Int. J. Adv. Appl. Math. and Mech. 3(2) (2015) 50 – 58 (ISSN: 2347-2529)

Journal homepage: www.ijaamm.com

IJAAMM

International Journal of Advances in Applied Mathematics and Mechanics

Application of Laplace Adomian Decomposition Method for the soliton solutions of Boussinesq-Burger equations

Research Article

Hardik S. Patel*, Ramakanta Meher

Applied Mathematics and Humanities Department, S.V.National Institute of Technology, Surat, Gujarat, India

Received 16 September 2015; accepted (in revised version) 07 November 2015

Abstract: Laplace Adomian Decomposition Method (LADM) is a combination of Adomian decomposition method (ADM) and Laplace Transform. It is an approximate analytical method, which can be adapted to solve system of nonlinear partial differential equations. In this paper, Boussinesq-Burger equation has been solved by using Laplace Adomian Decomposition Method which gives an approximate analytical solution that converges faster to the exact solution by using only few iterates of the recursive relation

MSC: 44A10 • 35R70

Keywords: Laplace Adomian Decomposition Method (LADM) • Boussinesq-Burger equation

1. Introduction

The soliton equation is one of the most prominent subjects in the field of nonlinear science. In the past several decades, a great number of efforts have been made to study various nonlinear soliton equations. The traditional methods of solving nonlinear wave equations include inverse scattering theory [1, 2], Backlund transformation [3–5], Darboux transformation [6] and Painlev expansion method [7], etc. With the rapid development of nonlinear science, some new powerful solving methods have been developed, such as homogeneous balance method [8], Jacobi elliptic function method [9], method of bifurcation [10], F-expansion method [11, 12] and some approximate method such as HPM and ADM method [13] and method of auxiliary equation [14], etc. The Laplace Adomian Decomposition Method (LADM) is an approximate analytical method, which can be adapted to solve nonlinear ordinary and partial differential equations. Khuri [15, 16] used this method for the approximate solution of a class of nonlinear ordinary differential equations. Elgazery [17] exploits this method to solve Falkner–Skan equation. This analytical technique basically illustrates how the Laplace transform be used to approximate the solutions of the nonlinear differential equations with the linearization of non-linear terms by using adomian polynomials. Jafari [22] discussed numerical solutions of telegraph and laplace equations on cantor sets using local fractional laplace decomposition method. Kumar [23] used RDT method for the Solutions of the coupled system of Burgers' equations and coupled Klein–Gordon equation. Pirzada [24] discussed Solution of fuzzy heat equations using Adomian Decomposition method.
Consider the nonlinear homogeneous Boussinesq–Burger equation:

$$u_t - \frac{1}{2}v_x + 2uu_x = 0 \tag{1}$$

$$v_t - \frac{1}{2}u_{xxx} + 2(uv)_x = 0, \quad 0 \le x \le 1 \tag{2}$$

Normally it arises in the study of fluid flow and describe the propagation of shallow water waves. Here x and t represents the normalized space and time respectively where $u(x,t)$ is the horizontal velocity field and $v(x,t)$ represent the height of water surface above a horizontal level at the bottom.

* Corresponding author.

E-mail address: hardy.nit@gmail.com (Hardik S. Patel), meher_ramakanta@yahoo.com (Ramakanta Meher)

2. Basic idea of Laplace Adomian Decomposition Method (LADM)

Consider the system of partial differential equations in operator form

$$\begin{aligned} &D_t u + R_1(u,v) + N_1(u,v) = g_1 \\ &D_t v + R_2(u,v) + N_2(u,v) = g_2 \end{aligned} \tag{3}$$

with initial conditions

$$\begin{aligned} &u(x,0) = f_1(x) \\ &v(x,0) = f_2(x) \end{aligned} \tag{4}$$

Where D_t is considered here as the first order partial differential operators, R_1 and R_2 are linear operator, N_1 and N_2 are nonlinear operators, g_1 andg_2 are inhomogenogenous terms. By applying Laplace transform to both side of Eq. (3) and using initial conditions (4), it obtain

$$\begin{aligned} &\mathscr{L}[D_t u] + \mathscr{L}[R_1(u,v)] + \mathscr{L}[N_1(u,v)] = \mathscr{L}[g_1] \\ &\mathscr{L}[D_t v] + \mathscr{L}[R_2(u,v)] + \mathscr{L}[N_2(u,v)] = \mathscr{L}[g_2] \end{aligned} \tag{5}$$

Using the differentiation property of Laplace transform, it gives

$$\begin{aligned} &\mathscr{L}[u] = \frac{f_1(x)}{s} + \frac{1}{s}\mathscr{L}[g_1] - \frac{1}{s}\mathscr{L}[R_1(u,v)] - \frac{1}{s}\mathscr{L}[N_1(u,v)] \\ &\mathscr{L}[v] = \frac{f_2(x)}{s} + \frac{1}{s}\mathscr{L}[g_2] - \frac{1}{s}\mathscr{L}[R_2(u,v)] - \frac{1}{s}\mathscr{L}[N_2(u,v)] \end{aligned} \tag{6}$$

