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Differential subordination results for Abbas-Starlike function in the upper half-plane

Research Article

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Abstract: In the present paper, we define new class of analytic functions in the upper half-plane $D = \{z \in \mathbb{C} : Re(z) > 0\}$. Also, by investigating appropriate classes of admissible functions, we obtain differential subordination results for functions belongs to this new class.

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1. Introduction

Let $\mathcal{H} = \mathcal{H}(D)$ stands for the class of functions $f : D \longrightarrow \mathbb{C}$ which are analytic in the upper half-plane $D = \{z \in \mathbb{C} : Re(z) > 0\}$ that have the hydrodynamic normalization (see [1, 2, 10])

$$\lim_{D \ni z \to \infty} (f(z) - z) = 0.$$

Stankiewicz [10] introduced two classes are the class $S^*(D)$ of starlike functions and the class $\mathcal{K}(D)$ of convex functions as follows:

$$S^*(D) = \left\{ f \in \mathcal{H}(D) : Re\left\{ \frac{f'(z)}{f(z)} \right\} < 0, \, z \in D \right\}$$

and

$$\mathcal{K}(D) = \left\{ f \in \mathcal{H}(D) : Re\left\{ \frac{f''(z)}{f'(z)} \right\} > 0, \, z \in D \right\}.$$

Next, we define new class for $f \in \mathcal{H}(D)$ as follows:

Definition 1.1.
A function $f \in \mathcal{H}(D)$ is said to be Abbas-starlike function, if it satisfies the geometric condition:

$$Re\left\{ \frac{(f'(z))^{\gamma}}{f(z)} \right\} < 0, \, \gamma \geq 1, \, z \in D.$$

We denote by A_{γ} the family of all Abbas-starlike functions in D. It is observed that for $\gamma = 1$, we have the class of starlike functions in D.

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With a view to recalling the principal of subordination between analytic functions in the upper half-plane, let f and g be members of the class $\mathcal{H}(D)$. The function f is said to be subordinate to g, or g is said to be superordinate to f, if there exists a function $\varphi \in \mathcal{H}(D)$ with $\varphi(D) \subset D$ such that $f(z) = g\left(\varphi(z)\right)$ $(z \in D)$. This subordination is denoted by $f \prec g$ or $f(z) \prec g(z)$ $(z \in D)$. Furthermore, if the function g is univalent in D, then we have (see [7])

$$f(z) \prec g(z) \iff f(D) \subset g(D).$$

Let Ω be any set in the complex plane $\mathbb{C}$. Also let $p \in \mathcal{H}(D)$ and $\psi : \mathbb{C}^3 \times D \longrightarrow \mathbb{C}$. Răducanu and Pascu [7] have extended the theory of differential subordinations in the unit disk U to the upper half-plane D. They determined properties of functions p that satisfy the following differential subordination:

$$\left\{\psi\left(p(z), p'(z), p''(z); z\right) : z \in D\right\} \subset \Omega.$$

In order to prove our next investigation, we shall need the following known results.

Definition 1.2 (Ră ducanu and Pascu [7]).
Let $\psi : \mathbb{C}^3 \times D \longrightarrow \mathbb{C}$ and the function h be univalent in D. If p is analytic in D and satisfies the following second-order differential subordination:

$$\psi\left(p(z), p'(z), p''(z); z\right) \prec h(z) \quad (z \in D), \tag{1}$$

then p is called a solution of the differential subordination (1). A univalent function q is called a dominant of the solutions of the differential subordination or more simply a dominant if $p(z) \prec q(z)$ for all p satisfying (1). A dominant $\check{q}$ that satisfies $\check{q}(z) \prec q(z)$ for all dominants q of (1) is said to be the best dominant.

