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A note on the paper "Hu et al., Common coupled fixed point theorems for weakly compatible mappings in fuzzy metric spaces, Fixed Point Theory and Applications 2013, 2013:220"

Research Note

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Abstract: The purpose of this remark is to point out that the main result proved in [1] is an immediate consequence of the result proved in [2]. We also point out that the results of [3] also generalizes the result of [1] .

MSC: 47H10 • 54H25

Keywords: Fuzzy metric spaces • Common coupled fixed points • Weakly compatible mappings

Theorem 3.2 of [1] establishes the existence and uniqueness of the common fixed point of the mappings $F : X \times X \to X$ and $g : X \to X$ within the framework of Fuzzy metric space. The reader should consult [1], [2], [3] for terms not specifically defined in this note.

The main result in [1, Theorem 3.2] is as below:

Theorem 1 ([1]). Let $(X, M, *)$ be a FM-Space, $*$ being continuous t-norm of H-type. Let $F : X \times X \to X$ and $g : X \to X$ be two weakly compatible mappings and there exists $\phi \in \Phi$ satisfying the following condition

(1) $M(F(x, y), F(u, v), \phi(t)) \geq M(gx, gu, t) * M(gy, gv, t)$, for all x, y, u, v in X and $t > 0$.

Suppose that $F(X \times X) \subseteq g(X)$ and $F(X \times X)$ or $g(X)$ is complete. Then F and g have a unique common fixed point in X.

Jain et al. [2, Theorem 3.2] proved the following main result for two pair of weak compatible mapping.

Theorem 2 ([2]). Let $(X, M, *)$ be a Fuzzy Metric Space, $*$ being continuous t-norm of H-type. Let $A : X \times X \to X$, $B : X \times X \to X$, $S : X \to X$, $T : X \to X$ be four mappings satisfying the following conditions:

(2) $A(X \times X) \subseteq T(X)$, $B(X \times X) \subseteq S(X)$,

(3) there exists $\phi \in \Phi$ such that $M(A(x, y), B(u, v), \phi(t)) \geq M(Sx, Tu, t) * M(Sy, Tv, t)$, for all x, y, u, v in X and $t > 0$.

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(4) the pairs (A, S) and (B, T) are weakly compatible,

(5) one of the subspaces $A(X \times X)$ or $T(X)$ and one of $B(X \times X)$ or $S(X)$ are complete.

Then there exists a unique point a in X such that

$$A(a, a) = S(a) = a = T(a) = B(a, a).$$

Remark 1. On taking $A = B = F$ and $S = T = g$ in Theorem 2 we can obtain Theorem 1.

In [3], Jain et al. introduced the notion of CLRg property to obtain the existence and uniqueness of common fixed point for the mappings $F : X \times X \to X$ and $g : X \to X$, in Fuzzy metric space.

Theorem 3 ([3]). Let $(X, M, *)$ be a FM-Space, $*$ being continuous t-norm of H-type. Let $F : X \times X \to X$ and $g : X \to X$ be two mappings and there exists $\phi \in \Phi$ satisfying condition (1) with the following conditions:

(6) the pair (F, g) is weakly compatible,

(7) the pair (F, g) satisfy CLRg property.

Then F and g have a coupled coincidence point in X. Moreover, there exists a unique point x in X such that $x = F(x, x) = g(x)$.

Remark 2. Comparing Theorem 3 with Theorem 1, we can see that Theorem 3 is a genuine generalization of Theorem 1.

Theorem 3 does not require the completeness of the space X or the range subspaces of the mappings F or g.

The condition $F(X \times X) \subseteq g(X)$ has been relaxed. Some interesting results can be seen in ([4]-[10]).

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