The Laplace Adomian decomposition method decomposes the unknown functions $u(x,t)$ and $v(x,t)$ by an infinite series of components as

$$\begin{aligned} &u(x,t) = \sum_{n=0}^{\infty} u_n(x,t) \\ &v(x,t) = \sum_{n=0}^{\infty} v_n(x,t) \end{aligned} \tag{7}$$

and the nonlinear operators $N_1(u,v)$ and $N_2(u,v)$ can be represented by an infinite series so called Adomian polynomials

$$\begin{aligned} &N_1(u,v) = \sum_{n=0}^{\infty} A_n \\ &N_2(u,v) = \sum_{n=0}^{\infty} B_n \end{aligned} \tag{8}$$

The Adomian polynomials can be generated for all forms of nonlinearity. They are determined by the following relations

$$\begin{aligned} &A_n = \frac{1}{n!}\left[\frac{d^n}{d\lambda^n}\left[N_1 \sum_{i=0}^{\infty}\left(\lambda^i u_i\right)\right]\right]_{\lambda=0} \\ &B_n = \frac{1}{n!}\left[\frac{d^n}{d\lambda^n}\left[N_2 \sum_{i=0}^{\infty}\left(\lambda^i v_i\right)\right]\right]_{\lambda=0} \end{aligned} \tag{9}$$

Substituting Eqs. (7) and (8) into Eq. (6), it gives

$$\begin{aligned} &\mathscr{L}\left[\sum_{n=0}^{\infty} u_n(x,t)\right] = \frac{f_1(x)}{s} + \frac{1}{s}\mathscr{L}[g_1] - \frac{1}{s}\mathscr{L}\left(R_1\left(\left[\sum_{n=0}^{\infty} u_n\right], \left[\sum_{n=0}^{\infty} v_n\right]\right)\right) - \frac{1}{s}\mathscr{L}\left(\sum_{n=0}^{\infty} A_n\right), \\ &\mathscr{L}\left[\sum_{n=0}^{\infty} v_n(x,t)\right] = \frac{f_2(x)}{s} + \frac{1}{s}\mathscr{L}[g_2] - \frac{1}{s}\mathscr{L}\left(R_2\left(\left[\sum_{n=0}^{\infty} u_n\right], \left[\sum_{n=0}^{\infty} v_n\right]\right)\right) - \frac{1}{s}\mathscr{L}\left(\sum_{n=0}^{\infty} B_n\right), \end{aligned} \tag{10}$$

Applying the linearity of Laplace transform in Eq. (10), it obtains the following recursively formula

$$\begin{aligned} &\sum_{n=0}^{\infty} \mathscr{L}[u_n(x,t)] = \frac{f_1(x)}{s} + \frac{1}{s}\mathscr{L}[g_1] - \frac{1}{s}\sum_{n=0}^{\infty} \mathscr{L}(R_1(u_n, v_n)) - \frac{1}{s}\sum_{n=0}^{\infty} \mathscr{L}(A_n), \\ &\sum_{n=0}^{\infty} \mathscr{L}[v_n(x,t)] = \frac{f_2(x)}{s} + \frac{1}{s}\mathscr{L}[g_2] - \frac{1}{s}\sum_{n=0}^{\infty} \mathscr{L}(R_1(u_n, v_n)) - \frac{1}{s}\sum_{n=0}^{\infty} \mathscr{L}(B_n), \end{aligned} \tag{11}$$

Matching both sides of Eq. (11), it yields the following iterative relation

$$\mathscr{L}[u_0] = \frac{f_1(x)}{s} + \frac{1}{s}\mathscr{L}[g_1]$$
$$\mathscr{L}[v_0] = \frac{f_2(x)}{s} + \frac{1}{s}\mathscr{L}[g_2] \tag{12}$$

$$\mathscr{L}[u_1] = -\frac{1}{s}\mathscr{L}[R_1(u_0, v_0)] - \frac{1}{s}\mathscr{L}[A_0]$$
$$\mathscr{L}[v_1] = -\frac{1}{s}\mathscr{L}[R_2(u_0, v_0)] - \frac{1}{s}\mathscr{L}[B_0] \tag{13}$$

for $k \geq 1$, the recursive relations are given by

$$\mathscr{L}[u_{k+1}] = -\frac{1}{s}\mathscr{L}[R_1(u_k, v_k)] - \frac{1}{s}\mathscr{L}[A_k]$$
$$\mathscr{L}[v_{k+1}] = -\frac{1}{s}\mathscr{L}[R_2(u_k, v_k)] - \frac{1}{s}\mathscr{L}[B_k] \tag{14}$$

Finally, by applying the inverse Laplace transform, we can evaluate u_k and v_k.

