta_7(X) - 2Sh\,\theta_3(X)\theta_4(X) - 2\,Sh\,\theta_1(X)\theta_6(X) - M^2\theta_7(X) - 2\,Sh\,\theta_0(X)\theta_7(X) - 2Sh\,\theta_2(X)\theta_5(X) = 0 \qquad \theta_8(0) = 0 \quad \theta_8(1) = 0 \tag{24}$$

$$P^9 : \frac{\hat{a}^2}{\hat{a}X^2}\theta_9(X) - 2\,Sh\,\theta_0(X)\theta_8(X) - 2Sh\,\theta_2(X)\theta_6(X) + M^2 G\,\varepsilon g\,\theta_8(X) - Sh\,\theta_4(X)^2 - 2Sh\,\theta_3(X)\theta_5(X) - 2Sh\,\theta_1(x)\,\theta_7(X) - M^2\theta_8(X) = 0 \qquad \theta_9(0) = 0 \quad \theta_9(1) = 0 \tag{25}$$

Solving Eqs. (11)-(13) with boundary conditions, we have for example $(M = 0.2\ ,\ \varepsilon g = 0.7\ ,\ Sh = 0.3\ ,\ G = 0.7)$:

$$\theta_0(x) = 1 \tag{26}$$

$$\theta_1(X) = 0.1462000000X^2 - 0.2924000000X \tag{27}$$

$$\theta_2(X) = 0.007558540000X^4 - 0.03023416000X^3 + 0.06046832000X \tag{28}$$

$$\theta_3(X) = 0.0003700550072X^6 - 0.002220330043X^5 + 0.002137444000X^4 + 0.006252424288X^3 - 0.01842572869X \quad (29)$$

$$\theta_4(X) = 0.00001593959384X^8 - 0.0001275167508x^7 + 0.0002210117096x^6 + 0.0004591642529X^5 - 0.0008840468384X^4 - 0.001905220347X^3 + 0.006395057376X \quad (30)$$

$$\theta_5(X) = 6.60995637010^{-7}X^{10} - 0.000006609956370X^9 + 0.00001764961222X^8 + 0.00001744205512X^7 - 0.00009141044309X^6 - 0.0001399151812X^5 + 0.0003607945965X^4 + 0.0006612489327X^3 - 0.002389298252X \quad (31)$$

$$\theta_6(X) = 2.64132440110^{-8}X^{12} - 3.16958928210^{-7}X^{11} + 0.000001140888524X^{10} + 2.12942122110^{-7}X^9 - 0.000007299879614X^8 - 0.000002496345373X^7 + 0.00003730616128X^6 + 0.00004856066354X^5 - 0.0001492043818X^4 - 0.0002470534392X^3 + 0.0009370552386X \quad (32)$$

$$\theta_7(X) = 1.03153604910^{-9}X^{14} - 1.44415046910^{-8}X^{13} + 6.57936162810^{-8}X^{12} - 3.85651514810^{-8}X^{11} - 4.39238159010^{-7}X^{10} + 2.99410022610^{-7}X^9 + 0.000002979205406X^8 - 4.44787618610^{-8}X^7 - 0.00001542773308X^6 - 0.00001814305982X^5 + 0.00006275414618X^4 + 0.00009689151168X^3 - 0.0003804260314X \quad (33)$$

$$\theta_8(X) = 3.95572825610^{-11}X^{16} - 6.32916521110^{-10}X^{15} + 3.49366424910^{-9}X^{14} - 4.60714301410^{-9}X^{13} - 2.21508094910^{-8}X^{12} + 4.46179232210^{-8}X^{11} + 1.65584512210^{-7}X^{10} - 2.21649509010^{-7}X^9 - 0.000001232032035X^8 + 3.20907792810^{-7}X^7 + 0.000006488778715X^6 + 0.000007115498969X^5 - 0.00002681526976X^4 - 0.00003933605163X^3 + 0.0001585028239X \quad (34)$$