Definition 1.3 (Miller and Mocanu [6]).
Let $Q(D)$ denote the set of functions $q \in \mathcal{H}(D)$ that are analytic and injective on $\bar{D}\backslash E(q)$, where

$$E(q) = \left\{\xi \in \partial D : \lim_{z \to \xi} q(z) = \infty\right\},$$

and are such that $q'(\xi) \neq 0$ for $\xi \in \partial D \backslash E(q)$.

Definition 1.4 (Răducanu and Pascu [7]).
Let Ω be a set in $\mathbb{C}$, $q \in Q(D)$. The class of admissible functions $\Psi_D\left[\Omega, q\right]$ consists of those functions $\psi : \mathbb{C}^3 \times D \longrightarrow \mathbb{C}$ that satisfy the following admissibility condition: $\psi(r, s, t; z) \notin \Omega$, whenever

$$r = q(\xi), \quad s = kq'(\xi) \quad and \quad Re\left\{\frac{t}{q'(\xi)}\right\} \geq k^2 Re\left\{\frac{q''(\xi)}{q'(\xi)}\right\},$$

where $z \in D$, $\xi \in \partial D \backslash E(q)$ and $k \geq 0$.

If $\psi : \mathbb{C}^2 \times D \longrightarrow \mathbb{C}$, then the admissibility condition reduces to the following form: $\psi(q(\xi), kq'(\xi); z) \notin \Omega$, where $z \in D$, $\xi \in \partial D \backslash E(q)$ and $k \geq 0$.

Lemma 1.1 (Răducanu and Pascu [7]).
Let $\psi \in \Psi_D\left[\Omega, q\right]$ *and* $p \in \mathcal{H}(D)$. *If*

$$\psi\left(p(z), p'(z), p''(z); z\right) \in \Omega \quad (z \in D),$$

then $p(z) \prec q(z)$.

In recent years, many authors obtained various interesting results associated with differential subordination and superordination in the unit disk, for example (see [3–5, 8, 9, 13–21]). Very recently, Tang et al. [11] (see also [12]) have investigated differential subordination for analytic functions in the upper half-plane. In this work, we consider certain suitable classes of admissible functions and derive some differential subordination properties of Abbas-starlike functions in the upper half-plane.

2. Main Results

Definition 2.1.
Let Ω be a set in $\mathbb{C}$ and $q \in Q(D) \cap \mathcal{H}(D)$. The class of admissible functions $\Phi_D[\Omega, q]$ consists of those functions $\phi : \mathbb{C}^3 \times D \longrightarrow \mathbb{C}$ that satisfy the following admissibility condition: $\phi(u, v, w; z) \notin \Omega$, whenever

$$u = q(\xi), \quad v = \frac{kq'(\xi)}{q(\xi)}, \; q(\xi) \neq 0 \quad and \quad Re\left\{\frac{u(w+v^2)}{q'(\xi)}\right\} \geq k^2 Re\left\{\frac{q''(\xi)}{q'(\xi)}\right\},$$

where $z \in D$, $\xi \in \partial D \backslash E(q)$ and $k \geq 0$.

Theorem 2.1.
Let $\phi \in \Phi_D[\Omega, q]$. If $f \in \mathcal{H}(D)$ satisfies

$$\left\{\phi\left(\frac{(f'(z))^\gamma}{f(z)}, \gamma\frac{f''(z)}{f'(z)} - \frac{f'(z)}{f(z)}, \gamma\frac{f'''(z)}{f'(z)} - \gamma\left(\frac{f''(z)}{f'(z)}\right)^2 - \frac{f''(z)}{f(z)} + \left(\frac{f'(z)}{f(z)}\right)^2; z\right) : z \in D\right\} \subset \Omega, \tag{2}$$

then

$$\frac{(f'(z))^\gamma}{f(z)} \prec q(z).$$

Proof. Let the analytic function p in D be defined by

$$p(z) = \frac{(f'(z))^\gamma}{f(z)}. \tag{3}$$

After some calculation using (3), we have

$$\frac{p'(z)}{p(z)} = \gamma\frac{f''(z)}{f'(z)} - \frac{f'(z)}{f(z)}. \tag{4}$$