3. Application of Laplace Adomian Decomposition Method to Boussinesq -Burger equation

Consider the general Boussinesq -Burger equation [19–21] of the form

$$u_t - \frac{1}{2}v_x + 2uu_x = 0 \tag{15}$$
$$v_t - \frac{1}{2}u_{xxx} + 2(uv)_x = 0, \quad 0 \leq x \leq 1 \tag{16}$$

with initial conditions

$$u(x,0) = \frac{ck}{2} + \frac{ck}{2}\tanh\left(\frac{-kx - \ln b}{2}\right) \tag{17}$$
$$v(x,0) = \frac{-k^2}{8}\sec h^2\left(\frac{kx + \ln b}{2}\right) \tag{18}$$

The exact solution of Eqs. (15) and (16) is given by [13]

$$u(x,t) = \frac{ck}{2} + \frac{ck}{2}\tanh\left(\frac{ck^2 t - kx - \ln b}{2}\right) \tag{19}$$
$$v(x,t) = \frac{-k^2}{8}\sec h^2\left(\frac{kx - ck^2 t + \ln b}{2}\right) \tag{20}$$

Applying Laplace transform to both sides of Eqs. (15)-(16) and using initial conditions (17) and (18), we have

$$\mathscr{L}[D_t u] = \frac{1}{2}\mathscr{L}\left[\frac{\partial v}{\partial x}\right] - 2\mathscr{L}\left[u\frac{\partial u}{\partial x}\right] \tag{21}$$
$$\mathscr{L}[D_t v] = \frac{1}{2}\mathscr{L}\left[\frac{\partial^3 u}{\partial x^3}\right] - 2\mathscr{L}\left[\frac{\partial(uv)}{\partial x}\right] \tag{22}$$

Using the differentiation property of Laplace transform, we get

$$s\mathscr{L}[u] = \frac{ck}{2} + \frac{ck}{2}\tanh\left(\frac{-kx - \ln b}{2}\right) + \frac{1}{2}\mathscr{L}\left[\frac{\partial v}{\partial x}\right] - 2\mathscr{L}\left[u\frac{\partial u}{\partial x}\right] \tag{23}$$
$$s\mathscr{L}[v] = \frac{-k^2}{8}\sec h^2\left(\frac{kx + \ln b}{2}\right) + \frac{1}{2}\mathscr{L}\left[\frac{\partial^3 u}{\partial x^3}\right] - 2\mathscr{L}\left[\frac{\partial(uv)}{\partial x}\right] \tag{24}$$

Or

$$\mathscr{L}[u] = \frac{1}{s}\left(\frac{ck}{2} + \frac{ck}{2}\tanh\left(\frac{-kx - \ln b}{2}\right)\right) + \frac{1}{2}\frac{1}{s}\mathscr{L}\left[\frac{\partial v}{\partial x}\right] - 2\frac{1}{s}\mathscr{L}\left[u\frac{\partial u}{\partial x}\right] \tag{25}$$
$$\mathscr{L}[v] = \frac{1}{s}\left(\frac{-k^2}{8}\sec h^2\left(\frac{kx + \ln b}{2}\right)\right) + \frac{1}{2}\frac{1}{s}\mathscr{L}\left[\frac{\partial^3 u}{\partial x^3}\right] - 2\frac{1}{s}\mathscr{L}\left[\frac{\partial(uv)}{\partial x}\right] \tag{26}$$

Here the nonlinear terms uu_x and $(uv)_x$ can be represented by an infinite series so called Adomian polynomials [18] as

$$uu_x = \sum_{n=0}^{\infty} A_n$$
$$(uv)_x = \sum_{n=0}^{\infty} B_n \tag{27}$$

Where $A_n's$ and $B_n's$ are the Adomian polynomials to be determined.
On substituting Eqs. (7) and (27) into Eqs. (25) and (26) respectively. it obtains

$$\mathscr{L}\left[\sum_{n=0}^{\infty} u_n\right] = \frac{1}{s}\left(\frac{ck}{2} + \frac{ck}{2}\tanh\left(\frac{-kx - \ln b}{2}\right)\right) + \frac{1}{2}\frac{1}{s}\mathscr{L}\left[\frac{\partial}{\partial x}\left(\sum_{n=0}^{\infty} v_n\right)\right] - 2\frac{1}{s}\mathscr{L}\left[\sum_{n=0}^{\infty} A_n\right] \tag{28}$$

$$\mathscr{L}\left[\sum_{n=0}^{\infty} v_n\right] = \frac{1}{s}\left(\frac{-k^2}{8}\sec h^2\left(\frac{kx + \ln b}{2}\right)\right) + \frac{1}{2}\frac{1}{s}\mathscr{L}\left[\frac{\partial^3}{\partial x^3}\left(\sum_{n=0}^{\infty} u_n\right)\right] - 2\frac{1}{s}\mathscr{L}\left[\sum_{n=0}^{\infty} B_n\right] \tag{29}$$

Table 1. The absolute error in the solution of Boussinesq-Burger equation using two terms approximation for LADM at various points with $c = \frac{1}{2}$, $k = -1$ and $b = 2$.