$$\vdots$$

The solution of this equation, when $p \to 1$, will be as follows

$$\theta(X) = \sum_{i=0}^{20} \lim_{P\to 1} P^i\, \theta_i(X)$$

$$= -0.245683629X + 2.150470710^{-28}X^{40} + 0.1462000X^2 - 0.025403688X^3 + 0.0090675505X^4 - 0.001865590X^5 + 0.00052608X^6 - 0.000112439X^7 + 0.00002839999544X^8 - 0.0000062384469X^9 + 0.00000148271601X^{10} + 1. - 3.28075304210^{-7}X^{11} + 7.53858384010^{-8}X^{12} - 1.66850876810^{-8}X^{13} + 3.75639300710^{-9}X^{14} - 8.28861718410^{-10}X^{15} + 1.842302640010^{-10}X^{16} - 4.04780617010^{-11}X^{17} + 8.91436436710^{-12}X^{18} - 1.95168698610^{-12}X^{19} + 4.27262710510^{-13}X^{20} - 9.29696790710^{-14}X^{21} + 2.02577922610^{-14}X^{22} - 4.41624540410^{-15}X^{23} + 9.55103338210^{-16}X^{24} - 2.05206797410^{-16}X^{25} + 4.48360812910^{-17}X^{26} - 9.81154093310^{-18}X^{27} + 2.06173650110^{-18}X^{28} - 4.33216868310^{-19}X^{29} + 9.85690578310^{-20}x^{30} - 2.21821778310^{-20}X^{31} + 4.31719551210^{-21}X^{32} - 8.19817565310^{-22}X^{33} + 2.02897120610^{-22}X^{34} - 5.46275436310^{-23}X^{35} + 1.14683487010^{-23}X^{36} - 1.65669713710^{-24}X^{37} + 1.55421771310^{-25}X^{38} - 8.60188293710^{-27}X^{39} \quad (M = 0.2\,g = 0.7, Sh = 0.3, G = 0.7) \quad (35)$$

4. Results and discussion

In this manuscript, the Homotopy Perturbation Method such as analytical technique is employed to find an analytical solution of the temperature distribution in a porous fin. Fig. 2 show comparison between the numerical solution and HPM solution for θ and θ' when Sh = 0.5 ,M = 0.4,G = 0.7 and different value of εg.
Fig. 3 illustrate the accuracy of HPM solution compare to numerical solution when, $\varepsilon g = 0.5$, $M = 0.3$, $Sh = 0.4$ and different value of G. Fig. 4 show comparison between numerical solution and HPM solution when$\varepsilon g = 0.3$, $G = 0.7$, $Sh = 0.3$ and different value of M.
Fig. 5 show comparison between the Numerical and HPM solution forθ and θ' at different value of Sh . According to Table 1 and Figs. 2, 3, 4, 5 , clearly show that the results by HPM are in excellent agreement with the exact solutions.

Table 1. The results of HAM and Numerical methods for $\theta(X)$ and $\theta'(X)$ for M = 0.3, $\varepsilon g = 0.4$, Sh = 0.5 and, G = 0.1