Further computations show that

$$\frac{p''(z)}{p(z)} - \left(\frac{p'(z)}{p(z)}\right)^2 = \left[\gamma\frac{f''(z)}{f'(z)} - \frac{f'(z)}{f(z)}\right]' = \gamma\frac{f'''(z)}{f'(z)} - \gamma\left(\frac{f''(z)}{f'(z)}\right)^2 - \frac{f''(z)}{f(z)} + \left(\frac{f'(z)}{f(z)}\right)^2. \tag{5}$$

We define the transformation from $\mathbb{C}^3$ to $\mathbb{C}$ by

$$u = r, \quad v = \frac{s}{r}, \quad w = \frac{rt - s^2}{r^2}. \tag{6}$$

Assume that

$$\psi(r, s, t; z) = \phi(u, v, w; z) = \phi\left(r, \frac{s}{r}, \frac{rt - s^2}{r^2}; z\right). \tag{7}$$

In view of (3), (4) and (5), we deduce from (7) that

$$\psi\left(p(z), p'(z), p''(z); z\right)$$
$$= \phi\left(\frac{(f'(z))^\gamma}{f(z)}, \gamma\frac{f''(z)}{f'(z)} - \frac{f'(z)}{f(z)}, \gamma\frac{f'''(z)}{f'(z)} - \gamma\left(\frac{f''(z)}{f'(z)}\right)^2 - \frac{f''(z)}{f(z)} + \left(\frac{f'(z)}{f(z)}\right)^2; z\right). \tag{8}$$

Hence (2) becomes

$$\psi\left(p(z), p'(z), p''(z); z\right) \in \Omega. \tag{9}$$

To complete the proof, we next show that the admissibility condition for $\phi \in \Phi_D[\Omega, q]$ is equivalent to the admissibility condition for ψ as given in Definition 1.2.
A computation using (6) yields

$$t = u(w + v^2).$$

Thus $\psi \in \Psi_D[\Omega, q]$ and by Lemma 1.1, we obtain $p(z) \prec q(z)$ or equivalently

$$\frac{(f'(z))^\gamma}{f(z)} \prec q(z).$$

This completes the proof of Theorem 2.1. □

We consider the special situation when $\Omega \neq \mathbb{C}$ is a simply connected domain and $\Omega = h(D)$ for some conformal mapping h of the half-plane D onto Ω. In this case the class $\Phi_D[h(D), q]$ is written as $\Phi_D[h, q]$. The following result is an immediate consequence of Theorem 2.1.

Theorem 2.2.
Let $\phi \in \Phi_D[h, q]$. If $f \in \mathcal{H}(D)$ satisfies

$$\phi\left(\frac{(f'(z))^\gamma}{f(z)}, \gamma\frac{f''(z)}{f'(z)} - \frac{f'(z)}{f(z)}, \gamma\frac{f'''(z)}{f'(z)} - \gamma\left(\frac{f''(z)}{f'(z)}\right)^2 - \frac{f''(z)}{f(z)} + \left(\frac{f'(z)}{f(z)}\right)^2; z\right) \prec h(z), \tag{10}$$

then

$$\frac{(f'(z))^\gamma}{f(z)} \prec q(z).$$

The next result is an extension of Theorem 2.1 to the case when the behavior of q on ∂D is not known.