(x,t)	$\lvert u_{Exact} - u_{LADM}\rvert$	$\lvert v_{Exact} - v_{LADM}\rvert$	$\lvert u_{Exact} - u_{HPM}\rvert$	$\lvert v_{Exact} - v_{HPM}\rvert$
(0.1,0.1)	4.0376E-05	5.4981E-05	4.0376E-05	5.4981E-05
(0.1,0.2)	1.5774E-04	2.2457E-04	1.5774E-04	2.2457E-04
(0.1,0.3)	3.4620E-04	5.1547E-04	3.4620E-04	5.1547E-04
(0.1,0.4)	5.9952E-04	9.3393E-04	5.9952E-04	9.3393E-04
(0.1,0.5)	9.1119E-04	1.4857E-03	9.1119E-04	1.4857E-03
(0.2,0.1)	3.4534E-05	6.1732E-05	3.4534E-05	6.1732E-05
(0.2,0.2)	1.3393E-04	2.5102E-04	1.3393E-04	2.5102E-04
(0.2,0.3)	2.9167E-04	5.7364E-04	2.9167E-04	5.7364E-04
(0.2,0.4)	5.0096E-04	1.0348E-03	5.0096E-04	1.0348E-03
(0.2,0.5)	7.5475E-04	1.6391E-03	7.5475E-04	1.6391E-03
(0.3,0.1)	2.8060E-05	6.7581E-05	2.8060E-05	6.7581E-05
(0.3,0.2)	1.0766E-04	2.7371E-04	1.0766E-04	2.7371E-04
(0.3,0.3)	2.3176E-04	6.2300E-04	2.3176E-04	6.2300E-04
(0.3,0.4)	3.9310E-04	1.1194E-03	3.9310E-04	1.1194E-03
(0.3,0.5)	5.8423E-04	1.7663E-03	5.8423E-04	1.7663E-03
(0.4,0.1)	2.1055E-05	7.2302E-05	2.1055E-05	7.2302E-05
(0.4,0.2)	7.9348E-05	2.9175E-04	7.9348E-05	2.9175E-04
(0.4,0.3)	1.6743E-04	6.6163E-04	1.6743E-04	6.6163E-04
(0.4,0.4)	2.7772E-04	1.1845E-03	2.7772E-04	1.1845E-03
(0.4,0.5)	4.0252E-04	1.8622E-03	4.0252E-04	1.8622E-03
(0.5,0.1)	1.3643E-05	7.5708E-05	1.3643E-05	7.5708E-05
(0.5,0.2)	4.9492E-05	3.0442E-04	4.9492E-05	3.0442E-04
(0.5,0.3)	9.9847E-05	6.8797E-04	9.9847E-05	6.8797E-04
(0.5,0.4)	1.5693E-04	1.2274E-03	1.5693E-04	1.2274E-03
(0.5,0.5)	2.1295E-04	1.9230E-03	2.1295E-04	1.9230E-03

Table 2. L_2 and L_∞ error norm for Boussinesq-Burger equation use two terms approximation for LADM and HPM at various points x.

x	Laplace Adomain Decomposition Method (LADM)				Homotopy Perturbation Method (HPM)			
	Error in case of two terms approximation for $u(x,t)$		Error in case of two terms approximation for $v(x,t)$		Error in case of two terms approximation for $u(x,t)$		Error in case of two terms approximation for $v(x,t)$	
	L_2	L_∞	L_2	L_∞	L_2	L_∞	L_2	L_∞
0.1	5.169E-4	9.111E-4	1.485E-3	1.485E-3	5.169E-4	9.111E-4	1.485E-3	1.485E-3
0.2	4.300E-4	7.547E-4	9.114E-4	1.639E-3	4.300E-4	7.547E-4	9.114E-4	1.6395-3
0.3	3.352E-4	5.842E-4	9.839E-4	1.766E-3	3.352E-4	5.842E-4	9.839E-4	1.766E-3
0.4	2.340E-4	4.025E-4	1.039E-3	1.862E-3	2.340E-4	4.025E-4	1.039E-3	1.862E-3
0.5	1.285E-4	2.129E-4	1.074E-3	1.923E-3	1.285E-4	2.129E-4	1.074E-3	1.923E-3

Table 3. The absolute error in the solution of Boussinesq – Burger equation using two terms approximation for LADM at various points with $c = \frac{1}{2}$, $k = -1$ and $b = 2$.

(x,t)	$\lvert u_{Exact} - u_{LADM}\rvert$	$\lvert v_{Exact} - v_{LADM}\rvert$	$\lvert u_{Exact} - u_{HPM}\rvert$	$\lvert v_{Exact} - v_{HPM}\rvert$
(0.1,0.1)	9.1140E-07	1.1915E-06	9.1140E-07	1.1915E-06
(0.1,0.2)	7.4086E-06	9.4169E-06	7.4086E-06	9.4169E-06
(0.1,0.3)	2.5391E-05	3.1365E-05	2.5391E-05	3.1365E-05
(0.1,0.4)	6.1082E-05	7.3295E-05	6.1082E-05	7.3295E-05
(0.1,0.5)	1.2100E-04	1.4097E-04	1.2100E-04	1.4097E-04
(0.2,0.1)	1.0246E-06	1.0629E-06	1.0246E-06	1.0629E-06
(0.2,0.2)	8.2995E-06	8.3474E-06	8.2995E-06	8.3474E-06
(0.2,0.3)	2.8349E-05	2.7620E-05	2.8349E-05	2.7620E-05
(0.2,0.4)	6.7973E-05	6.4101E-05	6.7973E-05	6.4101E-05
(0.2,0.5)	1.3421E-04	1.2241E-04	1.3421E-04	1.2241E-04
(0.3,0.1)	1.1227E-06	8.9460E-07	1.1227E-06	8.9460E-07
(0.3,0.2)	9.0676E-06	6.9635E-06	9.0676E-06	6.9635E-06
(0.3,0.3)	3.0880E-05	2.2828E-05	3.0880E-05	2.2828E-05
(0.3,0.4)	7.3821E-05	5.2465E-05	7.3821E-05	5.2465E-05
(0.3,0.5)	1.4533E-04	9.9167E-05	1.4533E-04	9.9167E-05
(0.4,0.1)	1.2022E-06	6.9110E-07	1.2022E-06	6.9110E-07
(0.4,0.2)	9.6832E-06	5.3051E-06	9.6832E-06	5.3051E-06
(0.4,0.3)	3.2885E-05	1.7133E-05	3.2885E-05	1.7133E-05
(0.4,0.4)	7.8397E-05	3.8748E-05	7.8397E-05	3.8748E-05
(0.4,0.5)	1.5391E-04	7.1974E-05	1.5391E-04	7.1974E-05
(0.5,0.1)	1.2600E-06	4.5970E-07	1.2600E-06	4.5970E-07
(0.5,0.2)	1.0121E-05	3.4311E-06	1.0121E-05	3.4311E-06
(0.5,0.3)	3.4283E-05	1.0739E-05	3.4283E-05	1.0739E-05
(0.5,0.4)	8.1517E-05	2.3445E-05	8.1517E-05	2.3445E-05
(0.5,0.5)	1.5962E-04	4.1831E-05	1.5962E-04	4.1831E-05