X	$\theta(X)$			$\theta'(X)$		
	HPM	NUM	Error	HPM	NUM	Error
0.00	1.000000000	1.000000000	0.000000	0.245683630	0.2456835880	4.10E-08
0.05	0.988078200	0.988078331	1.31E-07	0.231249680	0.2312497410	6.02E-08
0.10	0.976869122	0.976869187	6.49E-08	0.217170370	0.2171703780	6.40E-09
0.15	0.966355673	0.966355708	3.52E-08	0.203420460	0.2034204390	1.88E-08
0.20	0.956521988	0.956522043	5.51E-08	0.189975870	0.1899758670	5.10E-09
0.25	0.947353380	0.947353441	6.08E-08	0.176813630	0.1768136290	3.50E-09
0.30	0.938836286	0.938836346	5.93E-08	0.163911740	0.1639117370	6.20E-09
0.35	0.930958220	0.930958287	6.73E-08	0.151249110	0.151249110	1.50E-09
0.40	0.923707725	0.923707801	7.61E-08	0.138805460	0.1388054660	2.00E-09
0.45	0.917074349	0.917074425	7.59E-08	0.126561280	0.1265612770	2.00E-10
0.50	0.911048588	0.911048669	8.02E-08	0.114497690	0.1144976940	1.00E-09
0.55	0.905621874	0.905621961	8.75E-08	0.102596460	0.1025964670	4.60E-09
0.60	0.900786532	0.900786620	8.87E-08	0.090839875	0.0908398777	2.74E-09
0.65	0.896535762	0.896535855	9.26E-08	0.079210696	0.0792106994	3.31E-09
0.70	0.892863620	0.892863716	9.59E-08	0.067692109	0.0676921148	4.93E-09
0.75	0.889764984	0.889765078	9.36E-08	0.056267661	0.0562676618	6.00E-10
0.80	0.887235555	0.887235646	9.17E-08	0.044921200	0.0449211982	2.49E-09
0.85	0.885271828	0.885271929	1.01E-07	0.033636831	0.0336368341	2.77E-09
0.90	0.883871097	0.883871179	8.25E-08	0.022398856	0.0223988387	1.78E-08
0.95	0.883031428	0.883031465	3.68E-08	0.011191720	0.0111916668	6.21E-08
1.00	0.882751668	0.88275177	1.02E-07	0.000000000	0.0000000000	0.0000

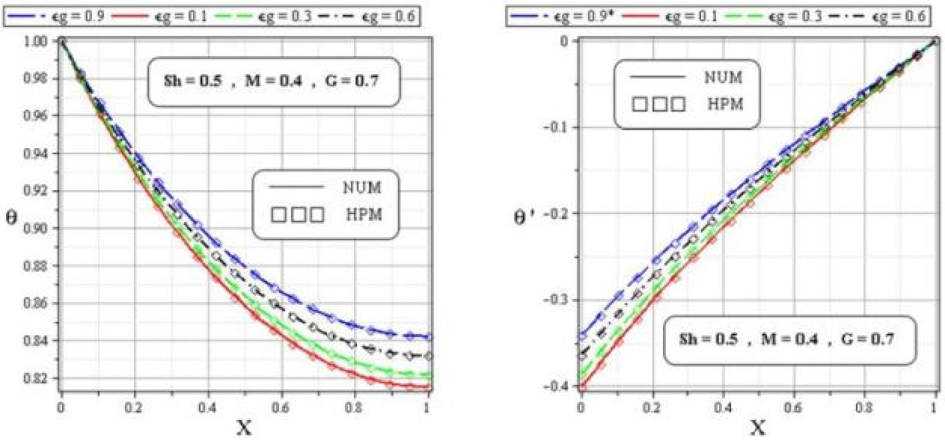

Fig. 2. The comparison between the Numerical and HPM solution for θ and θ' at different value of εg .