Theorem 2.3.
Let h and q be univalent in D with $q \in Q(D)$ and set $q_\rho(z) = q(\rho z)$ and $h_\rho(z) = h(\rho z)$. Suppose that $\phi : \mathbb{C}^3 \times D \longrightarrow \mathbb{C}$ satisfy one of the following conditions:
(1) $\phi \in \Phi_D[h, q_\rho]$ for some $\rho \in (0,1)$,
(2) there exists $\rho_0 \in (0,1)$ such that $\phi \in \Phi_D[h_\rho, q_\rho]$ for all $\rho \in (\rho_0, 1)$.
If $f \in \mathcal{H}(D)$ satisfies (10), then

$$\frac{(f'(z))^\gamma}{f(z)} \prec q(z).$$

Proof. Case (1) : By applying Theorem 2.1, we have $\frac{(f'(z))^\gamma}{f(z)} \prec q_\rho(z)$, since $q_\rho(z) \prec q(z)$, we get

$$\frac{(f'(z))^\gamma}{f(z)} \prec q(z).$$

Case (2) : Let $p(z) = \frac{(f'(z))^\gamma}{f(z)}$ and $p_\rho(z) = p(\rho z)$. Then

$$\phi\left(p_\rho(z), p'_\rho(z), p''_\rho(z); \rho z\right) = \phi\left(p(\rho z), p'(\rho z), p''(\rho z); \rho z\right) \in h_\rho(D).$$

By making use of Theorem 2.1 and the comment associated with

$$\phi\left(p(z), p'(z), p''(z); w(z)\right) \in \Omega,$$

where w is any function mapping D into D, with $w(z) = \rho z$, we have $p_\rho(z) \prec q_\rho(z)$ for $\rho \in (\rho_0, 1)$. By letting $\rho \to 1^-$, we obtain $p(z) \prec q(z)$. Thus

$$\frac{(f'(z))^\gamma}{f(z)} \prec q(z).$$

□

The next result yields the best dominant of the differential subordination (10):

Theorem 2.4.
Let h be univalent in D and $\phi : \mathbb{C}^3 \times D \longrightarrow \mathbb{C}$. Suppose that the differential equation:

$$\phi\left(q(z), \frac{q'(z)}{q(z)}, \frac{q''(z)}{q(z)} - \left(\frac{q'(z)}{q(z)}\right)^2; z\right) = h(z) \tag{11}$$

has a solution q and satisfies one of the following conditions:
(1) $q \in Q(D)$ and $\phi \in \Phi_D[h, q]$,
(2) q is univalent in D and $\phi \in \Phi_D[h, q_\rho]$ for some $\rho \in (0,1)$,
(3) q is univalent in D and there exists $\rho_0 \in (0,1)$ such that $\phi \in \Phi_D[h_\rho, q_\rho]$ for all $\rho \in (\rho_0, 1)$.
If $f \in \mathcal{H}(D)$ satisfies (10), then

$$\frac{(f'(z))^\gamma}{f(z)} \prec q(z).$$

and q is the best dominant.

Proof. By applying Theorem 2.2 and Theorem 2.3, we find that q is a dominant of (10). Since q satisfies (11), it is also a solution of (10) and hence q will be dominated by all dominants. Therefore q is the best dominant of (10). □

In the particular case $q(z) = z$ and in view of Definition 2.1, the class of admissible functions $\Phi_D[\Omega, q]$ denoted simply by $\Phi_D[\Omega, z]$ is described below:

Definition 2.2.
Let Ω be a set in $\mathbb{C}$. The class of admissible functions $\Phi_D[\Omega, z]$ consists of those functions $\phi : \mathbb{C}^3 \times D \longrightarrow \mathbb{C}$ such that

$$\phi\left(\mu, \frac{k}{\mu}, \frac{\mathcal{N}}{\mu} - \left(\frac{k}{\mu}\right)^2; z\right) \notin \Omega, \tag{12}$$

whenever $z \in D$, $\mu \in \mathbb{R} \setminus \{0\}$, $Re\{\mathcal{N}\} \geq 0$ and $k > 0$.