Table 4. L_2 and L_∞ error norm for Boussinesq – Burger equation use three terms approximation for LADM and HPM at various points x.

x	Laplace Adomain Decomposition Method (LADM)				Homotopy Perturbation Method (HPM)			
	Error in case of three terms approximation for $u(x,t)$		Error in case of three terms approximation for $v(x,t)$		Error in case of three terms approximation for $u(x,t)$		Error in case of three terms approximation for $v(x,t)$	
	L_2	L_∞	L_2	L_∞	L_2	L_∞	L_2	L_∞
0.1	6.176E-5	1.210E-4	7.255E-5	1.409E-4	6.176E-5	1.210E-4	7.255E-5	1.409E-4
0.2	6.856E-5	1.342E-4	6.313E-5	1.224E-4	6.856E-5	1.342E-4	6.313E-5	1.224E-4
0.3	7.430E-5	1.453E-4	5.129E-5	9.916E-5	7.430E-5	1.453E-4	5.129E-5	9.916E-5
0.4	7.875E-5	1.539E-4	3.742E-5	4.183E-5	7.875E-5	1.539E-4	3.742E-5	4.183E-5
0.5	8.173E-5	1.596E-4	2.203E-5	7.197E-5	8.173E-5	1.596E-4	2.203E-5	7.197E-5

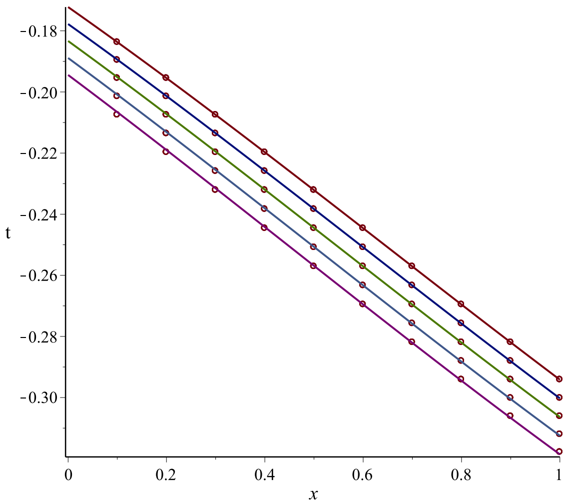

Fig. 1. Comparison of two terms LADM solution (Lines) and exact solution (Circle) of $u(x,t)$ for Boussinesq-Burger equation when $c = \frac{1}{2}$, $k = -1$ and $b = 2$.

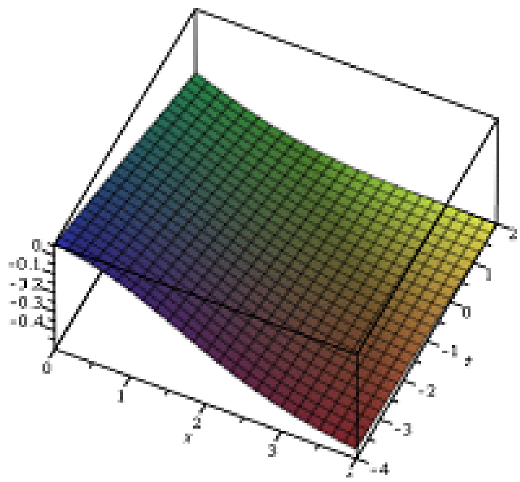

Fig. 2. One soliton approximate solution of $u(x,t)$ for Boussinesq-Burger equation when $c = \frac{1}{2}$, $k = -1$ and $b = 2$.