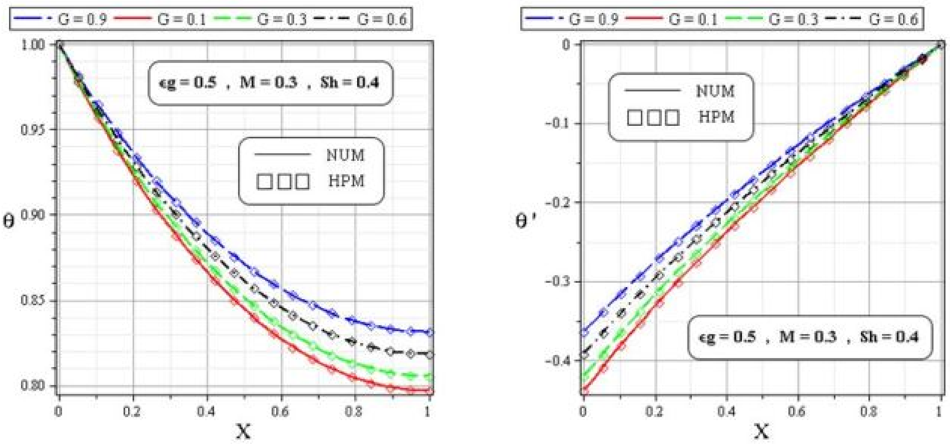

Fig. 3. The comparison between the Numerical and HPM solution for θ and θ' at different value of G .

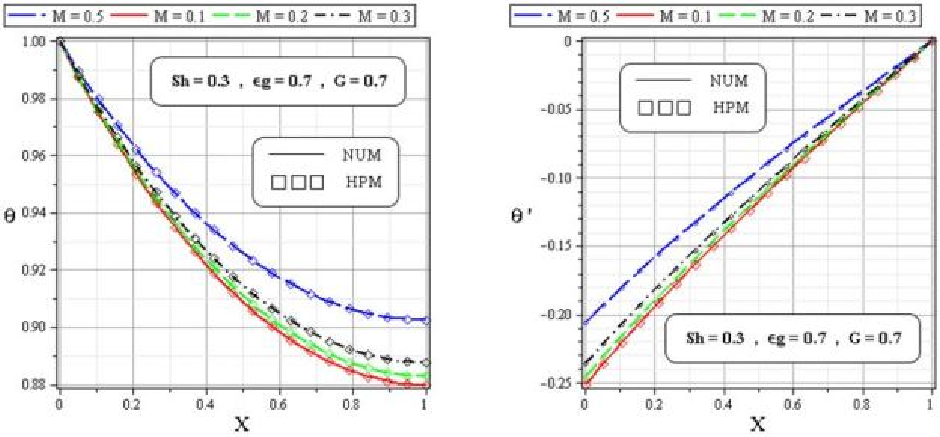

Fig. 4. The comparison between the Numerical and HPM solution for θ and θ' at different value of M .

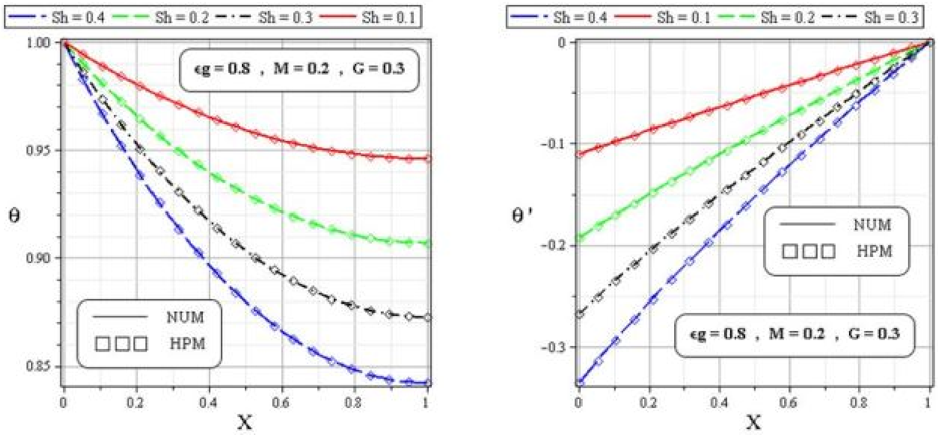

Fig. 5. The comparison between the Numerical and HPM solution for θ and θ' at different value of Sh .

5. Conclusion

In this work, the MHD on a Porous Fin to a Vertical Isothermal Surface with temperature dependent internal heat generation was analyzed using HPM. Also, this problem is solved by a numerical method. The Homotopy Perturbation Method is a powerful approach for solving nonlinear differential equation such as this problem. Also, it can be observed that there is good agreement between the present and numerical results.

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