Corollary 2.1.
Let $\phi \in \Phi_D[\Omega, z]$. If $f \in \mathcal{H}(D)$ satisfies

$$\phi\left(\frac{(f'(z))^\gamma}{f(z)}, \gamma\frac{f''(z)}{f'(z)} - \frac{f'(z)}{f(z)}, \gamma\frac{f'''(z)}{f'(z)} - \gamma\left(\frac{f''(z)}{f'(z)}\right)^2 - \frac{f''(z)}{f(z)} + \left(\frac{f'(z)}{f(z)}\right)^2; z\right) \in \Omega,$$

then

$$\frac{(f'(z))^\gamma}{f(z)} \prec z.$$

When $\Omega = q(D) = \{w : Re\{w\} > 0\}$, the class $\Phi_D[\Omega, z]$ is denoted for brevity by $\Phi_D[z]$, then Corollary 2.1 can now be rewritten in the following form:

Corollary 2.2.
Let $\phi \in \Phi_D[z]$. If $f \in \mathcal{H}(D)$ satisfies

$$Re\left\{\phi\left(\frac{(f'(z))^\gamma}{f(z)}, \gamma\frac{f''(z)}{f'(z)} - \frac{f'(z)}{f(z)}, \gamma\frac{f'''(z)}{f'(z)} - \gamma\left(\frac{f''(z)}{f'(z)}\right)^2 - \frac{f''(z)}{f(z)} + \left(\frac{f'(z)}{f(z)}\right)^2; z\right)\right\} > 0,$$

then

$$Re\left\{\frac{(f'(z))^\gamma}{f(z)}\right\} > 0.$$

Example 2.1.
Let the functions $G, T : D \longrightarrow \mathbb{C}$ be analytic in D and satisfy $Re\{G(z)\} \leq 0$ and $Re\{T(z)\} \leq 0$. Then the functions $\phi_1(u, v, w; z) = \frac{v}{u} + G(z)$ and $\phi_2(u, v, w; z) = v^2 - w + T(z)$ satisfy the admissibility condition (12) and hence from Corollary 2.1, we conclude that

$$Re\left\{\frac{\gamma f(z) f''(z) - (f'(z))^2}{(f'(z))^{\gamma+1}}\right\} > 0 \quad \Longrightarrow \quad Re\left\{\frac{(f'(z))^\gamma}{f(z)}\right\} > 0$$

and

$$Re\left\{\gamma(\gamma+1)\left(\frac{f''(z)}{f'(z)}\right)^2 + (1-2\gamma)\frac{f''(z)}{f(z)} - \gamma\frac{f'''(z)}{f'(z)}\right\} > 0 \quad \Longrightarrow \quad Re\left\{\frac{(f'(z))^\gamma}{f(z)}\right\} > 0.$$

Definition 2.3.
Let Ω be a set in $\mathbb{C}$ and $q \in Q(D) \cap \mathcal{H}(D)$. The class of admissible functions $\Phi_{D,1}[\Omega, q]$ consists of those functions $\phi : \mathbb{C}^2 \times D \longrightarrow \mathbb{C}$ that satisfy the following admissibility condition:

$$\phi(q(\xi), kq'(\xi); z) \notin \Omega,$$

where $z \in D$, $\xi \in \partial D \setminus E(q)$ and $k \geq 0$.

Theorem 2.5.
Let $\phi \in \Phi_{D,1}\left[\Omega, q\right]$. If $f \in \mathcal{H}(D)$ satisfies

$$\left\{\phi\left(\frac{\left(f'(z)\right)^{\gamma}}{f(z)}, \gamma\left(f'(z)\right)^{\gamma-1}\frac{f''(z)}{f(z)} - \frac{\left(f'(z)\right)^{\gamma+1}}{\left(f(z)\right)^{2}}; z\right) : z \in D\right\} \subset \Omega, \tag{13}$$

then

$$\frac{\left(f'(z)\right)^{\gamma}}{f(z)} \prec q(z).$$

Proof. Let the analytic function p in D be defined by

$$p(z) = \frac{\left(f'(z)\right)^{\gamma}}{f(z)}. \tag{14}$$

Differentiating (14), with respect to z, we have

$$p'(z) = \gamma\left(f'(z)\right)^{\gamma-1}\frac{f''(z)}{f(z)} - \frac{\left(f'(z)\right)^{\gamma+1}}{\left(f(z)\right)^{2}}. \tag{15}$$