where the Adomian polynomials are given by

$$
\begin{aligned}
A_0 &= u_0 \frac{\partial u_0}{\partial x},\\
A_1 &= u_1 \frac{\partial u_0}{\partial x} + u_0 \frac{\partial u_1}{\partial x},\\
A_2 &= u_2 \frac{\partial u_0}{\partial x} + u_1 \frac{\partial u_1}{\partial x} + u_0 \frac{\partial u_2}{\partial x},\\
A_3 &= u_3 \frac{\partial u_0}{\partial x} + u_2 \frac{\partial u_1}{\partial x} + u_1 \frac{\partial u_2}{\partial x} + u_0 \frac{\partial u_3}{\partial x},\\
&\vdots
\end{aligned}
\tag{30}
$$

and

$$
\begin{aligned}
B_0 &= \frac{\partial}{\partial x}(u_0 v_0),\\
B_1 &= \frac{\partial}{\partial x}(u_0 v_1 + u_1 v_0),\\
B_2 &= \frac{\partial}{\partial x}(u_0 v_2 + u_1 v_1 + u_2 v_0),\\
B_3 &= \frac{\partial}{\partial x}(u_0 v_3 + u_1 v_2 + u_2 v_1 + u_3 v_0),\\
&\vdots
\end{aligned}
\tag{31}
$$

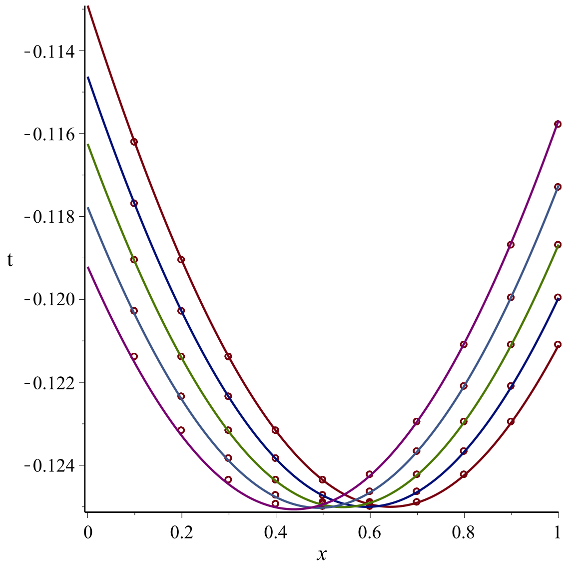

Fig. 3. Comparison of two terms LADM solution (Lines) and exact solution (Circle) of $v(x,t)$ for Boussinesq-Burger equation when $c=\frac{1}{2}$, $k=-1$ and $b=2$.

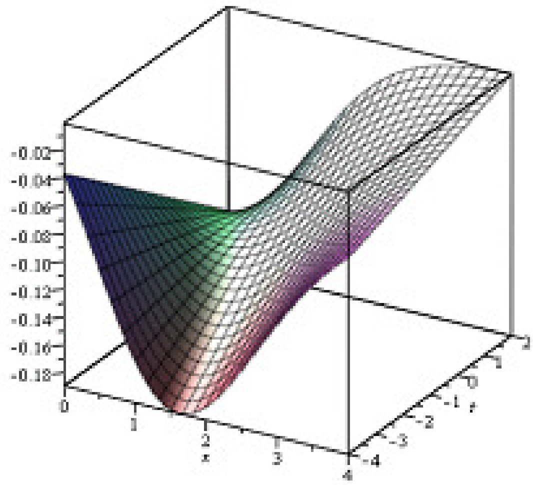

Fig. 4. One soliton approximate solution of $v(x,t)$ for Boussinesq-Burger equation when $c=\frac{1}{2}$, $k=-1$ and $b=2$.

Now from Eq. (28), we obtain the following recursively formula as

$$\begin{aligned}\mathscr{L}[u_0] &= \frac{1}{s}\left(\frac{ck}{2}+\frac{ck}{2}\tanh\left(\frac{-kx-\ln b}{2}\right)\right)\\ \mathscr{L}[v_0] &= \frac{1}{s}\left(\frac{-k^2}{8}\sec h^2\left(\frac{kx+\ln b}{2}\right)\right)\end{aligned} \tag{32}$$

$$\begin{aligned}\mathscr{L}[u_1] &= \frac{1}{2}\frac{1}{s}\mathscr{L}\left[\frac{\partial v_0}{\partial x}\right]-2\frac{1}{s}\mathscr{L}[A_0]\\ \mathscr{L}[v_1] &= \frac{1}{2}\frac{1}{s}\mathscr{L}\left[\frac{\partial^3 u_0}{\partial x^3}\right]-2\frac{1}{s}\mathscr{L}[B_0]\end{aligned} \tag{33}$$

For $k \geq 1$, the recursive relations are given by

$$\begin{aligned}\mathscr{L}[u_{k+1}] &= \frac{1}{2}\frac{1}{s}\mathscr{L}\left[\frac{\partial v_k}{\partial x}\right]-2\frac{1}{s}\mathscr{L}[A_k]\\ \mathscr{L}[v_{k+1}] &= \frac{1}{2}\frac{1}{s}\mathscr{L}\left[\frac{\partial^3 u_k}{\partial x^3}\right]-2\frac{1}{s}\mathscr{L}[B_k]\end{aligned} \tag{34}$$

By applying the inverse Laplace transform in Eqs. (32) and (33), it obtains

$$\begin{aligned}u_0 &= -\frac{1}{4}-\frac{1}{4}\tanh\left(\frac{1}{2}x-\frac{1}{2}\ln(2)\right)\\ v_0 &= -\frac{1}{8}\operatorname{sech}^2\left(\frac{1}{2}x-\frac{1}{2}\ln(2)\right)\end{aligned} \tag{35}$$

$$\begin{aligned}u_1 &= -\frac{1}{16}\frac{t}{\cosh^2\left(\frac{1}{2}x-\frac{1}{2}\ln(2)\right)}\\ v_1 &= \frac{0.0625\,t\sinh\left(\frac{x}{2}-0.3465735903\right)}{\cosh\left(\frac{x}{2}-0.3465735903\right)^3}\end{aligned} \tag{36}$$

$$u_2 = \frac{1.525878906\times 10^{11}\, t^2 \begin{pmatrix} 0.06152343751\cosh^6\left(\frac{x}{2}\right) - 0.06298828124\cosh^4\left(\frac{x}{2}\right) \\ -0.06347656255\cosh^5\left(\frac{x}{2}\right)\sinh\left(\frac{x}{2}\right) \\ +0.03027343753\cosh^3\left(\frac{x}{2}\right)\sinh\left(\frac{x}{2}\right) + 0.008972167959\cosh^2\left(\frac{x}{2}\right) \\ -0.001403808597\cosh\left(\frac{x}{2}\right)\sinh\left(\frac{x}{2}\right) - 0.00009155273440 \end{pmatrix}}{\left(26.51650430\cosh\left(\frac{x}{2}\right) - 8.838834765\sinh\left(\frac{x}{2}\right)\right)^8}$$

$$v_2 = -\frac{3.814697266\times 10^{12}\, t^2 \begin{pmatrix} -0.01398331282\cosh^2\left(\frac{x}{2}\right)\sinh\left(\frac{x}{2}\right) - 0.1444942327\cosh^5\left(\frac{x}{2}\right) \\ +0.09891899063\sinh\left(\frac{x}{2}\right)\cosh^4\left(\frac{x}{2}\right) - 0.002168708245\cosh\left(\frac{x}{2}\right) \\ +0.04971844564\cosh^3\left(\frac{x}{2}\right) + 0.0001402647122\sinh\left(\frac{x}{2}\right) \\ +0.08907888173\cosh^7\left(\frac{x}{2}\right) - 0.08769781373\cosh^6\left(\frac{x}{2}\right)\sinh\left(\frac{x}{2}\right) \end{pmatrix}}{\left(26.51650430\cosh\left(\frac{x}{2}\right) - 8.838834765\sinh\left(\frac{x}{2}\right)\right)^9}$$

$$\vdots \tag{37}$$

Hence, the approximate solutions of Boussinesq -Burger equation is given by

$$\begin{aligned} u(x,t) =& u_0(x,t) + u_1(x,t) + u_2(x,t) + \ldots \\ =& -\frac{1}{4} - \frac{1}{4}\tanh\left(\frac{1}{2}x - \frac{1}{2}\ln(2)\right) - \frac{1}{16}\frac{t}{\cosh^2\left(\frac{1}{2}x - \frac{1}{2}\ln(2)\right)} \\ & - \frac{1.525878906\times 10^{11}\, t^2 \begin{pmatrix} 0.06152343751\cosh^6\left(\frac{x}{2}\right) - 0.06298828124\cosh^4\left(\frac{x}{2}\right) \\ -0.06347656255\cosh^5\left(\frac{x}{2}\right)\sinh\left(\frac{x}{2}\right) \\ +0.03027343753\cosh^3\left(\frac{x}{2}\right)\sinh\left(\frac{x}{2}\right) + 0.008972167959\cosh^2\left(\frac{x}{2}\right) \\ -0.001403808597\cosh\left(\frac{x}{2}\right)\sinh\left(\frac{x}{2}\right) - 0.00009155273440 \end{pmatrix}}{\left(26.51650430\cosh\left(\frac{x}{2}\right) - 8.838834765\sinh\left(\frac{x}{2}\right)\right)^8} + \cdots \end{aligned} \tag{38}$$

$$\begin{aligned} v(x,t) =& v_0(x,t) + v_1(x,t) + v_2(x,t) + \ldots \\ =& -\frac{1}{8}\operatorname{sech}^2\left(\frac{1}{2}x - \frac{1}{2}\ln(2)\right) + \frac{0.06250000000\, t\sinh\left(\frac{x}{2} - 0.3465735903\right)}{\cosh\left(\frac{x}{2} - 0.3465735903\right)^3} \\ & - \frac{3.814697266\times 10^{12}\, t^2 \begin{pmatrix} -0.01398331282\cosh^2\left(\frac{x}{2}\right)\sinh\left(\frac{x}{2}\right) - 0.1444942327\cosh^5\left(\frac{x}{2}\right) \\ +0.09891899063\sinh\left(\frac{x}{2}\right)\cosh^4\left(\frac{x}{2}\right) - 0.002168708245\cosh\left(\frac{x}{2}\right) \\ +0.04971844564\cosh^3\left(\frac{x}{2}\right) + 0.0001402647122\sinh\left(\frac{x}{2}\right) \\ +0.08907888173\cosh^7\left(\frac{x}{2}\right) - 0.08769781373\cosh^6\left(\frac{x}{2}\right)\sinh\left(\frac{x}{2}\right) \end{pmatrix}}{\left(26.51650430\cosh\left(\frac{x}{2}\right) - 8.838834765\sinh\left(\frac{x}{2}\right)\right)^9} + \cdots \end{aligned} \tag{39}$$

Eqs. (38) and (39) represents the approximate solution of Eqs. (15) and (16) having it's numerical values have been represented in Table 1 and Table 3.