We next define the transformation from $\mathbb{C}^2$ to $\mathbb{C}$ by

$$u = r, \quad v = s.$$

Setting

$$\psi(r, s; z) = \phi(u, v; z) = \phi(r, s; z). \tag{16}$$

If we use the equations (14) and (15), it follows from (16) that

$$\psi\left(p(z), p'(z); z\right) = \phi\left(\frac{\left(f'(z)\right)^{\gamma}}{f(z)}, \gamma\left(f'(z)\right)^{\gamma-1}\frac{f''(z)}{f(z)} - \frac{\left(f'(z)\right)^{\gamma+1}}{\left(f(z)\right)^{2}}; z\right). \tag{17}$$

By using (13) and (17), we find that

$$\psi\left(p(z), p'(z); z\right) \in \Omega.$$

Thus, from (16), we see that the admissibility condition for $\phi \in \Phi_{D,1}\left[\Omega, q\right]$ in Definition 2.3 is equivalent to the admissibility condition for ψ as given in Definition 1.2. Therefore $\psi \in \Psi_D\left[\Omega, q\right]$ and by Lemma 1.1, we obtain $p(z) \prec q(z)$ or equivalently

$$\frac{\left(f'(z)\right)^{\gamma}}{f(z)} \prec q(z).$$

□

In the case $\Omega \neq \mathbb{C}$ is a simply connected domain with $\Omega = h(D)$ for some conformal mapping h of D onto Ω, the class $\Phi_{D,1}\left[h(D), q\right]$ is written as $\Phi_{D,1}\left[h, q\right]$. The following result is an immediate consequence of Theorem 2.5.

Theorem 2.6.
Let $\phi \in \Phi_{D,1}\left[h, q\right]$. If $f \in \mathcal{H}(D)$ satisfies

$$\phi\left(\frac{\left(f'(z)\right)^{\gamma}}{f(z)}, \gamma\left(f'(z)\right)^{\gamma-1}\frac{f''(z)}{f(z)} - \frac{\left(f'(z)\right)^{\gamma+1}}{\left(f(z)\right)^{2}}; z\right) \prec h(z), \tag{18}$$

then

$$\frac{\left(f'(z)\right)^{\gamma}}{f(z)} \prec q(z).$$

We extend Theorem 2.6 to the case when the behavior of q on ∂D is not known.

Theorem 2.7.
Let q be univalent function in D with $q \in Q(D)$. Let $\phi \in \Phi_{D,1}\left[h, q_\rho\right]$ for some $\rho \in (0,1)$ such that $q_\rho(z) = q(\rho z)$. If $f \in \mathcal{H}(D)$ satisfies (18), then

$$\frac{\left(f'(z)\right)^\gamma}{f(z)} \prec q(z).$$

As special case, when $q(z) = z$, we obtain the following Corollary.

Corollary 2.3.
Let Ω be a set in $\mathbb{C}$ and let $\phi : \mathbb{C}^2 \times D \longrightarrow \mathbb{C}$ satisfy $\phi\left(\mu, k; z\right) \notin \Omega$, whenever $z \in D$, $\mu \in \mathbb{R}$ and $k \geq 0$. If $f \in \mathcal{H}(D)$ satisfies

$$\phi\left(\frac{\left(f'(z)\right)^\gamma}{f(z)}, \gamma\left(f'(z)\right)^{\gamma-1}\frac{f''(z)}{f(z)} - \frac{\left(f'(z)\right)^{\gamma+1}}{\left(f(z)\right)^2}; z\right) \in \Omega,$$

then

$$Re\left\{\frac{\left(f'(z)\right)^\gamma}{f(z)}\right\} > 0.$$

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