4. Conclusion

Here Laplace Adomain Decomposition Method (LADM) has been successfully developed and applied for solution of Boussinesq-Burger equation. By keen observation we concluded that the Laplace Adomain Decomposition Method (LADM) is a powerful and efficient technique that can be used for finding approximate analytical solution of a system of nonlinear partial differential equation.

Acknowledgment

The authors are thankful to the Department of Applied Mathematics and Humanities, Sardar Vallabhbhai National Institute of Technology, Surat, for providing research facilities.

References

[1] M. J. Ablowitz, P. A. Clarkson, Solitons, nonlinear evolution equations and inverse scattering, Vol. 149, Cambridge university press, 1991.
[2] M. Wadati, Stochastic korteweg-de vries equation, Journal of the Physical Society of Japan 52 (8) (1983) 264–2648.
[3] G. L. Lamb Jr, Elements of soliton theory, New York, Wiley-Interscience, 300 p. 29–95: 1 (1980) 29.
[4] M. Wadati, H. Sanuki, K. Konno, Relationships among inverse method, backlund transformation and an infinite number of conservation laws, Progress of theoretical physics 53 (2)(1975) 419–436.
[5] K. Konno, M. Wadati, Simple derivation of backlund transformation from riccati form of inverse method, Progress of Theoretical Physics 53 (6) (1975) 1652–1656.
[6] V. B. Matveev, M. A. Salle, Darboux transformations and solitons, Springer, 1991.
[7] F. Cariello, M. Tabor, Painleve expansions for nonintegrable evolution equations, Physica D: Nonlinear Phenomena 39 (1) (1989) 77–94.
[8] M. Wang, Y. Zhou, Z. Li, Application of a homogeneous balance method to exact solutions of nonlinear equations in mathematical physics, Physics Letters A 216 (1) (1996) 67–75.
[9] S. Liu, Z. Fu, S. Liu, Q. Zhao, Jacobi elliptic function expansion method and periodic wave solutions of nonlinear wave equations, Physics Letters A 289 (1) (2001) 69-âĂŞ74.
[10] J. Li, L. Zhang, Bifurcations of traveling wave solutions in generalized pochhammer–chree equation, Chaos, Solitons & Fractals 14 (4) (2002) 581–593.
[11] Y. Zhou, M. Wang, Y. Wang, Periodic wave solutions to a coupled kdv equations with variable coefficients, Physics Letters A 308 (1) (2003) 31–36.
[12] J.-L. Zhang, M.-L. Wang, Y.-M. Wang, Z.-D. Fang, The improved f-expansion method and its applications, Physics Letters A 350 (1) (2006) 103–109.
[13] S. M. El-Sayed, D. Kaya, An application of the adm to seven-order sawada–kotara equations, Applied mathematics and computation 157 (1) (2004) 93-âĂŞ101.
[14] Sirendaoreji, A new auxiliary equation and exact travelling wave solutions of nonlinear equations, Physics Letters A 356 (2) (2006) 124–130.
[15] S. A. Khuri, A laplace decomposition algorithm applied to a class of nonlinear differential equations, Journal of Applied Mathematics 1 (4) (2001) 141–155.
[16] S. Khuri, A new approach to bratus problem, Applied Mathematics and Computation 147 (1) (2004) 131–136.
[17] N. S. Elgazery, Numerical solution for the falkner-skan equation, Chaos, Solitons & Fractals 35 (4) (2008) 738âĂŞ-746.
[18] G. Adomian, Solving frontier problems of physics: The decomposition method kluwer academic publisher, Boston, MA.
[19] J. Zhang, Y. Wu, X. Li, Quasi-periodic solution of the (2+ 1)-dimensional boussinesq-burgers soliton equation, Physica A: Statistical Mechanics and its Applications 319 (2003) 213–232.
[20] Z. Liang, Z. Li-Feng, L. Chong-Yin, Some new exact solutions of jacobian elliptic function about the generalized boussinesq equation and boussinesq? burgers equation, Chinese Physics B 17 (2) (2008) 403.
[21] A. A. Rady, M. Khalfallah, On soliton solutions for boussinesq-burgers equations, Communications in Nonlinear Science and Numerical Simulation 15 (4) (2010) 886–894.
[22] H. Jafari, & H. K. Jassim, Numerical solutions of telegraph and laplace equations on cantor sets using local fractional laplace decomposition method, International Journal of Advances in Applied Mathematics and Mechanics 2(3) (2015) 144–151.
[23] A. Kumar, & R. Arora, Solutions of the coupled system of Burgers' equations and coupled Klein–Gordon equation by RDT Method. International Journal of Advances in Applied Mathematics and Mechanics 1(2) (2013) 103–115.
[24] U. M. Pirzada, & D. C. Vakaskar, Solution of fuzzy heat equations using Adomian Decomposition method, International Journal of Advances in Applied Mathematics and Mechanics 3(1) (2015) 87–91.